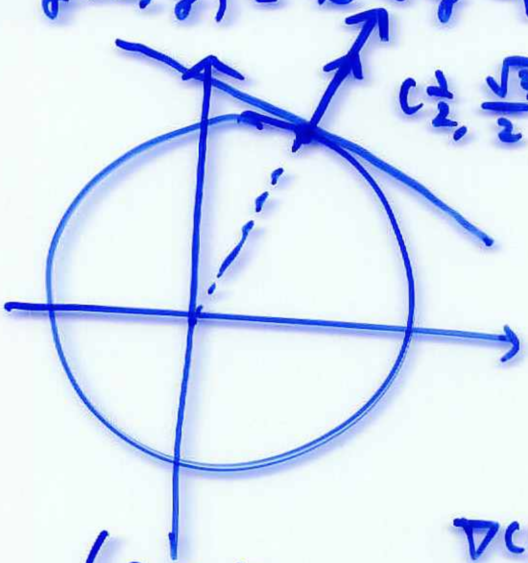
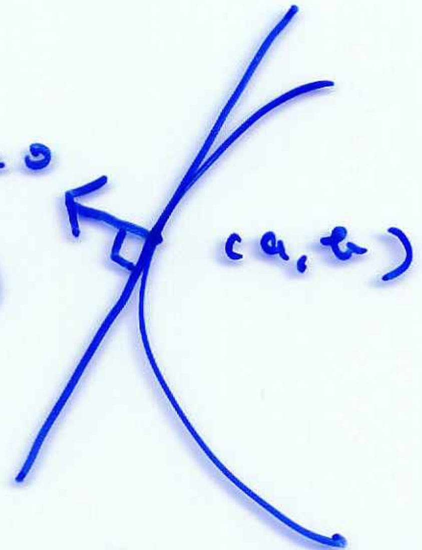
CT273 復習

例 $g(x, y) = x^2 + y^2 - 1 = 0$





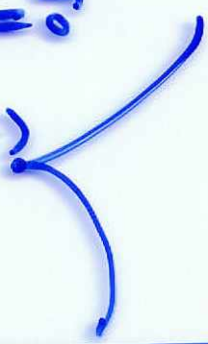
$g(x, y) = 0$

$$\nabla(g) = \begin{pmatrix} 2x \\ 2y \end{pmatrix}$$

$$\nabla(g)(a, b) = \begin{pmatrix} g_x(a, b) \\ g_y(a, b) \end{pmatrix}$$

$\nabla(g)(a, b) = \vec{0}$

なる (a, b)



接線の方程式

$$g_x(a, b)(x - a) + g_y(a, b)(y - b) = 0$$

$g_y(a, b) \neq 0$ とする.

$$y = \boxed{-\frac{g_x(a, b)}{g_y(a, b)}}(x - a) + b$$

← 傾き.

演習 8.10

$$g = 4x^2 - 4xy + y^2 - x - 2y + 2 = 0$$

at $\left(\frac{2}{5}, \frac{4}{5}\right)$

$$g_x = 8x - 4y + 0 - 1 - 0 + 0$$
$$= 8x - 4y - 1$$
$$g_y = 0 - 4x \cdot 1 + 2y - 0 - 2 + 0$$
$$= -4x + 2y - 2$$
$$g_x\left(\frac{2}{5}, \frac{4}{5}\right) = \frac{16}{5} - \frac{16}{5} - 1 = -1$$
$$g_y\left(\frac{2}{5}, \frac{4}{5}\right) = -\frac{8}{5} + \frac{8}{5} - 2 = -2$$
$$\nabla(g)\left(\frac{2}{5}, \frac{4}{5}\right) = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$
$$(-1)\left(x - \frac{2}{5}\right) + (-2)\left(y - \frac{4}{5}\right) = 0$$

