演習 1.2

(1) $a_0 = 1, \quad a_{n+1} = 3a_n - 2$
$(n = 0, 1, 2, \cdots)$

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1) $\boxed{a_0, \cdots, a_n} \longrightarrow a_{n+1}$ 帰納的な定義

2) a_0 : given

recursive な定義.

特性方程式

$\lambda = 3\lambda - 2$ ⇝ 解 λ

$a_n = \lambda \ (n = 0, 1, \cdots)$
or
$a_{n+1} = 3a_n - 2$ の解

$\lambda = 1$

$$\begin{array}{rrcl} & a_{n+1} & = & 3a_n - 2 \\ -) & 1 & = & 3 \cdot 1 - 2 \\ \hline & a_{n+1} - 1 & = & 3(a_n - 1) \end{array}$$

⇝ $\{a_n - 1\}$ は公比 3 の等比数列.

$a_n - 1 = 3^n (a_0 - 1)$

$a_0 - 1 \xrightarrow{\times 3} a_1 - 1 \xrightarrow{\times 3} a_2 - 1, \cdots \xrightarrow{\times 3} a_n - 1$

$a_n = 1 + 3^n (1 - 1) = 1$

(2) $a_0 = 1,\ a_{n+1} = -2a_n + 1$

特性方程式' $\lambda = -2\lambda + 1 \rightsquigarrow \lambda = \frac{1}{3}$

$3\lambda = 1$

$$
\begin{array}{rl}
 & a_{n+1} = -2a_n + 1 \\
-) & \frac{1}{3} = -2 \times \frac{1}{3} + 1 \\ \hline
 & a_{n+1} - \frac{1}{3} = (-2)\left(a_n - \frac{1}{3}\right)
\end{array}
$$

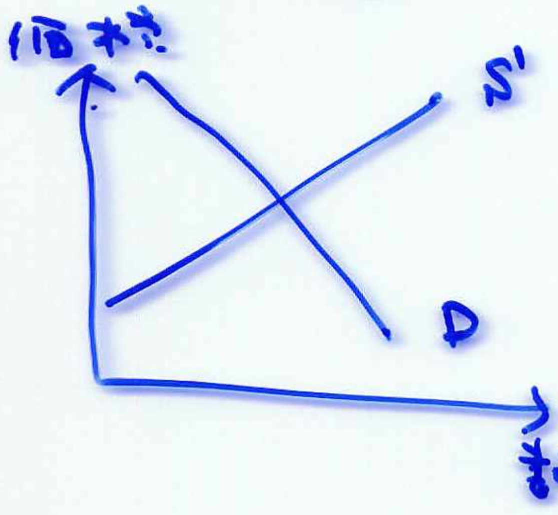
$\left\{ a_n - \frac{1}{3} \right\}$ は '公比が (-2) の等比数列'.

$$a_n - \frac{1}{3} = (-2)^n \left(a_0 - \frac{1}{3}\right)$$

$$\rightsquigarrow a_n = \frac{1}{3} + (-2)^n \left(1 - \frac{1}{3}\right)$$

$$= \frac{1}{3} + (-2)^n \cdot \frac{2}{3}.$$

CT 35 page.



需要曲線

$D: \ p = aD + b$

供給曲線

$S': \ p = a'S' + b'$

仮定 $a \neq a'$

$x_t \longrightarrow p_t = a x_t + b.$

第t期の供給量　　第t期の価格

$$x_{t+1} = \frac{1}{a'}(p_t - b')$$

$$x_{t+1} = \frac{1}{a'}(a x_t + b - b')$$

$$= \frac{a}{a'} x_t + \frac{b - b'}{a'}$$

$\neq 1$

特性方程式

$$\lambda = \frac{a}{a'} \lambda + \frac{b - b'}{a'}$$

$$\lambda = \lambda_0 := \frac{b - b'}{a' - a}$$

$\{x_t - \lambda_0\}$ は公比 $\frac{a}{a'}$ の等比数列.

