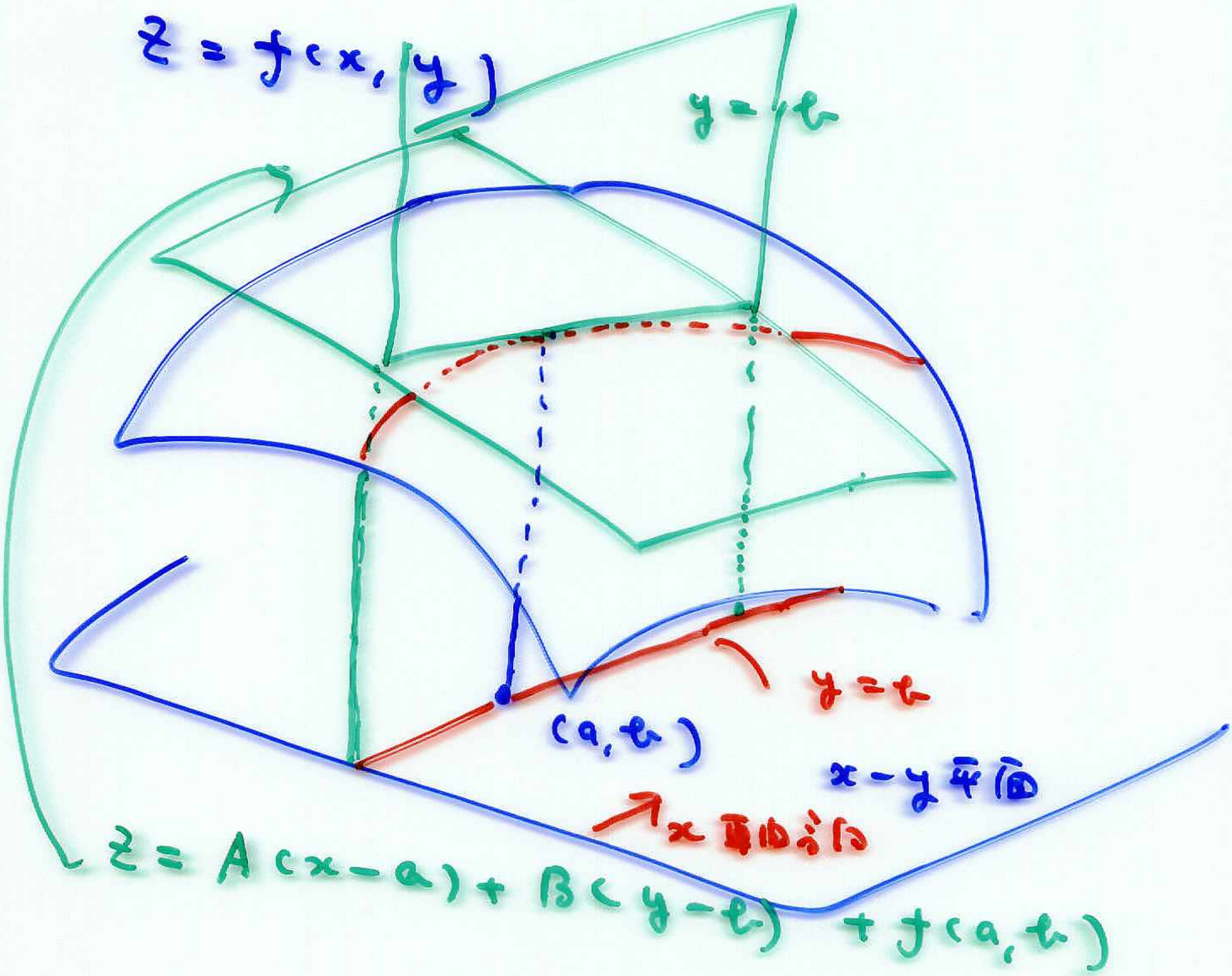
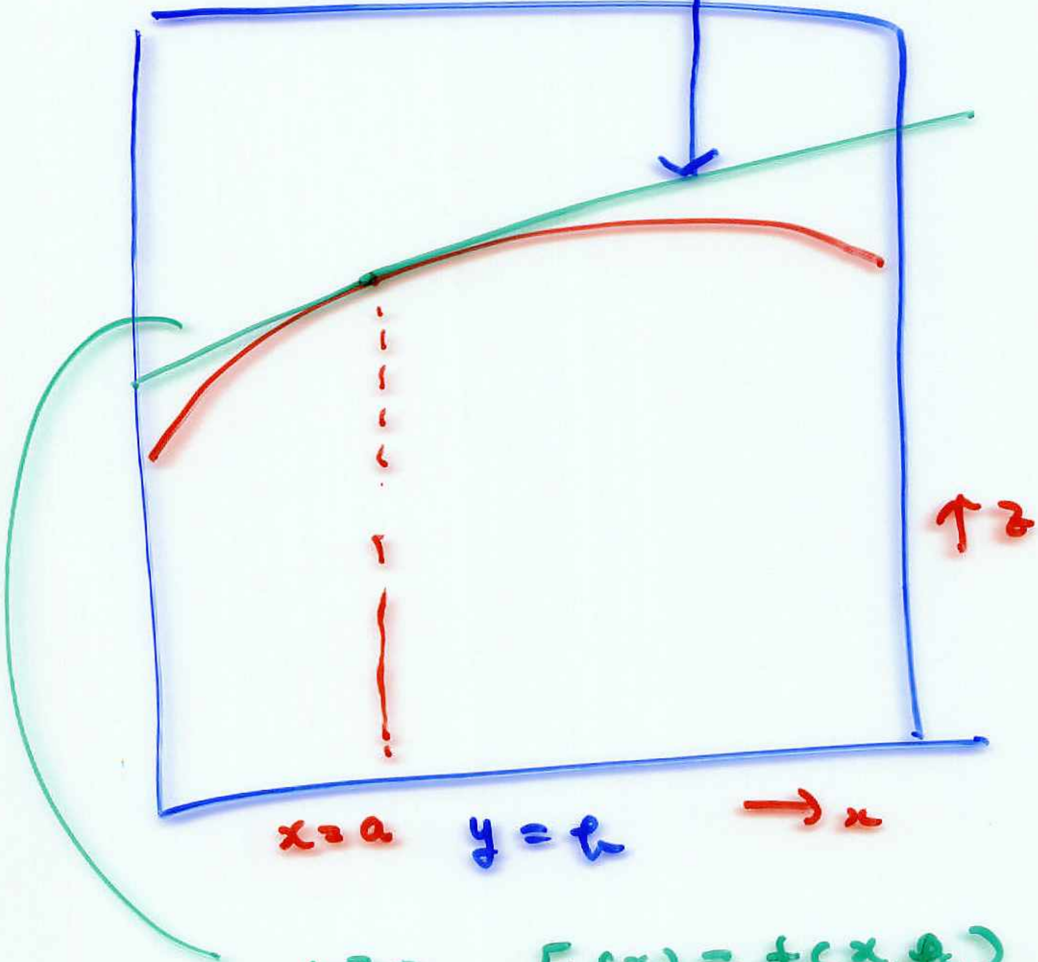