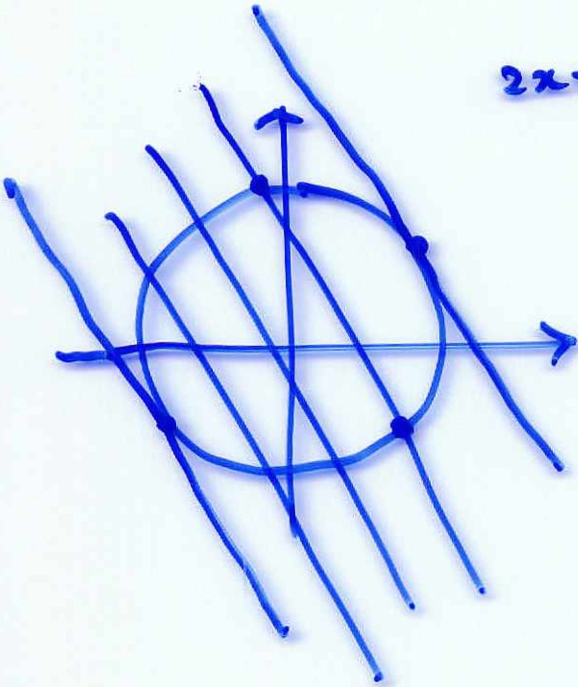
Lagrange の未定乗数法

例4　$g(x, y) = x^2 + y^2 - 1 = 0$

上を (x, y) が動く

$z = f(x, y) = 2x + y$

の最大・最小を考える



$2x + y = C$

$g = x^2 + y^2 - 1 = 0$ の下で

$t = 2x + y$

陰関数の定理

$g(a, b) = 0$

$g_y(a, b) \neq 0$

(a, b) の近くで

$\Big[$ $((x, y)$ が $g(x, y) = 0$ 上にある$)$

$\Leftrightarrow \quad y = \varphi(x)$

が満たされるように $\varphi : (a-\delta, a+\delta) \longrightarrow \mathbb{R}$

がある.