$$x_t - \lambda_0 = \left(\frac{a}{a'}\right)^t (x_0 - \lambda_0)$$

$$x_t = \lambda_0 + \left(\frac{a}{a'}\right)^t (x_0 - \lambda_0)$$

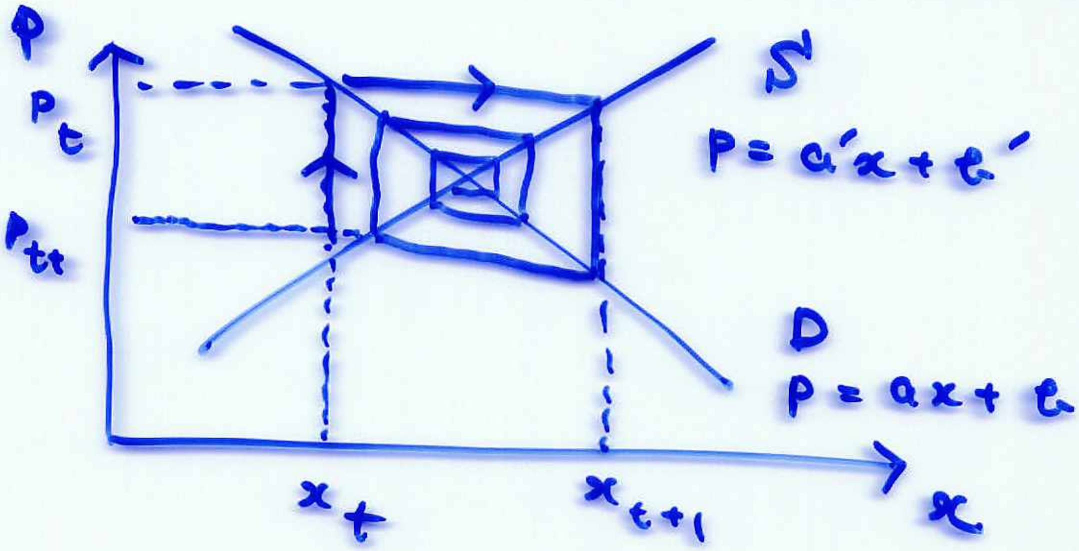
$\left|\frac{a}{a'}\right| < 1$ とする

$$\boxed{\begin{array}{c} |r| < 1 \\ r^n \to 0 \end{array}}$$

$$x_t \to \lambda_0 = \frac{b - b'}{a' - a}$$

$$p_t = a x_t + b \longrightarrow a\lambda_0 + b.$$

$$(x_t, p_t) \longrightarrow (\lambda_0, a\lambda_0 + b)$$



2階の差分方程式. cf 13 page

$$a_{n+2} - 3a_{n+1} + 2a_n = 0$$

特性方程式 $\lambda^2 - 3\lambda + 2 = 0$

$$\lambda^2 - 3\lambda + 2 = (\lambda - 2)(\lambda - 1)$$

$$\begin{cases} \alpha + \beta = 3 \\ \alpha\beta = 2 \end{cases}$$

$\lambda = 1, 2$

$\alpha = 1,\ \beta = 2$

$$a_{n+2} - (\alpha+\beta)a_{n+1} + \alpha\beta\, a_n = 0$$

$\lambda^2 + A\lambda + B = 0$ の2解 α, β.

$$\begin{cases} \alpha + \beta = -A \\ \alpha\beta = B \end{cases}$$

$$\begin{cases} a_{n+2} - \alpha a_{n+1} = \beta(a_{n+1} - \alpha a_n) \\ a_{n+2} - \beta a_{n+1} = \alpha(a_{n+1} - \beta a_n) \end{cases}$$

1) $\{a_{n+1} - \alpha a_n\}$ は公比 β の等比数列.

2) $\{a_{n+1} - \beta a_n\}$ は公比 α ——

$$a_{n+1} - \alpha a_n = \beta^n(a_1 - \alpha a_0)$$

$$-)\ a_{n+1} - \beta a_n = \alpha^n(a_1 - \beta a_0)$$

$$(\beta - \alpha)a_n = \beta^n(a_1 - \alpha a_0) - \alpha^n(a_1 - \beta a_0)$$

$\beta \neq \alpha$

$$a_n = \frac{\beta^n - \alpha^n}{\beta - \alpha}a_1 - \alpha\beta\frac{\beta^{n-1} - \alpha^{n-1}}{\beta - \alpha}a_0$$

$$a_{n+2} + p a_{n+1} + q a_n = 0$$

a_0, a_1 : given

$$a_2 = -p a_1 - q a_0$$

$$a_3 = -p a_2 - q a_1$$

$$\vdots$$

特性方程式 $\lambda^2 + p\lambda + q = 0$

2解 α, β.