$z = f(x, y)$
$y = b$
$y = b$
(a, b)
x-y 平面
x 軸方向
$z = A(x-a) + B(y-b) + f(a, b)$

$z = A(x-a) + f(a,b)$



仮定 $F(x) = f(x,b)$

$$F'(a) = \frac{\partial f}{\partial x}(a,b)$$

$$\longrightarrow \quad A = \frac{\partial f}{\partial x}(a,b)$$

同様に $B = \dfrac{\partial f}{\partial y}(a,b)$

<u>まとめ</u> 接平面

$$z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$

CT 272 p.

134 $z = xy$

$$z_x = y \cdot 1 = y, \quad z_y = x \cdot 1 = x.$$

at $(1,1)$ $\quad z_x(1,1) = 1, \quad z_y(1,1) = 1$

$$z = 1 \cdot (x-1) + 1 \cdot (y-1) + 1$$

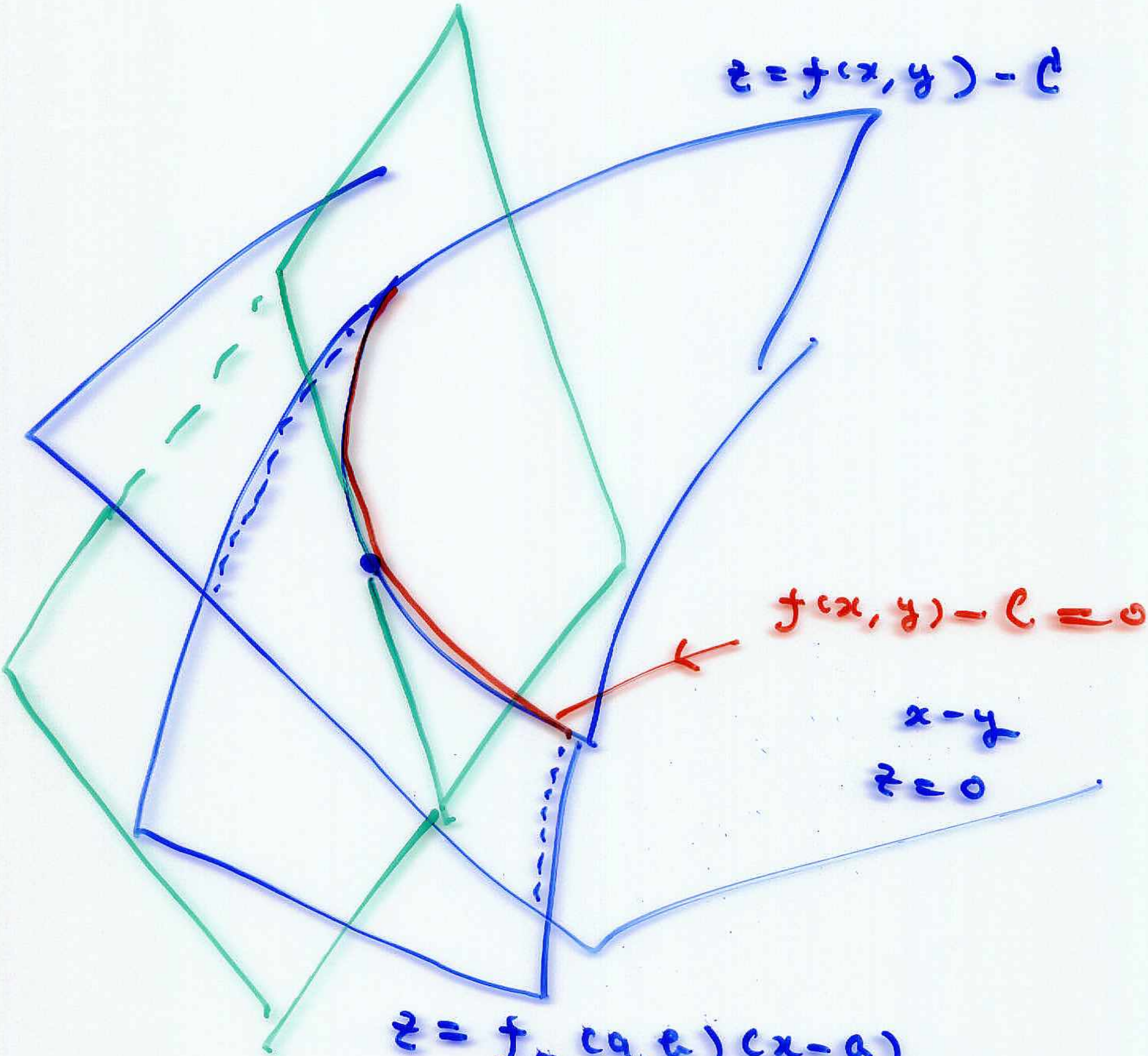
134 $z = f(x,y) = 3x^2 - 2y^2$

$$f_x = 6x, \quad f_y = -4y$$

at $(1,-1)$ $\quad f_x(1,-1) = 6$

$$f_y(1,-1) = +4$$

$$z = 6(x-1) + 4(y+1) + 1$$



$$z = f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

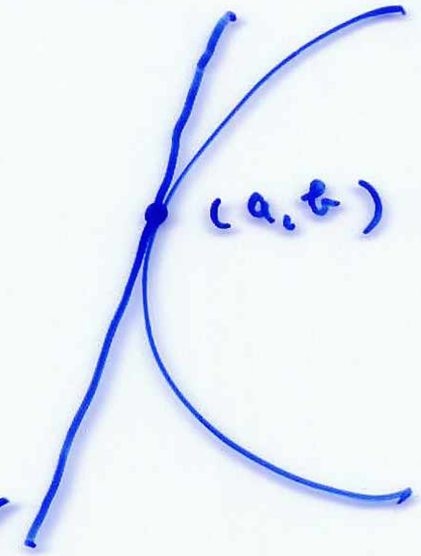
$z = 0$ とおくと

$$f_x(a, b)(x - a) + f_y(a, b)(y - b) = 0$$

$f(x, y) = C$

例1

$x^2 + y^2 - 1 = 0$



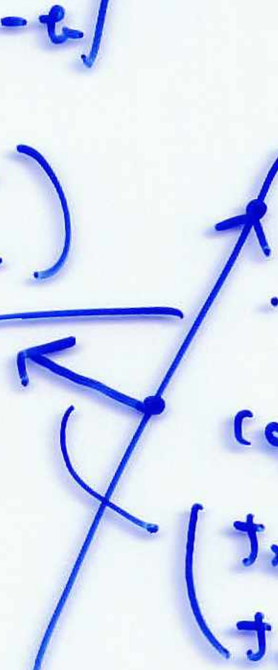
$f(a, b) = C$

この方程式を求める.

$$f_x(a, b)(x - a) + f_y(a, b)(y - b) = 0$$

$$\begin{pmatrix} f_x(a, b) \\ f_y(a, b) \end{pmatrix} \cdot \begin{pmatrix} x - a \\ y - b \end{pmatrix} = 0$$