$g : C^1 \Rightarrow \varphi : C^1$

$\Big($ g_x, g_y : 存在

g, g_x, g_y : 連続

φ' が存在して

φ, φ' が連続

$$\varphi'(t) = -\frac{g_x(t, \varphi(t))}{g_y(t, \varphi(t))}$$

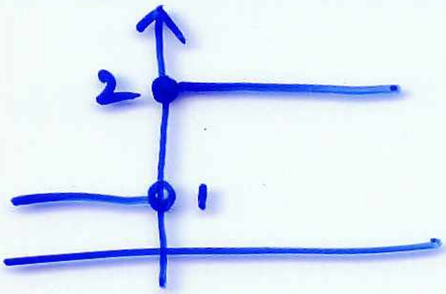
特に $t = a$, $\varphi(a) = b$

$$\varphi'(a) = -\frac{g_x(a, b)}{g_y(a, b)}$$

連続性　$y = f(x)$ が $x = a$ で連続

$\Longleftrightarrow$ $x_n \to a$ $(n \to +\infty)$ ならば $f(x_n) \to f(a)$

定義

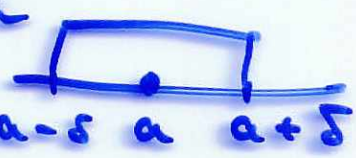


$-\frac{1}{n} \to 0$

$f(-\frac{1}{n}) = 1 \to 1 \neq \underset{\underset{2}{\parallel}}{f(0)}$

f は $x = 0$ で不連続

$f(a) > 0$　f：$x = a$ で連続

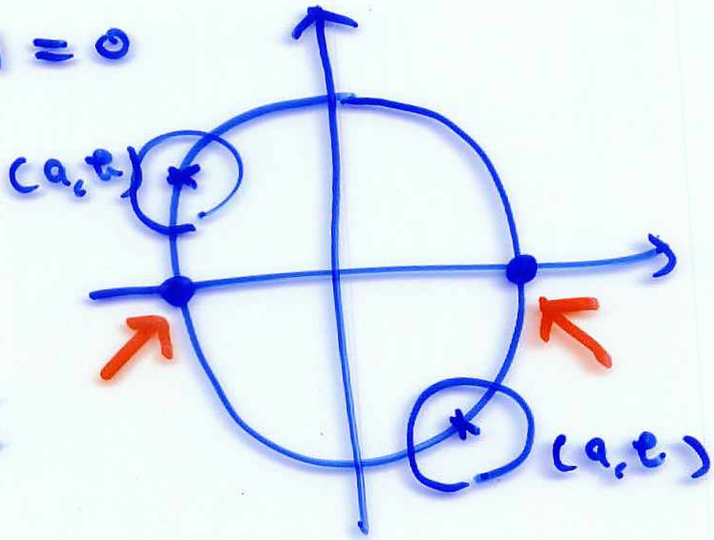


$\Longrightarrow$ 小さな $\delta > 0$ があって

$f(t) > 0$ $(a - \delta < t < a + \delta)$

$g(x,y) = x^2 + y^2 - 1 = 0$

$g_y = 2y$



$b > 0$

$y = \sqrt{1-x^2}$

$b < 0$

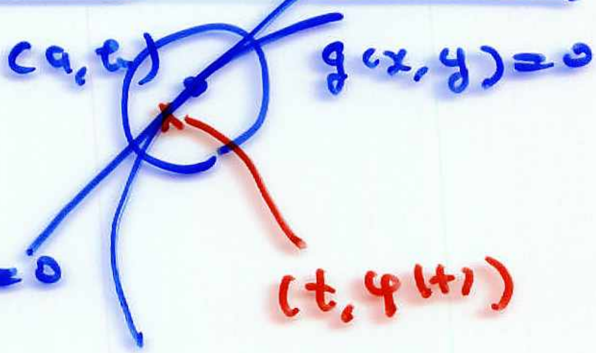
$y = -\sqrt{1-x^2}$

接線の傾きは $-\dfrac{g_x(a,b)}{g_y(a,b)}$

注 g: C^1 級

$g(a,b) = 0$

$g_y(a,b) \neq 0$



(a,b) の近くで $C : g(x,y) = 0$

は $y = \varphi(x)$

と書ける

$\rightsquigarrow g(t, \varphi(t)) \equiv 0 \quad (t = a$ の近くで$)$

$$\boxed{\begin{aligned} F(t) &= f(x(t), y(t)) \\ F'(t) &= f_x\, x'(t) + f_y\, y'(t) \end{aligned}}$$

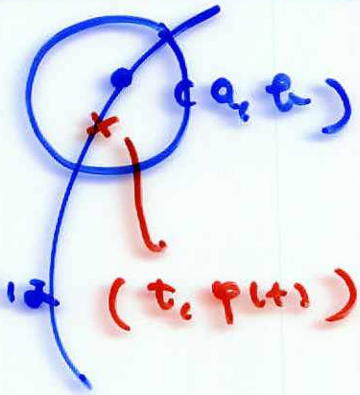
$\longrightarrow g_x(t, \varphi(t)) \cdot 1 + g_y(t, \varphi(t)) \cdot \varphi'(t) \equiv 0$

(a,b) の近くで $g_y(t, \varphi(t)) \neq 0$ $\quad \varphi'(a) = -\dfrac{g_x(a,b)}{g_y(a,b)}$

$g_y(x,y) \neq 0$ $\nwarrow$ g_y の連続性.

C: $g(x,y)=0$ を (x,y) が動く.
$z=f(x,y)$ の挙動を考える

(a,b) で極値をとるとする.



$g_y(a,b) \neq 0$

(a,b) の近くで C: $g(x,y)=0$ は

$$y=\varphi(x)$$

と書けるとする.

$$F(t)=f(t,\varphi(t))$$

$$\longrightarrow F'(a)=0$$

$$F'(t)=f_x(t,\varphi(t))\cdot 1+f_y(t,\varphi(t))\cdot\varphi'(t)$$

$$F'(a)=f_x(a,b)+f_y(a,b)\,\varphi'(a)=0$$

$$\varphi'(a) = -\frac{g_x(a,b)}{g_y(a,b)}$$

Lagrange の未定乗数法

$$=f_x(a,b)-\underbrace{\boxed{\frac{f_y(a,b)}{g_y(a,b)}}}_{=\lambda}\,g_x(a,b)=0$$

$$\begin{cases} f_x(a,b)=\lambda\, g_x(a,b) \\ f_y(a,b)=\lambda\, g_y(a,b) \end{cases}$$

