証明が必要.

$$\lim_{t \to +\infty} \left(1 + \frac{1}{t}\right)^t = e$$

$$\lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^n = e$$ <u>eの定義</u>

$\alpha > 0$

$$\left(1 + \frac{\alpha}{n}\right)^n = \left\{\left(1 + \frac{1}{\frac{n}{\alpha}}\right)^{\frac{n}{\alpha}}\right\}^{\alpha}$$

$t_n = \frac{n}{\alpha}$ とおくと $n \to +\infty$ のとき $t_n \to +\infty$

$$s_n = \left(1 + \frac{1}{t_n}\right)^{t_n} \longrightarrow e \quad (n \to \infty)$$

$\lim_{t \to \infty} f(t) = A \iff$ 任意の数列 $\{t_n\}$

$t_n \to +\infty$ のとき

$\Longrightarrow f(t_n) \to A$

$(n \to +\infty)$

---

s^{α} は $s = e$ で連続

$$s_n^{\alpha} \longrightarrow e^{\alpha} \quad (n \to +\infty.)$$

年利 α

$1 \longrightarrow 1+\alpha$

$1 \longrightarrow \left(1+\frac{\alpha}{2}\right)^2$ 半年複利

$\left(1+\frac{\alpha}{3}\right)^3$ $\frac{1}{3}$年複利

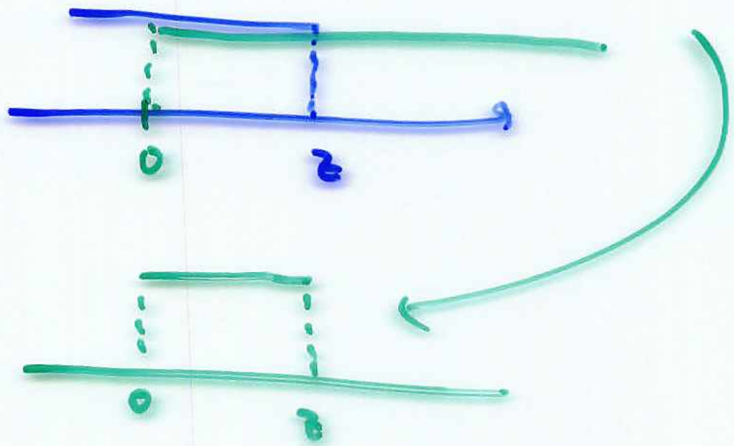
$\left(1+\frac{\alpha}{n}\right)^n$ $\frac{1}{n}$年 —

$\downarrow$

e^{α} 瞬間複利.

$z \geq 0$

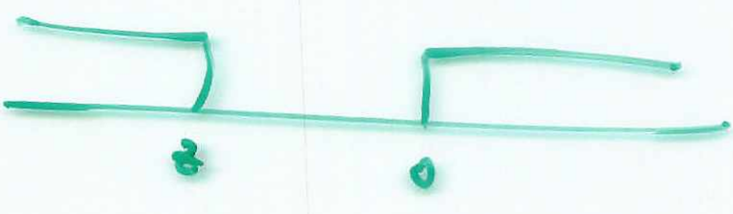
$Y(z-y) \cdot Y(y)$



μ

m

$z < 0$



$$\int_0^{+\infty} f(z) = 1$$

$$\shortparallel$$

$$\frac{\alpha^{\lambda+\mu}}{\Gamma(\lambda)\Gamma(\mu)} \cdot I \cdot \underbrace{\int_0^{+\infty} z^{\lambda+\mu-1} e^{-\alpha z}\, dz}_{\shortparallel \atop \frac{\Gamma(\lambda+\mu)}{\alpha^{\lambda+\mu}}}$$

$$\int_0^{+\infty} \frac{\alpha^{\lambda+\mu}}{\Gamma(\lambda+\mu)} z^{\lambda+\mu-1} e^{-\alpha z}\, dz = 1$$

Z : $\lambda+\mu$, α の Γ に従う.

$$\overset{B(\lambda,\mu)}{\overset{\shortparallel}{I}} \cdot \frac{\Gamma(\lambda+\mu)}{\Gamma(\lambda)\Gamma(\mu)} = 1$$