$$\begin{pmatrix} f_x(a, b) \\ f_y(a, b) \end{pmatrix} \perp \begin{pmatrix} x - a \\ y - b \end{pmatrix}$$



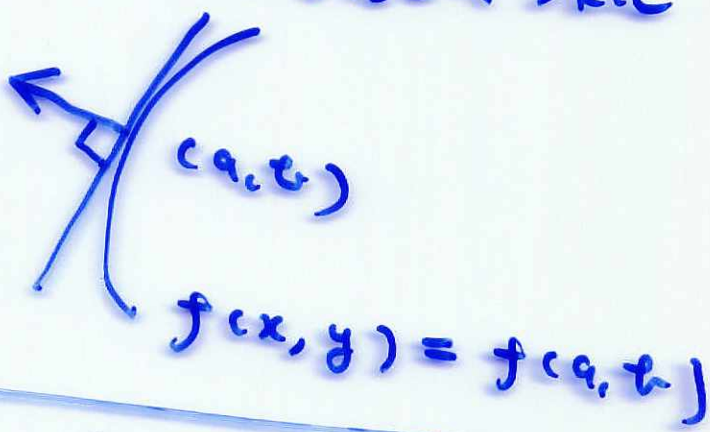
$$\begin{pmatrix} x - a \\ y - b \end{pmatrix}$$

$$\begin{pmatrix} f_x(a, b) \\ f_y(a, b) \end{pmatrix}$$

$$\nabla(f)(a,b) = \begin{pmatrix} f_x(a,b) \\ f_y(a,b) \end{pmatrix}$$

nabla

f の (a, b) における勾配ベクトル



例

$$\underbrace{x^2 - xy + y^2}_{\overset{\shortparallel}{f}} = 1 \quad \text{at } (1, 0)$$

$$f_x = 2x - y, \quad f_y = -x + 2y$$

$$\nabla(f)(1,0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$2(x-1) - 1\cdot(y-0) = 0$$

例 8.17

$$g = x^2 - 3xy + y^2 + 1 = 0$$

$(1, 2)$ $\qquad 1 - 3 \cdot 1 \cdot 2 + 4 + 1 = 0$

$$g_x = 2x - 3y \cdot 1 = 2x - 3y$$

$$g_y = 0 - 3x \cdot 1 + 2y = -3x + 2y$$

$$g_x(1, 2) = -4, \quad g_y(1, 2) = 1$$

接線は $-4(x-1) + 1 \cdot (y-2) = 0$

$U \subset \mathbb{R}^2$ 開. $f : U \longrightarrow \mathbb{R}$

f が U 上で C^1 級.

$\Longleftrightarrow$ U の各点 (a,b) で f_x, f_y が存在して

f, f_x, f_y が (a,b) で連続

$(x,y) \longrightarrow (a,b)$ のとき

$f(x,y) \longrightarrow f(a,b)$

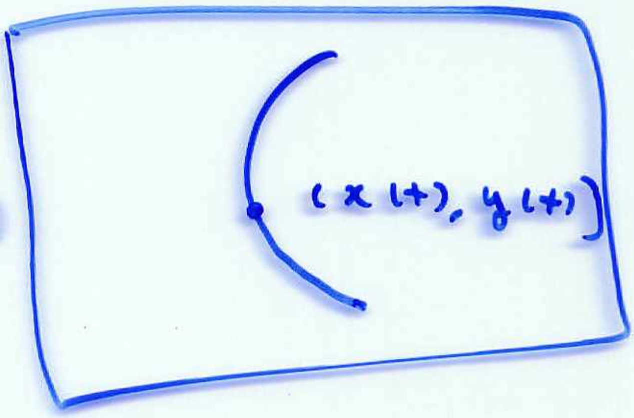
$\Longleftrightarrow$ f は (a,b) で連続.

定理 f が U 上 C^1 とする.