<u>$\alpha \neq \beta$ のとき</u>

$$a_n = \frac{\beta^n - \alpha^n}{\beta - \alpha} a_1 + \alpha\beta \frac{\alpha^{n-1} - \beta^{n-1}}{\beta - \alpha} a_0$$

1) <u>$\alpha = \beta$ のとき</u>

2) $D = p^2 - 4q < 0$ のとき. $\alpha, \beta \notin \mathbb{R}$

<u>演習</u>

$$\alpha = \cos\theta + i\sin\theta$$

$$\alpha' = \cos\theta' + i\sin\theta'$$

$\alpha\alpha' = ?$ 　　加法定理.

$\dfrac{1}{\alpha} = ?$

演習 1) $a_{n+2} - 4a_{n+1} + 3a_n = 0$

を満たす $\{a_n\}$ を a_0, a_1 を使って表す.

2) $a_{n+2} + a_{n+1} - 2a_n = 0$

について同様.

$$A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = (\vec{a} \ \vec{b})$$

$$A \begin{pmatrix} x \\ y \end{pmatrix} = x\vec{a} + y\vec{b}$$

$$= x\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + y\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$= \begin{pmatrix} x a_1 + y b_1 \\ x a_2 + y b_2 \end{pmatrix}$$

補足

$$A = \begin{pmatrix} \mathbb{a} \\ \mathbb{b} \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}$$

Blackboard Boldface.

$$A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \mathbb{a} \\ \mathbb{b} \end{pmatrix} \vec{x} = \begin{pmatrix} \mathbb{a}\vec{x} \\ \mathbb{b}\vec{x} \end{pmatrix}$$

$$(a_1, a_2)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = a_1 x_1 + a_2 x_2$$

$$A(\vec{b}_1, \vec{b}_2) = (\boxed{A\vec{b}_1} \ \boxed{A\vec{b}_2})$$

$$\begin{pmatrix} \mathbb{a}_1 \\ \mathbb{a}_2 \end{pmatrix} (\vec{b}_1, \vec{b}_2) = \begin{pmatrix} \mathbb{a}_1 \vec{b}_1 & \mathbb{a}_1 \vec{b}_2 \\ \mathbb{a}_2 \vec{b}_1 & \mathbb{a}_2 \vec{b}_2 \end{pmatrix}$$

証 7.1.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_1 \end{pmatrix}$$ 1行と2行の交換

$$\begin{pmatrix} \lambda & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \lambda x_1 \\ x_2 \end{pmatrix}$$ 1行を λ 倍.

$$\begin{pmatrix} 1 & 0 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ \lambda x_2 \end{pmatrix}$$ 2行を λ 倍.

$$\begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ \lambda x_1 + x_2 \end{pmatrix}$$ 1行の λ 倍を 2行に加える.

$$\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + \lambda x_2 \\ x_2 \end{pmatrix}$$ 2行の λ 倍を 1行に加える

7.2 行基本変形.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = \begin{pmatrix} a_2 & b_2 \\ a_1 & b_1 \end{pmatrix}$$ 1行と2行の交換

$$\begin{pmatrix} \lambda & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = \begin{pmatrix} \lambda a_1 & \lambda b_1 \\ a_2 & b_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ \lambda a_2 & \lambda b_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ \lambda a_1 + a_2 & \lambda b_1 + b_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = \begin{pmatrix} a_1 + \lambda a_2 & b_1 + \lambda b_2 \\ a_2 & b_2 \end{pmatrix}$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad B = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} ad - bc & 0 \\ 0 & -bc + ad \end{pmatrix}$$

$$= (ad - bc) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} ad - bc & 0 \\ 0 & -bc + ad \end{pmatrix} \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= (ad - bc) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad |A| = ad - bc$$

$$AB = BA = (ad - bc) I_2 \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$|A| = ad - bc \neq 0$ とする.