$$\nabla(f)(a,b)=\lambda\,\nabla(g)(a,b)$$

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定理 $C: g(x,y)=0$ の下で

$$z=f(x,y)$$

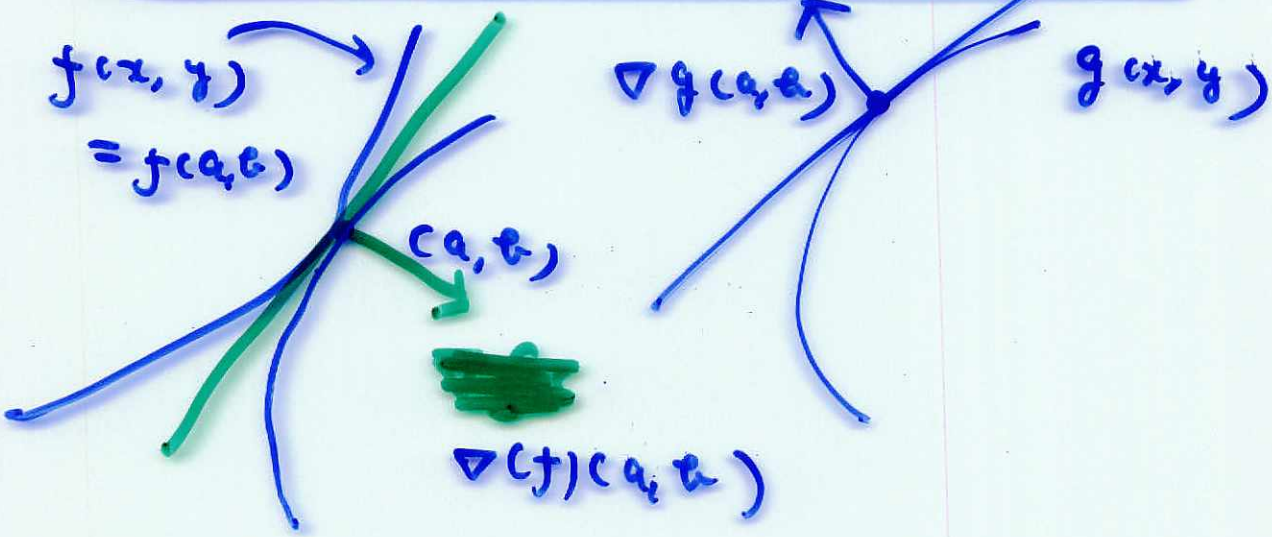
を考える. $(a,b)\in C$ で極値をとり

$$g_y(a,b)\neq 0 \text{ OR } g_x(a,b)\neq 0$$

が成立すると仮定.

$$\nabla(f)(a,b)=\lambda\nabla(g)(a,b)$$

を満たす λ が存在.



⟶ 定理の状況で

$g(x,y)=0$ と $f(x,y)=f(a,b)$

接している

(a,b) で

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$$g(x, y) = x^2 + y^2 - 1 = 0$$

$$z = f(x, y) = 2x + y$$

$$\nabla(g) = \begin{pmatrix} 2x \\ 2y \end{pmatrix}$$

$$\nabla(f) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(x, y) で極値をとるとする.

$$\begin{cases} 2 = 2\lambda x \\ 1 = 2\lambda y \\ x^2 + y^2 = 1 \end{cases}$$

$\lambda = 0$ は有り得ない..

$$x = \frac{1}{\lambda}, \quad y = \frac{1}{2\lambda}$$

$$\frac{1}{\lambda^2} + \frac{1}{4\lambda^2} = 1 \qquad \frac{1}{\lambda^2} \cdot \frac{5}{4} = 1$$

$$\lambda^2 = \frac{5}{4}$$

$$(x, y) = \left(\pm \frac{2}{\sqrt{5}}, \pm \frac{1}{\sqrt{5}}\right) \qquad \lambda = \pm \frac{\sqrt{5}}{2}$$

複号同順.

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$g(x,y) = x^2 + y^2 - 1 = 0$ の下で

$z = f(x,y) = xy.$

$\nabla(g) = \begin{pmatrix} 2x \\ 2y \end{pmatrix}, \quad \nabla(f) = \begin{pmatrix} y \\ x \end{pmatrix}$

$$\begin{cases} y = \lambda \cdot 2x & \cdots ① \\ x = \lambda 2y. & \cdots ② \\ x^2 + y^2 = 1. \end{cases}$$

② より $x = 0$ ✗

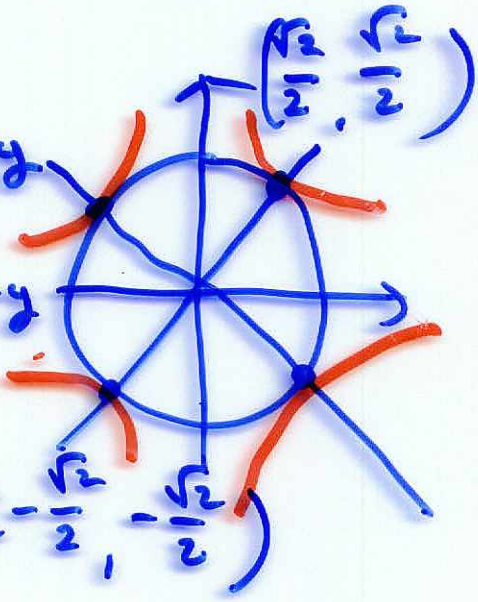
① に $x = 2\lambda y$ を代入

$y = 4\lambda^2 y. \iff y = 0$ OR $4\lambda^2 = 1$

$\lambda = \pm \frac{1}{2}.$

$\lambda = \frac{1}{2}$ のとき ① より $x = y$

$\lambda = -\frac{1}{2}$ のとき ② より $x = -y$



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290 page 例 演習 8.13.

1 $x, y > 0$ $\quad x + y = 1$ の下で

$z = x^{\frac{1}{2}} y^{\frac{1}{4}}.$

十分条件

$F'(a)=0,\ F''(a) \underset{<}{>} 0$

$\Rightarrow$ $F(t)$ は $t=a$ で

極小.

大

$F(t) = f(t, \varphi(t))$ を用いて $F''(a) \gtrless 0$

を考える.