$\Longrightarrow$ f は $(a,b) \in U$ で全微分可能

$z = f(x,y)$

$F(t) = f(x(t), y(t))$



問題 $F'(t) = ?$

$$t = 0 \qquad (x(0), y(0)) = (a, b)$$

$$t \to 0$$

$$\frac{F(t) - F(0)}{t - 0} = \frac{f(x(t), y(t)) - f(a, b)}{t}$$

$$= \frac{1}{t}\Big\{ f_x(a, b)(x(t) - \underset{x(0)}{\overset{\|}{a}}) + f_y(a, b)(y(t) - \underset{y(0)}{\overset{\|}{b}})$$

$$+ \cancel{f(a, b)} + \beta(x(t), y(t)) - \cancel{f(a, b)} \Big\}$$

$$= f_x(a, b) \boxed{\frac{x(t) - x(0)}{t}} + f_y(a, b) \boxed{\frac{y(t) - y(0)}{t}}$$

$$\to x'(0) \qquad\qquad \to y'(0)$$

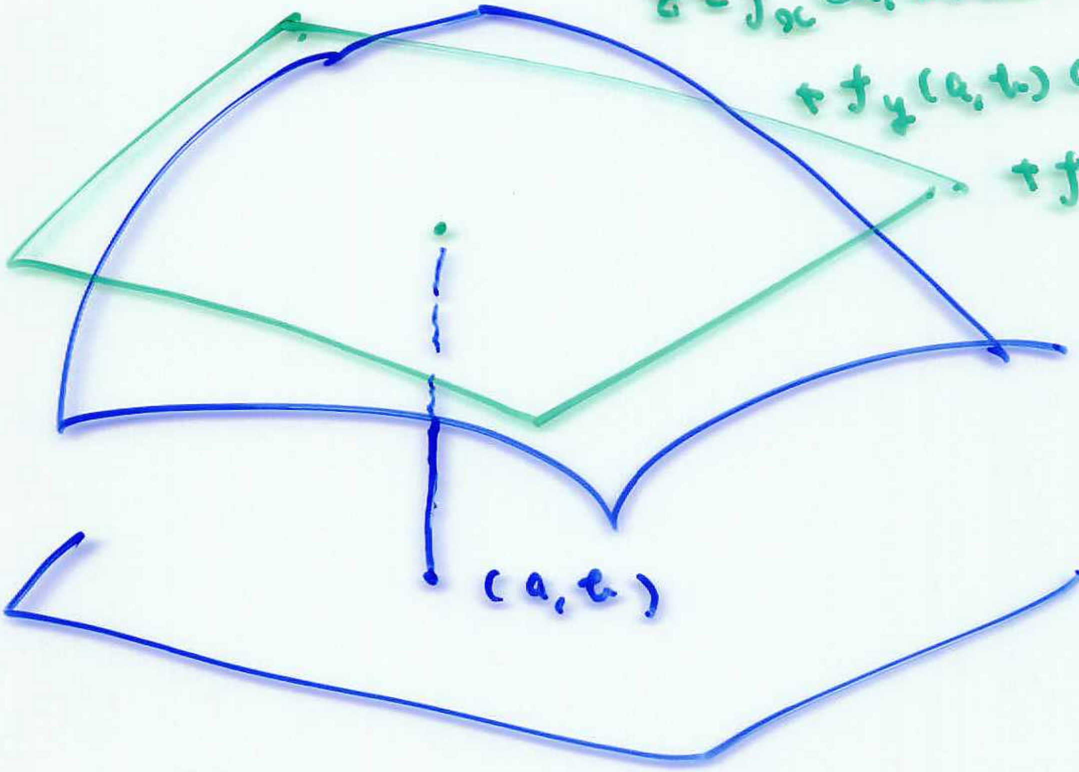
$$+ \boxed{\frac{\beta(x(t), y(t))}{t}} \to 0$$

後に示す.

$$\boxed{F'(0) = f_x(a, b)\, x'(0) + f_y(a, b)\, y'(0)}$$

$$z = f(x, y)$$

$$z = f_x(a, b)(x-a) + f_y(a, b)(y-b) + f(a, b)$$

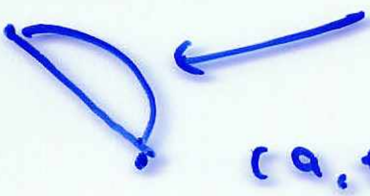


$$\beta(x, y) = f(x, y) - \{ f_x(a, b)(x-a) + f_y(a, b)(y-b) + f(a, b) \}$$

定義: $f(x, y)$ が (a, b) で(全)微分可能とは

$$\frac{\beta(x, y)}{\sqrt{(x-a)^2 + (y-b)^2}} \to 0 \quad \left((x, y) \to (a, b) \right)$$

(x, y)



$x(t)$, $y(t)$: 微分可能 $\longrightarrow$ 連続

$t \to 0$ のとき
$$x(t) \to x(0) = a$$
$$y(t) \to y(0) = b$$

$$\leadsto (x(t), y(t)) \longrightarrow (a, b)$$

$$\left| \frac{\beta(x(t), y(t))}{t} \right| = \underbrace{\left| \frac{\beta(x(t), y(t))}{\sqrt{(x(t)-a)^2 + (y(t)-b)^2}} \right|}_{\to 0}$$

$$\times \underbrace{\sqrt{\left(\frac{x(t) - \overset{=x(0)}{a}}{t}\right)^2 + \left(\frac{y(t) - \overset{=y(0)}{b}}{t}\right)^2}}_{\longrightarrow \sqrt{(x'(0))^2 + (y'(0))^2}}$$