$X := \dfrac{1}{|A|} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ とおく.

$AX = XA = I_2$.

$X = A^{-1}$

A の逆行列.

CT 209 page

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$AX = XA = I_2$ をみたす2次正方行列は __あれば__ 1つしかない

$$\left.\begin{aligned} AX &= XA = I_2 \\ AY &= YA = I_2 \end{aligned}\right\} \Rightarrow X = Y$$

$$AX = I_2 \leadsto Y(AX) = YI_2 = Y$$

$$Y(AX) \overset{\text{結合則}}{=} (YA)X = I_2X = X$$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$AI_2 = I_2A = A$$

A, B, C : 2次正方行列

$$(AB)C = A(BC)$$

結合則

$AB = BA$ は成立するとは限らない…

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\rightsquigarrow \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ には逆行列はない.

・ $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\det(A) = ad - bc \neq 0$ のとき.

A には逆行列あり

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

・ $\det(A) = 0$ の時には？

$\det(A) = 0$ とする

$A\vec{x} = \vec{0}$ かつ $\vec{x} \neq \vec{0}$ をみたす

$\vec{x} \in \mathbb{R}^2$ が存在.

A に逆行列 X があるとする.

$A\vec{x} = \vec{0}$ $\quad XA = AX = I_2$

$X(A\vec{x}) = X\vec{0} = \vec{0}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$= (XA)\vec{x} = I_2\vec{x} = \vec{x}$

A に逆行列がある $\Rightarrow (A\vec{x} = \vec{0} \Rightarrow \vec{x} = \vec{0})$

まとめ $\det(A) = 0$

$\Rightarrow$ 逆行列はない…

定義 Aが正則とは

$$AX = XA = I_2$$

を満たす2次正方行列Xが存在する。

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① $\det(A) \neq 0$

② A：正則

①⇒②　$A^{-1} = \dfrac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(②⇒①) ≡ 対偶

not ① ⇒ not ②

$\det(A) = 0$　逆行列が存在しない。

斉次方程式。

③ $(A\vec{x} = \vec{0} \Rightarrow \vec{x} = \vec{0})$

自明解。

③⇒① ≡ not ① ⇒ not ③

$\det(A) = 0 \Rightarrow A\vec{x} = \vec{0}$ かつ $\vec{x} \neq \vec{0}$ をみたす $\vec{x}$ がある。

(x_n, y_n)

$$\begin{cases} x_{n+1} = a x_n + b y_n + \alpha \\ y_{n+1} = c x_n + d y_n + \beta \end{cases}$$

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix} + \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \vec{0}$ の場合を考える.

$\vec{x}_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$

$$\boxed{\vec{x}_{n+1} = A \vec{x}_n .}$$

$$\vec{x}_0 \xrightarrow{A\cdot} \vec{x}_1 \xrightarrow{A\cdot} \vec{x}_2 \longrightarrow \cdots \xrightarrow{A\cdot} \vec{x}_n$$

初期値.

$$\vec{x}_n = A^n \vec{x}_0$$

A^n をどのように求めるか？

2通りの方法.

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $A^2 = A \cdot A$

$$A^2 - (a+d)A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} - (a+d)\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} a^2 + bc & \boxed{ab + bd} \\ \boxed{ac + cd} & bc + d^2 \end{pmatrix} - \begin{pmatrix} a(a+d) & b(a+d) \\ c(a+d) & d(a+d) \end{pmatrix}$$

$ab + bd = b(a+d)$

$ac + cd = c(a+d)$

$$= \begin{pmatrix} bc - ad & 0 \\ 0 & bc - ad \end{pmatrix}$$

$$= -|A| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^2 - (a+d)A = -|A| I_2$$

ケーリー・ハミルトンの定理.

$$A^2 - (a+d)A + |A| I_2 = O_2$$

$O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ zero 行列.