$x^2 = y$

$$dy = 2x\, dx$$
$$= 2\sqrt{y}\, dx$$
$$dx = \frac{1}{2\sqrt{y}}\, dy$$

$$f_X(\xi) = \int_{-\infty}^{+\infty} \underbrace{e^{ix\xi}}_{\cos x\xi + \sqrt{-1}\sin x\xi} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx.$$

$$= \int_{-\infty}^{+\infty} \cos x\xi \, \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx + \sqrt{-1} \underbrace{\int_{-\infty}^{+\infty} \sin x\xi \, \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx}_{= 0}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \cos x\xi \, \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx$$

$$\sum_{n=1}^{+\infty} \int_a^b |f_n(x)|\, dx < +\infty$$

$$\Longrightarrow \int_a^b \sum_{n=1}^{+\infty} f_n(x)\, dx$$

$$= \sum_{n=1}^{+\infty} \int_a^b f_n(x)\, dx.$$

$$\int_{-\infty}^{+\infty} \underbrace{e^{-\frac{x^2}{2}} \cdot e^{ix\xi}}_{\displaystyle \parallel \atop \displaystyle e^{-\frac{1}{2}(x-\frac{i}{2}\xi)^2 - \frac{1}{2}\xi^2}} dx$$

$$= e^{-\frac{1}{2}\xi^2} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\boxed{(x - \frac{i}{2}\xi)}^2} dx \qquad (x-\tfrac{i}{2}\xi) = \tau$$

$$= e^{-\frac{1}{2}\xi^2} \underbrace{\int_{-\infty}^{+\infty} e^{-\frac{1}{2}\tau^2} d\tau}_{\displaystyle \parallel \atop \displaystyle \sqrt{2\pi}}$$

難かしい内容を
使う.

$$Z = \sigma X + m \qquad \sigma > 0$$

$$\varphi_Z(\xi) = e^{im\xi} \varphi_X(\sigma\xi)$$

$$W \sim N_{m_1+\cdots+m_n,\ \sigma^2}$$

$$\sigma^2 = \sigma_1^2 + \cdots + \sigma_n^2$$

$$\varphi_Z(\xi) = \varphi_W(\xi)$$

$$Z \sim f(z),\quad W \sim g(w)$$

$$\varphi_Z(\xi) \equiv \varphi_W(\xi)$$

$$\Longrightarrow \quad \int_a^b f(z) = \int_a^b g(w)\,dw$$

$f(z) \equiv g(z)$ とはいえない.

<u>レポート</u>

1) $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ 収束することを示せ.

2) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ のとき

$\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t = e$ を示せ.

X, Y 集合

$$f : X \longrightarrow Y$$
$$\quad\quad\quad\quad\; \cup$$
$$\quad\quad\quad\quad\; A \text{ 部分集合}$$

Aの逆像

$$f^{-1}(A) = \{ x \in X ;\ f(x) \in A \}$$
$$\cap$$
$$X$$

$A_1, A_2 \subset Y$.

$$f^{-1}(A_1 \cap A_2) = f^{-1}(A_1) \cap f^{-1}(A_2)$$

$$\boxed{\begin{array}{l} y \in A_1 \cap A_2 \\ \Longleftrightarrow y \in A_1 \text{ かつ } y \in A_2 \end{array}}$$

$$\begin{aligned}
& x \in f^{-1}(A_1 \cap A_2) \\
\Longleftrightarrow\ & f(x) \in A_1 \cap A_2 \\
\Longleftrightarrow\ & f(x) \in A_1 \text{ かつ } f(x) \in A_2 \\
\Longleftrightarrow\ & x \in f^{-1}(A_1) \text{ かつ } x \in f^{-1}(A_2) \\
\Longleftrightarrow\ & x \in f^{-1}(A_1) \cap f^{-1}(A_2)
\end{aligned}$$

$X \longrightarrow Y$ $\qquad$ $A_1, A_2 \subset Y$.

$y \in A_1 \cup A_2$

$\iff y \in A_1$ OR

$y \in A_2$

レポート 3)

$$f^{-1}(A_1 \cup A_2)$$

$$= f^{-1}(A_1) \cup f^{-1}(A_2)$$

を示せ.