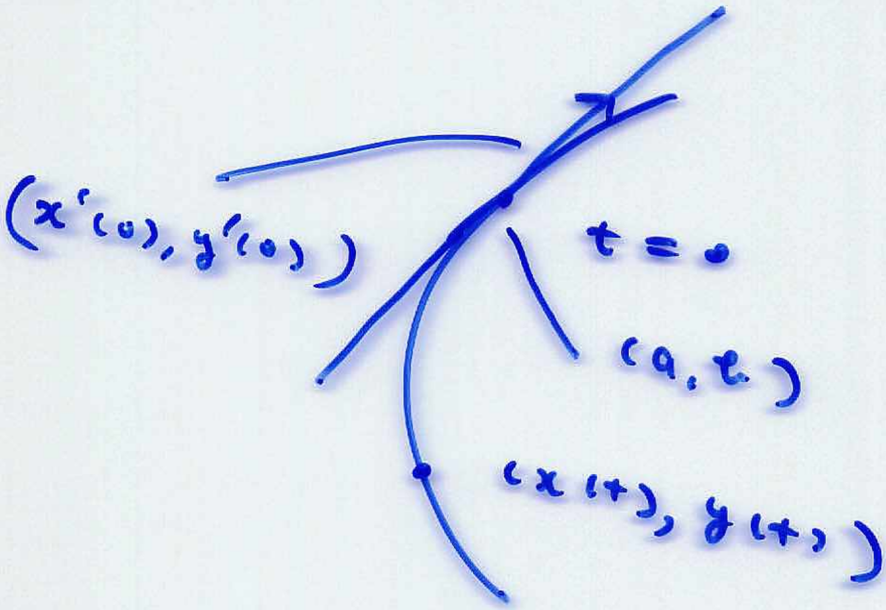
$\to 0$ $(t \to 0)$

$$F(t) = f(x(t), y(t))$$

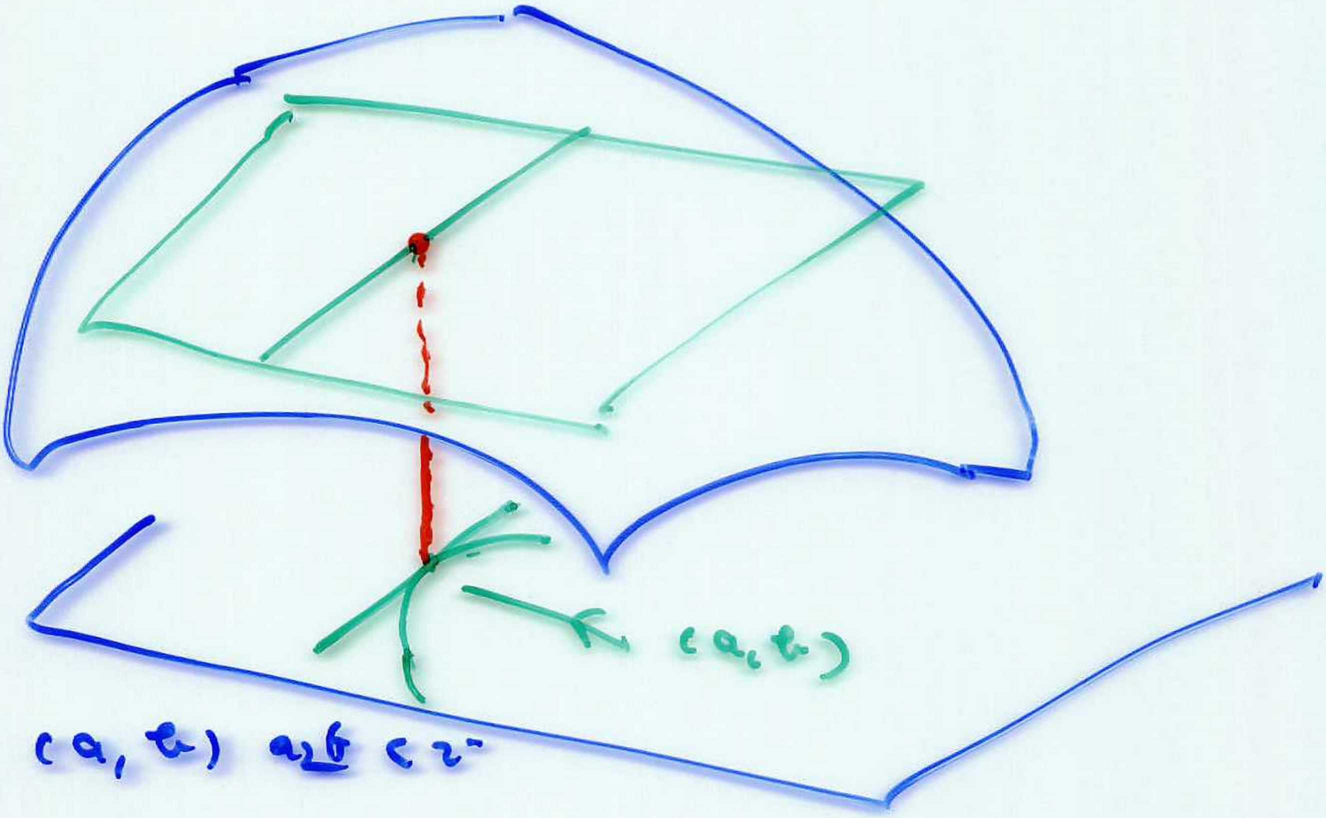
$$F'(t) = f_x(x(t), y(t))\, x'(t) + f_y(x(t), y(t))\, y'(t)$$

$z = f(x, y) \quad x = x(t),\ y = y(t)$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$



$$(x(t), y(t)) \fallingdotseq (a, b) + (x'(0), y'(0))\, t$$



(a, b) における

$$f(x, y) \doteqdot f_x(a, b)(x-a) + f_y(a, b)(y-b) + f(a, b)$$

$$\overset{F(t)}{\overset{\parallel}{f(x(t), y(t))}} \doteqdot f_x(a, b)\, x'(0)\, t + f_y(a, b)\, y'(0)\, t + f(a, b)$$

$$\boxed{\begin{aligned} x(t) &\doteqdot a + x'(0)\, t \\ y(t) &\doteqdot b + y'(0)\, t \end{aligned}}$$

$$= \boxed{\left(f_x(a, b)\, x'(0) + f_y(a, b)\, y'(0) \right)} t + f(a, b)$$

$$\overset{?}{F'(0)}$$

(2) $a_{n+2} + a_{n+1} - 2a_n = 0$

特性方程式

$$\lambda^2 + \lambda - 2 = 0$$

$(\lambda+2)(\lambda-1) = 0$ から $\lambda = -2, 1$

$$a_{n+2} - ((-2)+1)a_{n+1} + (-2)\cdot 1\, a_n = 0$$

$$\begin{cases} a_{n+2} + 2a_{n+1} = a_{n+1} + 2a_n \\ a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) \end{cases}$$

$$a_{n+1} + 2a_n = a_1 + 2a_0$$

$$-)\ \ a_{n+1} - a_n = 2^n(a_1 - a_0)$$

$$3a_n = (a_1 + 2a_0) - 2^n(a_1 - a_0)$$

から

$$a_n = \frac{1}{3}(1 - 2^n)a_1 + \frac{1}{3}(2 + 2^n)a_0$$

演習

$$a_{n+2} - 4a_{n+1} + 3a_n = 0$$

特性方程式

$$\lambda^2 - 4\lambda + 3 = 0$$

$$(\lambda - 3)(\lambda - 1) = 0 \text{ より } \lambda = 1, 3$$

の特性根

$$a_{n+2} - (1+3)a_{n+1} + 1 \cdot 3\, a_n = 0$$

$$\begin{cases} a_{n+2} - a_{n+1} = 3(a_{n+1} - a_n) \\ a_{n+2} - 3a_{n+1} = a_{n+1} - 3a_n \end{cases}$$