CT 218 page

$$a_{n+2} - 4a_{n+1} - 5a_n = 0$$

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \quad \longleftrightarrow$$

$$A^2 - 4A - 5I_2 = O$$

$$\lambda^2 - 4\lambda - 5 = 0 \quad \leftarrow \text{Aの固有方程式.}$$

$$\| $$

$$(\lambda - 5)(\lambda + 1) \qquad \lambda = 5, -1$$

$$A^2 - (5 + (-1))A + 5 \cdot (-1) I_2 = O_2$$

$$\begin{cases} A^2 - 5A = (-1)(A - 5I_2) \\ A^2 + A = 5(A + I_2) \end{cases}$$

$$\begin{cases} A(A - 5I_2) = (-1)(A - 5I_2) \\ A(A + I_2) = 5(A + I_2) \end{cases}$$

$$\begin{aligned} A^2(A - 5I_2) &= A \cdot A(A - 5I_2) \\ &= A \cdot (-1)(A - 5I_2) \\ &= (-1)A(A - 5I_2) \\ &= (-1) \cdot (-1)(A - 5I_2) \\ &= (-1)^2 (A - 5I_2) \end{aligned}$$

$$A^3(A - 5I_2) = A \cdot A^2 (A - 5I_2)$$

$$= A \cdot (-1)^2 (A - 5I_2) = (-1)^2 \cdot (-1)(A - 5I_2)$$

$$\begin{cases} A(A-5I_2) = (-1)(A-5I_2) \\ A(A+I_2) = 5\,(A+I_2) \end{cases}$$

$$\begin{cases} A^n(A-5I_2) = (-1)^n(A-5I_2) \\ A^n(A+I_2) = 5^n(A+I_2) \end{cases}$$

$$\begin{array}{rl} & \begin{cases} A^{n+1} - 5A^n = (-1)^n(A-5I_2) \\ A^{n+1} + A^n = 5^n(A+I_2) \end{cases} \\ - & \\ \hline \end{array}$$

$$-6A^n = (-1)^n(A-5I_2) - 5^n(A+I_2)$$

$$A^n = -\frac{1}{6}\left\{ \underline{\qquad\qquad\qquad} \right\}$$

固有方程式に重根がある場合は問題.

220page.

<u>演習</u> $A = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$ に対して

A^n を求める.

命題 P, Q, R

≡ P, Q, R の TF によらず値が等しい

$$P \text{ OR } (Q \text{ AND } R) \equiv (P \text{ OR } Q) \text{ and } (P \text{ OR } R)$$

P	Q	P OR Q	P and Q
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

T : 真 TRUE

F : 偽 False

P	Q	R	T_z	T_b
T	T	T	T	T
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Q and R	T_z	P OR Q	P OR R	T_b

「数学力をどうつけるか」 芳沢光雄（ちくま新書）

演習 8.9 (1)

$$z = x^2 + xy + y^2 - 4x - 2y$$

$$\begin{cases} z_x = 2x + y \cdot 1 + 0 - 4 - 0 \\ \quad = 2x + y - 4 = 0 \\ z_y = 0 + x \cdot 1 + 2y - 0 - 2 \\ \quad = x + 2y - 2 = 0 \end{cases}$$

極小

$$\begin{cases} 2x + y = 4 \\ x + 2y = +2. \end{cases}$$

$$\begin{array}{rrrr} & 2 & 1 & 4 \\ -) & 2 & 4 & +4 \\ \hline & 0 & -3 & 0 \end{array}$$

$y = 0$, $x = -2y + 2 = 2$

---

(2) $z = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

$$\begin{cases} z_x = 4x^3 + 0 - 4x + 4y \cdot 1 - 0 \\ \quad = 4x^3 - 4x + 4y = 0 \\ z_y = 0 + 4y^3 - 0 + 4x \cdot 1 - 4y \\ \quad = 4y^3 + 4x - 4y = 0 \end{cases}$$

$$\begin{cases} x^3 - x + y = 0 \quad \cdots ① \\ y^3 + x - y = 0 \quad \cdots ② \end{cases}$$

①+② $\quad x^3 + y^3 = 0$ が… 必・要.

$$(x+y)(x^2 - xy + y^2) = 0$$

Ⅰ $x + y = 0$ のとき.