差分方程式

$$a_{n+2} + p a_{n+1} + q a_n = 0$$

・ 特性方程式

$$\lambda^2 + p\lambda + q = 0$$

判別式 $D = p^2 - 4q = 0$ の場合.

$$(\lambda - \alpha)^2 = 0$$

$$p = -2\alpha, \quad q = \alpha^2.$$

$$a_{n+2} - 2\alpha a_{n+1} + \alpha^2 a_n = 0$$

$$(a_{n+2} - \alpha a_{n+1}) = \alpha (a_{n+1} - \alpha a_n)$$

$\{a_{n+1} - \alpha a_n\}$ は公比 α の等比数列.

$$a_{n+1} - \alpha a_n = \alpha^n (a_1 - \alpha a_0)$$

$\alpha \neq 0$ とする. 両辺を α^{n+1} で割る.

$$\frac{a_{n+1}}{\alpha^{n+1}} - \frac{a_n}{\alpha^n} = \underbrace{\alpha^{-1}(a_1 - \alpha a_0)}$$

$\left\{\frac{a_n}{\alpha^n}\right\}$ は等差数列.

$$\frac{a_n}{\alpha^n} = \frac{a_0}{1} + n\alpha^{-1}(a_1 - \alpha a_0)$$

$$\rightarrow a_n = a_0\alpha^n + n\alpha^{n-1}(a_1 - \alpha a_0)$$

信用創造

銀行に預金 d.　預金準備率 x

$$d \begin{cases} \nearrow \ xd & \text{中央銀行} \\ \searrow \ (1-x)d & \text{貸し出し} \end{cases}$$

$$(1-x)d \begin{cases} \nearrow \ x(1-x)d & \text{中央銀行} \\ \searrow \ (1-x)^2 d & \text{貸し出し} \end{cases}$$

還元

貸し出し総額

$$(1-x)d + (1-x)^2 d + (1-x)^3 d + \cdots$$

$$= d(1-x)\{1 + (1-x) + (1-x)^2 + \cdots\}$$

$$= d(1-x)\frac{1}{1-(1-x)}$$

$|r|<1$

$$1 + r + r^2 + \cdots = \frac{1}{1-r}$$

$$= d\frac{1-x}{x}.$$

月日経 24page.

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}, \quad \vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} \in \mathbb{R}^N$$

$$\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2 + \cdots + x_N y_N$$

$$\vec{x} \cdot \vec{y} = (\vec{x}, \vec{y})$$

$$\|\vec{x}\|^2 = (\vec{x}, \vec{x})$$

$$= x_1^2 + x_2^2 + \cdots + x_N^2$$

$$\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_N^2}$$

$\vec{x}$ のノルム

大きさ

$$\vec{x} + \vec{y} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_N + y_N \end{pmatrix}, \quad \lambda \vec{x} = \begin{pmatrix} \lambda x_1 \\ \lambda x_2 \\ \vdots \\ \lambda x_N \end{pmatrix}$$

$$(\vec{x} + \vec{y}, \vec{z}) = (\vec{x}, \vec{z}) + (\vec{y}, \vec{z})$$

$$(\vec{x}, \vec{y} + \vec{z}) = (\vec{x}, \vec{y}) + (\vec{x}, \vec{z})$$

$$(\lambda \vec{x}, \vec{y}) = (\vec{x}, \lambda \vec{y}) = \lambda (\vec{x}, \vec{y})$$

$$(\vec{x}, \vec{y}) = (\vec{y}, \vec{x})$$

$$\|\vec{x}+\vec{y}\|^2 = \|\vec{x}\|^2 + 2(\vec{x},\vec{y}) + \|\vec{y}\|^2$$

$$
\begin{aligned}
\text{左} &= (\vec{x}+\vec{y},\ \vec{x}+\vec{y}) \\
&= (\vec{x},\ \vec{x}+\vec{y}) + (\vec{y},\ \vec{x}+\vec{y}) \\
&= (\vec{x},\vec{x}) + (\vec{x},\vec{y}) \\
&\quad + (\vec{y},\vec{x}) + (\vec{y},\vec{y}) \\
&= \|\vec{x}\|^2 + 2(\vec{x},\vec{y}) + \|\vec{y}\|^2 = \text{右}
\end{aligned}
$$

193p.

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

平均

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

x_1	y_1
x_2	y_2
$\vdots$	$\vdots$
x_N	y_N

$$V(x) = \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2$$

$$\mathrm{Cov}(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})$$

共分散

$$\vec{x} = \frac{1}{\sqrt{N}}\begin{pmatrix} x_1-\bar{x} \\ x_2-\bar{x} \\ \vdots \\ x_N-\bar{x} \end{pmatrix},\quad \vec{y} = \frac{1}{\sqrt{N}}\begin{pmatrix} y_1-\bar{y} \\ y_2-\bar{y} \\ \vdots \\ y_N-\bar{y} \end{pmatrix}$$