$\{a_{n+1} - a_n\}$ は公比3の等比数列.

$$a_{n+\cancel{2}} - a_{n} = 3^n(a_1 - a_0)$$

$$-)\ a_{n+\cancel{2}} - 3a_{n} = 1^n(a_1 - 3a_0)$$

$$2a_n = 3^n(a_1 - a_0) - (a_1 - 3a_0)$$

$$a_n = \frac{1}{2}(a_1 - a_0)3^n + \frac{1}{2}(3a_0 - a_1)$$

$$\rightsquigarrow a_n = \frac{1}{2}(3^n - 1)a_1 - \frac{1}{2}(3^n - 3)a_0$$

$$a_n = c\lambda^n$$

$$c\lambda^{n+2} - 4c\lambda^{n+1} + 3c\lambda^n = 0$$

$$= c\lambda^n(\lambda^2 - 4\lambda + 3)$$

$\rightsquigarrow \quad \lambda = 1, 3$

$b_n = c_1, \quad c_n = c_2 3^n$ gap.

$\rightsquigarrow a_n = c_1 + c_2 3^n.$

関于 n 一般的表示.

gap 有

$$\begin{array}{rl} a_0 = & c_1 + c_2 \\ -) \; a_1 = & c_1 + 3c_2 \\ \hline a_0 - a_1 = & -2c_2 \end{array}$$

$$c_2 = \frac{1}{2}(a_1 - a_0)$$

$$\begin{aligned} c_1 &= a_0 - c_2 \\ &= a_0 - \frac{1}{2}(a_1 - a_0) \\ &= \frac{3}{2}a_0 - \frac{1}{2}a_1 \end{aligned}$$

$$a_{n+2} - 4a_{n+1} + 3a_n = 0$$

$\{\alpha_n\}\{\beta_n\}$ が解　　$\{c_1\alpha_n\}$ も解

$$c_1\alpha_{n+2} - 4c_1\alpha_{n+1} + 3c_1\alpha_n = 0$$

$$+\big)\ c_2\beta_{n+2} - 4c_2\beta_{n+1} + 3c_2\beta_n = 0$$

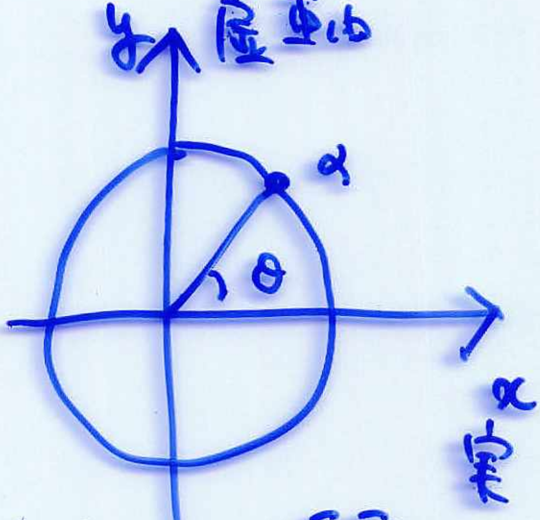
$$(c_1\alpha_{n+2} + c_2\beta_{n+2})$$
$$- 4(c_1\alpha_{n+1} + c_2\beta_{n+1})$$
$$+ 3(c_1\alpha_n + c_2\beta_n) = 0$$

$\rightsquigarrow \{c_1\alpha_n + c_2\beta_n\}$ も解.

$\alpha = \cos\theta + i\sin\theta$

$\alpha' = \cos\theta' + i\sin\theta'$

$\alpha \leftrightarrow (\cos\theta, \sin\theta)$



Gauss平面.

複素平面.

$$
\begin{aligned}
\alpha\alpha' &= (\cos\theta + i\sin\theta) \times (\cos\theta' + i\sin\theta') \\
&= (\cos\theta\cos\theta' - \sin\theta\sin\theta') + i(\cos\theta\sin\theta' + \cos\theta'\sin\theta) \\
&= \cos(\theta+\theta') + i\sin(\theta+\theta')
\end{aligned}
$$

「博士の愛した数式」 小川洋子

日本数学会出版賞.

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{i\theta} \cdot e^{i\theta'} = e^{i(\theta+\theta')}$

$$e^t = 1 + t + \frac{1}{2!} t^2 + \frac{1}{3!} t^3 + \cdots$$

右辺を使って　$z \in \mathbb{C}$

$$e^z := 1 + z + \frac{1}{2!} t^2 + \frac{1}{3!} t^3 + \cdots$$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$z = it$

$$e^{it} = 1 + it + \frac{1}{2!} (it)^2 + \frac{1}{3!} (it)^3 + \frac{1}{4!} (it)^4 + \cdots$$

$$= \left(1 - \frac{1}{2!} t^2 + \frac{1}{4!} t^4 - \frac{1}{6!} t^6 + \cdots\right)$$

$$+ i \left(t - \frac{1}{3!} t^3 + \frac{1}{5!} t^5 - + \cdots\right)$$

$$= \cos t + i \sin t$$

$$e^{it} = \cos t + i \sin t$$

$$\alpha = \cos\theta + i\sin\theta$$

$$\alpha\cdot\alpha = \cos(2\theta) + i\sin(2\theta)$$

$$\alpha^3 = \cos(3\theta) + i\sin(3\theta)$$

$$\alpha^n = \cos(n\theta) + i\sin(n\theta)$$

$$a_{n+2} + p\,a_{n+1} + q\,a_n = 0$$

$$\lambda^2 + p\lambda + q = 0 \qquad p, q \in \mathbb{R}$$

2解 α, β が $\alpha \neq \beta$

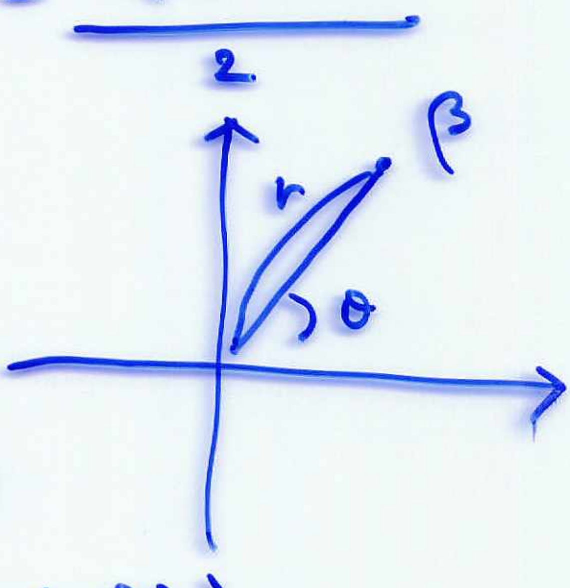
$$a_n = a_1 \frac{\beta^n - \alpha^n}{\beta - \alpha} + a_0 \frac{\alpha\beta(\alpha^{n-1} - \beta^{n-1})}{\beta - \alpha}$$