$y = -x$ を① に代入

$$x^3 - 2x = 0$$

$$x(x^2 - 2) = 0$$

$$x = 0, \pm\sqrt{2}.$$

$$y = 0, \mp\sqrt{2}.$$

$$(0, 0), (\pm\sqrt{2}, \mp\sqrt{2})$$

$\alpha\beta = 0$

$\Updownarrow$

$\alpha = 0$

OR

$\beta = 0$

Ⅱ $x^2 - xy + y^2 = 0$

$$\| $$

$$\left(x - \frac{y}{2}\right)^2 + \frac{3}{4}y^2$$

$A, B \geqq 0$

$A + B = 0$

$\Downarrow$

$A = B = 0$

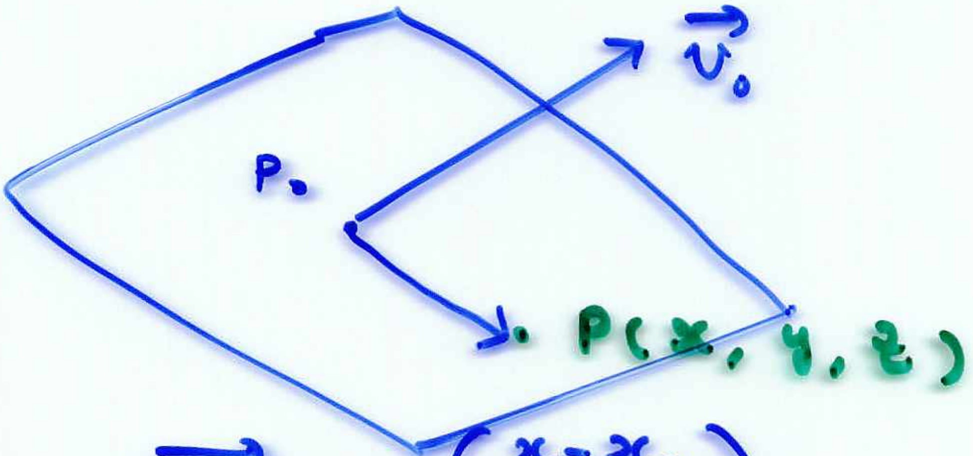
$x = \frac{y}{2}$ かつ $y = 0$

$\rightsquigarrow x = y = 0$

$\mathbb{R}^3$ 中 平面

通る1点 $P_0 = (x_0, y_0, z_0)$ + 法線方向.

$\longrightarrow \vec{v_0} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \neq \vec{0}$



$$\overrightarrow{P_0P} = \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}$$

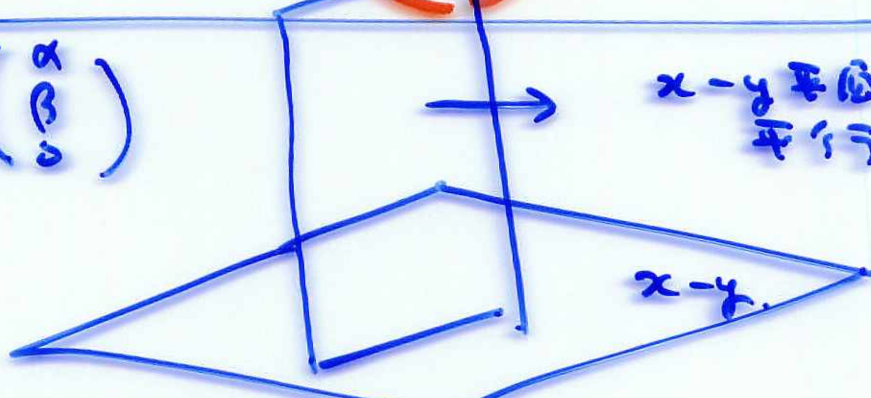
$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = 0$$