(証明) 全ての $\lambda \in \mathbb{R}$ に対して

$$0 \leqq \|\lambda \vec{x} - \vec{y}\|^2$$
$$= \|\lambda \vec{x}\|^2 + 2(\lambda \vec{x}, -\vec{y}) + \|-\vec{y}\|^2$$
$$= \lambda^2 \|\vec{x}\| - 2\lambda(\vec{x}, \vec{y}) + \|\vec{y}\|^2$$

$$\|\lambda \vec{x}\| = |\lambda| \, \|\vec{x}\|$$

(I) $\vec{x} = \vec{0}$

CS は OK

(II) $\vec{x} \neq \vec{0}$

$\|\vec{x}\| > 0$

$\vec{x} = \vec{0} \iff \|\vec{x}\| = 0$

$$f(\lambda) = A\lambda^2 + B\lambda + C$$

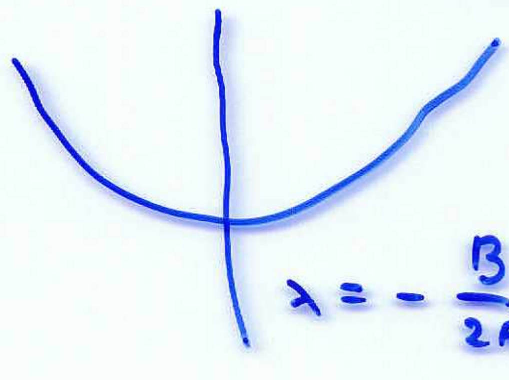
$A > 0$ とする.

($f(\lambda)$ が全ての $\lambda \in \mathbb{R}$ で $\geqq 0$ OK)

$$\iff D = B^2 - 4AC \leqq 0$$

$$-\frac{B^2 - 4AC}{4A}$$
$$\parallel$$

$$f(\lambda) = A\left(\lambda + \frac{B}{2A}\right)^2 + C - \frac{B^2}{4A}$$



$$\lambda = -\frac{B}{2A}$$

$$-\frac{B^2 - 4AC}{4A} \geqq 0$$

($A > 0$

$\iff B^2 - 4AC \leqq 0$

$V(x) = \|\vec{x}\|^2$, $V(y) = \|\vec{y}\|^2$

$\mathrm{Cov}(x, y) = (\vec{x}, \vec{y})$

$$\rho_{xy} = \frac{\mathrm{Cov}(x, y)}{\sqrt{V(x)}\sqrt{V(y)}} = \frac{(\vec{x}, \vec{y})}{\|\vec{x}\| \cdot \|\vec{y}\|}$$

x と y の相関係数.

$|\rho_{xy}| \leqq 1$

$$|\rho_{xy}| = \frac{|(\vec{x}, \vec{y})|}{\|\vec{x}\| \cdot \|\vec{y}\|}$$

$|\rho_{xy}| \sim 1$

((x, y) は直線状に分布する

コーシー・シュワルツの不等式.

$\vec{x}, \vec{y} \in \mathbb{R}^N$ に対し

$|(\vec{x}, \vec{y})| \leqq \|\vec{x}\| \cdot \|\vec{y}\|$

$$(x_1 y_1 + x_2 y_2 + \cdots + x_N y_N)^2 \leqq (x_1^2 + \cdots + x_N^2)(y_1^2 + \cdots + y_N^2)$$

$$f(\lambda) = \underbrace{\|\vec{x}\|^2}_{\vee \atop 0}\lambda^2 - 2(\vec{x}, \vec{y})\lambda + \|\vec{y}\|^2 \geqq 0$$

全λに

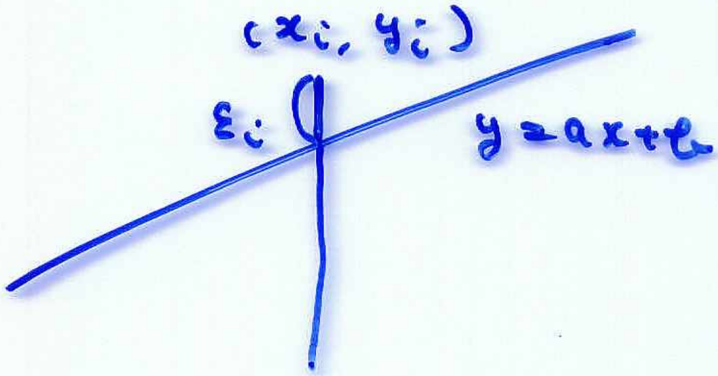
$$4(\vec{x}, \vec{y})^2 - 4\|\vec{x}\|^2 \cdot \|\vec{y}\|^2 \leqq 0$$

$$(\vec{x}, \vec{y})^2 \leqq \|\vec{x}\|^2 \|\vec{y}\|^2$$

$$|(\vec{x}, \vec{y})| \leqq \|\vec{x}\| \cdot \|\vec{y}\|$$

回帰分析.