$D = p^2 - 4q < 0$

$$x = \frac{-p \pm \sqrt{D}}{2} = \frac{-p \pm i\sqrt{-D}}{2}$$

$$\beta = -\frac{p}{2} + i\frac{\sqrt{-D}}{2}$$

$$\alpha = -\frac{p}{2} - i\frac{\sqrt{-D}}{2}$$



$$\beta = r(\cos\theta + i\sin\theta)$$

$$\alpha = r(\cos\theta - i\sin\theta)$$

$$\beta^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

$$\alpha^n = r^n(\cos(n\theta) - i\sin(n\theta))$$

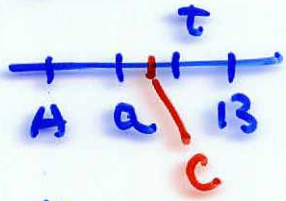
$t \in \mathbb{R}$

$$e^t = 1 + t + \frac{1}{2!}t^2 + \frac{1}{3!}t^3 + \cdots\cdots$$

CT 126 P.

Taylorの定理 $f \in C^\infty((A,B))$

(A,B)の各点で何回でも微分可能



$$f(t) = f(a) + f'(a)(t-a) + \frac{f''(a)}{2!}(t-a)^2 +$$

$$+ \frac{1}{3!}f^{(3)}(a)(t-a)^3 + \cdots\cdots$$

$$+ \frac{1}{n!}f^{(n)}(a)(t-a)^n$$

$$+ \frac{1}{(n+1)!}f^{(n+1)}(c)(t-a)^{n+1}$$

剰余項

あるc tとaの間

$f(t) = e^t, \ f'(t) = e^t, \ \cdots, \ f^{(n)}(t) = e^t$ $\quad 0$

$f^{(n)}(0) = 1$

$a = 0$

$$e^t = 1 + t + \frac{1}{2!}t^2 + \cdots + \frac{1}{n!}t^n + \boxed{\frac{e^c}{(n+1)!}t^{n+1}}$$

これを満たすcが0とtの間にある.

$n \to \infty \quad \boxed{} \longrightarrow 0$

$$\beta^n - \alpha^n = 2 r^n i \sin(n\theta)$$

$$\beta - \alpha = 2 r i \sin\theta, \quad \alpha\beta = r^2$$

$$a_n = a_1 r^{n-1} \frac{\sin(n\theta)}{\sin\theta}$$

$$= a_0 r^n \frac{\sin(n-1)\theta}{\sin\theta}$$

$$a_{n+2} + p\, a_{n+1} + q\, a_n = 0$$

$\lambda^2 + p\lambda + q = 0$ 特性方程式

$$D = p^2 - 4q < 0$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad |A| = ad - bc$$

$$A^2 - (a+d)A + |A|\, I_2 = O_2$$

(ケーリー・ハミルトンの定理)

固有方程式

$$\lambda^2 - (a+d)\lambda + |A| = 0$$

$$\left(= |\lambda I_2 - A| \right) \leftarrow \text{行列式.}$$

$$A(A - 7I_2) = (-5)(A - 7I_2)$$

$$\begin{aligned} A^2(A - 7I_2) &= A \cdot A(A - 7I_2) \\ &= A \cdot (-5)(A - 7I_2) \\ &= (-5)\, A(A - 7I_2) \\ &= (-5)(-5)(A - 7I_2) \end{aligned}$$

$A = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$ は

$$A^2 - 2A - 35 I_2 = O_2$$

を満たす. 対応する固有方程式は

$$\lambda^2 - 2\lambda - 35 = 0$$

$$\text{i.e.} \quad (\lambda - 7)(\lambda + 5) = 0$$

$$\lambda = 7, -5$$

$$A^2 - (7 + (-5)) A + (-5) \cdot 7 I_2 = O_2$$

$$\begin{cases} A(A - 7I_2) = (-5)(A - 7I_2) \\ A(A + 5I_2) = 7(A + 5I_2) \end{cases}$$

$$\begin{cases} A^n(A - 7I_2) = (-5)^n (A - 7I_2) \\ A^n(A + 5I_2) = 7^n (A + 5I_2) \end{cases}$$

$$-12 A^n = (-5)^n (A - 7I_2) - 7^n (A + 5I_2)$$

$$A^n = \frac{1}{12} 7^n (A + 5I_2) - \frac{1}{12} (-5)^n (A - 7I_2)$$

---

220 p. (7.40)

固有方程式が重根を持たない場合.

$$\left(B\vec{v}=\vec{0} \Longrightarrow \vec{v}=\vec{0}\right)$$

$$\Longleftrightarrow \quad \det(B) \neq 0 .$$

$$\left\{\begin{array}{l} \text{ある } \vec{v} \neq \vec{0} \text{ に対し } \quad B\vec{v}=\vec{0} \\ \Longleftrightarrow \quad \det(B)=0 \end{array}\right.$$

CT 223 p.