$$\alpha (x - x_0) + \beta (y - y_0) + \gamma (z - z_0) = 0$$

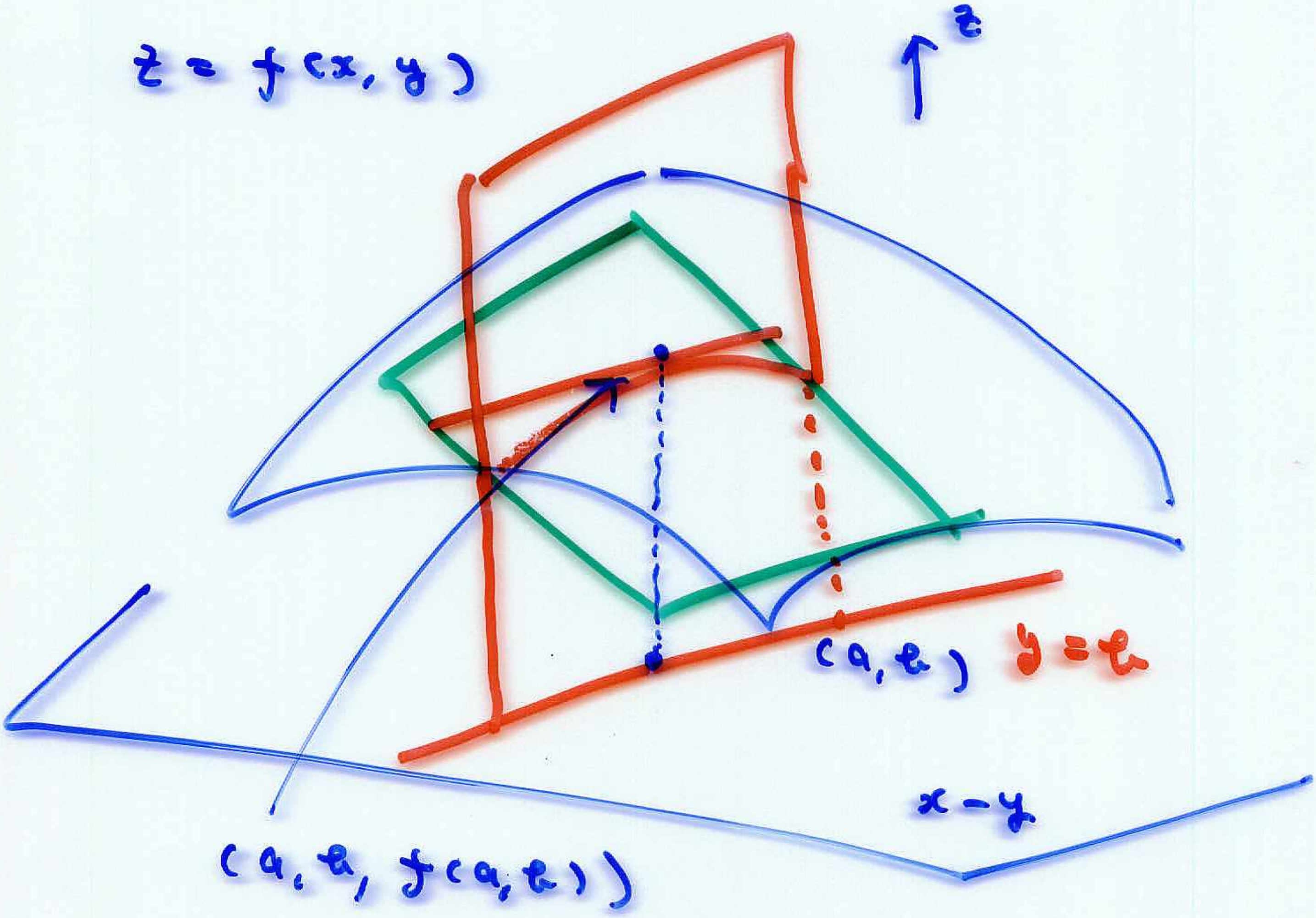
$\gamma \neq 0$ のとき.