モデル $y = ax + b$



x_1	y_1
x_2	y_2
$\vdots$	$\vdots$
x_N	y_N
説明変数	目的変数

$\varepsilon_i = y_i - (ax_i + b)$ 誤差.

① $\bar{\varepsilon} = 0$

$\quad \shortparallel$

$\bar{y} - a\bar{x} - b$

$\varepsilon = y - (ax + b)$

$$\boxed{\bar{y} = a\bar{x} + b}$$

② $V(\varepsilon)$：最小..

$$\varepsilon_i = y_i - ax_i - (\bar{y} - a\bar{x})$$
$$= (y_i - \bar{y}) - a(x_i - \bar{x})$$

$$\vec{\varepsilon} = \frac{1}{\sqrt{N}}\left(\varepsilon_i - \bar{\varepsilon}''\right)^{0}$$

$$= \frac{1}{\sqrt{N}}\left((y_i - \bar{y}) - a(x_i - \bar{x})\right)$$

$$= \vec{y} - a\vec{x} \qquad \vec{x} = \frac{1}{\sqrt{N}}\begin{pmatrix} x_1 - \bar{x} \\ \vdots \\ x_N - \bar{x} \end{pmatrix}$$

$$V(\varepsilon) = \|\vec{\varepsilon}\|^2$$

$$= \|\vec{y} - a\vec{x}\|^2$$

$$= a^2\|\vec{x}\|^2 - 2a(\vec{x}, \vec{y}) + \|\vec{y}\|^2$$

$$= \|\vec{x}\|^2\left(a - \frac{(\vec{x}, \vec{y})}{\|\vec{x}\|^2}\right)^2 + \|\vec{y}\|^2 - \frac{(\vec{x}, \vec{y})^2}{\|\vec{x}\|^2}$$

$$\left.\begin{aligned} a &= \frac{(\vec{x}, \vec{y})}{\|\vec{x}\|^2} = \frac{\mathrm{cov}(x, y)}{V(x)} \\ b &= \bar{y} - a\bar{x} \end{aligned}\right]$$

$y = ax + b$ 回帰直線.

このとき. $V(\varepsilon) = \|\vec{y}\|^2 - \dfrac{(\vec{x}, \vec{y})^2}{\|\vec{x}\|^2}$

$$= \|\vec{y}\|^2\left(1 - \frac{(\vec{x}, \vec{y})^2}{\|\vec{y}\|^2\|\vec{x}\|^2}\right)$$

$$= V(y)\left(1 - \rho_{xy}^2\right)$$

$|\rho_{xy}| \to 1$ のとき $V(\varepsilon) \to 0$

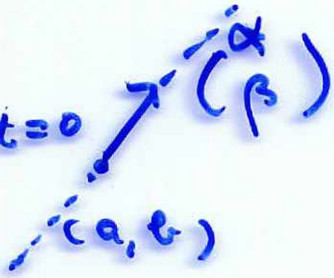
$$V(\varepsilon) = \frac{1}{N}\sum_{i=1}^{1} \varepsilon_i^2 \qquad \bar{\varepsilon} = 0$$

データが回帰直線に近づく

$z=f(x,y)$ が (a,b) で極値をとるとする.

$$\rightsquigarrow \frac{\partial f}{\partial x}(a,b)=\frac{\partial f}{\partial y}(a,b)=0$$

$$F(t)=f(a+\alpha t,\ b+\beta t) \qquad t=0$$



$$F'(t)=f_x(\quad)\cdot\alpha + f_y\cdot\beta$$

$$F'(0)=f_x(a,b)\,\alpha+f_y(a,b)\,\beta=0$$

$$F'(t)=f_x(a+\alpha t,\ b+\beta t)\cdot\alpha + f_y(\quad)\cdot\beta \qquad \boxed{f_{xy}=(f_x)_y}$$

$$F''(t)=\alpha(f_{xx}\cdot\alpha+f_{xy}\cdot\beta)+\beta(f_{yx}\cdot\alpha+f_{yy}\cdot\beta) \qquad f_{xy}=f_{yx}$$

$$=\alpha^2 f_{xx}+2\alpha\beta f_{xy}+\beta^2 f_{yy}$$

$$F''(0)=\alpha^2 f_{xx}(a,b)+2\alpha\beta f_{xy}(a,b)+\beta^2 f_{yy}(a,b)$$

$$(\vec{x}, \vec{y}) = x_1 y_1 + x_2 y_2 = {}^t\vec{x}\,\vec{y}$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \vec{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \qquad = (x_1, x_2)\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$${}^t\vec{x} = (x_1, x_2)$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad {}^tA = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$A = \begin{pmatrix} \mathfrak{a}_1 \\ \mathfrak{a}_2 \end{pmatrix} = (\vec{a}_1, \vec{a}_2)$$