A : 2次正方行列.

$$= \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$$

$$\boxed{A\vec{v} = \lambda\vec{v} \text{ となる } \vec{v} \text{ を求める.}}$$

$\vec{v} \neq \vec{0}$

$$A\vec{v} = \lambda I_2 \vec{v} \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\lambda I_2 \vec{v} - A\vec{v} = \vec{0} \qquad I_2\vec{v} = \vec{v}$$

$$\left\{\begin{array}{l} (\lambda I_2 - A)\vec{v} = \vec{0} \\ \text{かつ } \vec{v} \neq \vec{0} \end{array}\right.$$

λ : 固有値

$\vec{v} \neq \vec{0}$: 固有ベクトル

$$\longrightarrow |\lambda I_2 - A| = 0$$

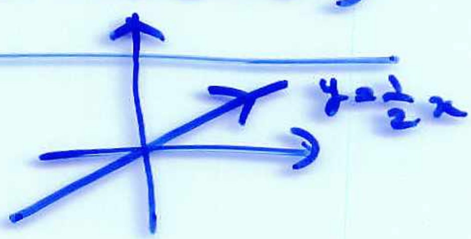
$$\left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix} \right| = \begin{vmatrix} \lambda-1 & -12 \\ -3 & \lambda-1 \end{vmatrix}$$

$$= (\lambda-1)^2 - (-3)(-12)$$

$$= \lambda^2 - 2\lambda - 35 = (\lambda-7)(\lambda+5)$$

$\lambda = 7$ のとき

$$7I_2 - A = \begin{pmatrix} 6 & -12 \\ -3 & 6 \end{pmatrix}$$



$$\begin{pmatrix} 6 & -12 \\ -3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \iff \left(\begin{array}{l} 6x - 12y = 0 \\ \text{かつ} \\ -3x + 6y = 0 \end{array}\right.$$

$$\iff x - 2y = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2y \\ y \end{pmatrix} = y\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\underline{\lambda = -5}$ $\quad -5I_2 - A = \begin{pmatrix} -6 & -12 \\ -3 & -6 \end{pmatrix}$

$$\begin{pmatrix} -6 & -12 \\ -3 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff \begin{matrix} -6x - 12y = 0 \\ -3x - 6y = 0 \end{matrix}$$

$$\iff x + 2y = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2y \\ y \end{pmatrix} = y\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\vec{v_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix},\ \vec{v_2} = \begin{pmatrix} -2 \\ 1 \end{pmatrix},\ P = (\vec{v_1}\ \vec{v_2})$$

$$A\vec{v_1} = 7\vec{v_1},\quad A\vec{v_2} = -5\vec{v_2}$$

$$AP = A(\vec{v_1}\ \vec{v_2}) = (A\vec{v_1}\ A\vec{v_2})$$

$$= (7\vec{v_1}\ -5\vec{v_2}) = (\vec{v_1}\ \vec{v_2})\begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}$$

$$= P\begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}$$

<u>実は Pは正則である.</u>

$$|P| = \begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix} = 4 \neq 0,\ P \text{正則}.$$

$$AP = P\begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}$$

$$\underbrace{AP\cdot P^{-1}}_{\parallel} = P\begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}P^{-1}$$

$$A\cdot PP^{-1} = A\cdot I_2 = A$$

Aは対角化可能.

$$\rightsquigarrow \quad A = P\begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}P^{-1}.$$

$$\begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \begin{pmatrix} \alpha' & 0 \\ 0 & \beta' \end{pmatrix} = \begin{pmatrix} \alpha\alpha' & 0 \\ 0 & \beta\beta' \end{pmatrix}$$

$$A = P \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix} P^{-1}.$$

$$A^2 = P \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix} \underbrace{P^{-1} \cdot P}_{= I_2} \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix} P^{-1}$$

$(AB)C$
$= A(BC)$
結合則.

$$= P \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix} P^{-1}$$

$$= P \begin{pmatrix} 7^2 & 0 \\ 0 & (-5)^2 \end{pmatrix} P^{-1}$$

$$A^3 = P \begin{pmatrix} 7^2 & 0 \\ 0 & (-5)^2 \end{pmatrix} \underbrace{P^{-1} P}_{= I_2} \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix} P^{-1}$$

$$= P \begin{pmatrix} 7^2 & 0 \\ 0 & (-5)^2 \end{pmatrix} \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix} P^{-1}$$

$$= P \begin{pmatrix} 7^3 & 0 \\ 0 & (-5)^3 \end{pmatrix} P^{-1}$$

$$A^n = P \begin{pmatrix} 7^n & 0 \\ 0 & (-5)^n \end{pmatrix} P^{-1}$$

演習 7.5 をやっておく.

1) 数列の漸化式, $a_{n+2}+pa_{n+1}+qa_n=0$

2) ベクトルの大きさ, 内積 (2次形式) の特性根が重根のとき.

3) 偏微分の応用.

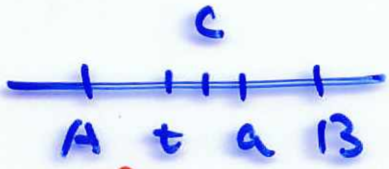
① 停留点が極大・極小となる条件

② Lagrange の未定乗数法

f: (A, B) 上 微分可能.

$f'(t) \equiv 0$

$\Rightarrow$ $f(t) \equiv C$



(証明) $f(t) - f(a) = \overset{0}{\overset{\|}{f'(c)}}(t-c)$

となる c が t と a の間にあり.

108p. 平均値の定理.

$f(t) = f(a)$

$\alpha \in \mathbb{R}$

$\mathbb{R}$上 $f'(t) = \alpha f(t)$ をみたす

$$\left(f(t)\,e^{-\alpha t}\right)' = f'(t)\left(e^{-\alpha t}\right) + f(t)\left(e^{-\alpha t}\right)'$$

$$= \alpha f(t)\,e^{-\alpha t} + f(t)\left(-\alpha e^{-\alpha t}\right)$$

$$= 0$$

$(fg)' = f'g + fg'$

$(e^t)' = e^t$

$y = f(x)$

$y = f(x(t))$ $\quad \dfrac{dy}{dt} = \dfrac{df}{dx} \cdot \dfrac{dx}{dt}$

$$\left(e^{-\alpha t}\right)' = e^{-\alpha t} \cdot \frac{d(-\alpha t)}{dt}$$

$$= -\alpha\, e^{-\alpha t}$$

$x = -\alpha t$

$f'(t) = \alpha f(t) \rightsquigarrow \left(f(t)\,e^{-\alpha t}\right)' \equiv 0$

$\rightsquigarrow f(t)\,e^{-\alpha t} \equiv f(0)$

$\rightsquigarrow f(t) = f(0)\,e^{\alpha t}$