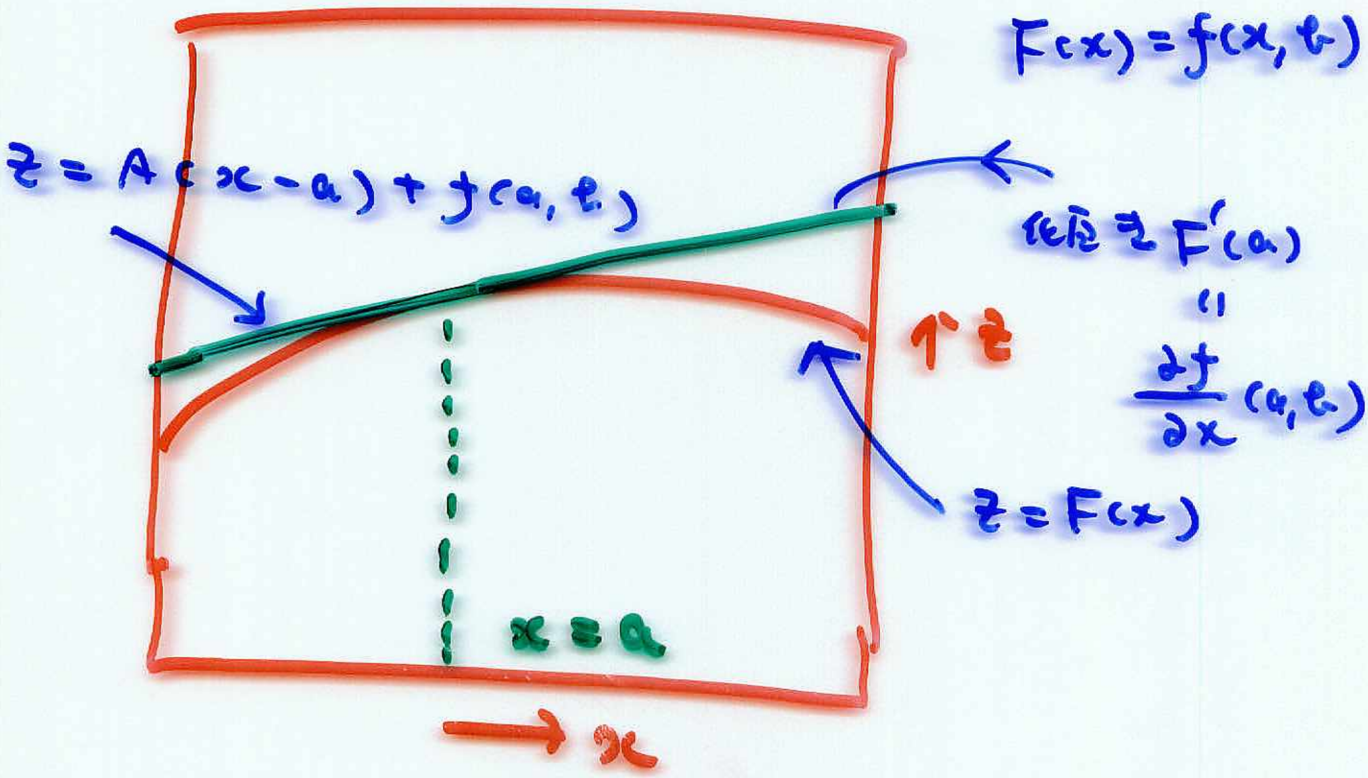
$$z = \underbrace{-\frac{\alpha}{\gamma}}_{=A} (x - x_0) \underbrace{-\frac{\beta}{\gamma}}_{=B} (y - y_0) + z_0$$

$\gamma = 0$ $\quad \begin{pmatrix} \alpha \\ \beta \\ 0 \end{pmatrix} \longrightarrow$ $x-y$ 平面に平行



平面は $x-y$ に垂直.


z = f(x, y)
z
(a, b)
y = b
x-y
(a, b, f(a, b))


F(x) = f(x, b)
z = A(x − a) + f(a, b)
傾き = F'(a)
= ∂f/∂x (a, b)
z
z = F(x)
x = a
x

接平面

$$z = A(x-a) + B(y-b) + f(a,b)$$

とおく

$y=b$ の断面

$$z = A(x-a) + f(a,b)$$

$$\boxed{A = \frac{\partial f}{\partial x}(a,b)}$$

同様に $\quad B = \frac{\partial f}{\partial y}(a,b)$

<u>まとめ</u>

$$z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$