$${}^tA = ({}^t\mathfrak{a}_1, {}^t\mathfrak{a}_2)$$

$$\mathfrak{a} = (a_1, a_2)$$

$${}^t\mathfrak{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad = \begin{pmatrix} {}^t\vec{a}_1 \\ {}^t\vec{a}_2 \end{pmatrix}$$

$$(A\vec{x}, \vec{y}) = (\vec{x}, {}^tA\vec{y})$$

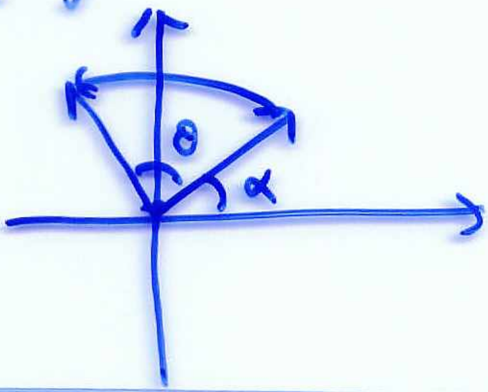
$$(\vec{x}, \vec{y} \in \mathbb{R}^2)$$

$$P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$P r \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} = r \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

$$= r \begin{pmatrix} \cos\theta\cos\alpha - \sin\theta\sin\alpha \\ \sin\theta\cos\alpha + \cos\theta\sin\alpha \end{pmatrix}$$

$$= r \begin{pmatrix} \cos(\theta+\alpha) \\ \sin(\theta+\alpha) \end{pmatrix}$$



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233 page $A = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$ を対角化. [illegible]

$${}^tA = A \text{ とする } \alpha \neq \beta.$$

$$\boxed{\left.\begin{aligned} A\vec{v}_1 &= \alpha\vec{v}_1 \\ A\vec{v}_2 &= \beta\vec{v}_2 \end{aligned}\right) \Longrightarrow (\vec{v}_1, \vec{v}_2) = 0}$$

$$(A\vec{v}_1, \vec{v}_2) = (\vec{v}_1, {}^tA\vec{v}_2) = (\vec{v}_1, A\vec{v}_2)$$

$$= (\alpha\vec{v}_1, \vec{v}_2) \qquad\qquad \| \;\; (\vec{v}_1, \beta\vec{v}_2)$$

$$= \alpha(\vec{v}_1, \vec{v}_2) \qquad\qquad \| \;\; \beta(\vec{v}_1, \vec{v}_2)$$

$$(\alpha - \beta)(\vec{v}_1, \vec{v}_2) = 0 \rightsquigarrow (\vec{v}_1, \vec{v}_2) = 0$$

$$\alpha - \beta \neq 0$$

A: 対称行列. $\Rightarrow$ $A = P \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} P^{-1}$

P: 回転行列

$$A = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$$

$$|\lambda I_2 - A| = \begin{vmatrix} \lambda - 5 & -2 \\ -2 & \lambda - 2 \end{vmatrix}$$

$$= \lambda^2 - 7\lambda + 6 = (\lambda - 6)(\lambda - 1)$$

<u>$\lambda = 1$</u> $\begin{pmatrix} -4 & -2 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \Leftrightarrow 2x + y = 0$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -2x \end{pmatrix} = x\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

<u>$\lambda = 6$</u> $\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \Leftrightarrow x - 2y = 0$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2y \\ y \end{pmatrix} = y\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 0$$

$$P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$ 回転.

$$\vec{v_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \rightsquigarrow \vec{p_1} = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ -2 \end{pmatrix} \qquad 1$$

$$\vec{v_2} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \rightsquigarrow \vec{p_2} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad 7$$

$$P = (\vec{p_1}\ \vec{p_2}) = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$$

回転

$$AP = (A\vec{p_1}\ \ A\vec{p_2})$$
$$= (1\cdot\vec{p_1}\ \ 7\vec{p_2}) = (\vec{p_1}\ \vec{p_2})\begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix}$$
$$= P\begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix} \rightsquigarrow A = P\begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix}P^{-1}.$$

$$P^{-1} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$$

$$P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

回転行列 P に対しては $\qquad = {}^tP.$

$$P^{-1} = {}^tP.$$

特に $\ {}^tPP = P\,{}^tP = I_2$

236p.

$A = \begin{pmatrix} 2 & -2 \\ -2 & 1 \end{pmatrix}$ を回転行列で対角化する.

$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ の対角化. → 月曜.

(月) ① 対称行列の対角化 +
2次形式. 233〜

② 極値問題
停留点がいつ極小、極大になるか.
278〜

③ 4章の話