復習のこと

$$e^{ix\xi} e^{-\lambda x} = e^{(i\xi-\lambda)x}$$

$$\left\{ \frac{1}{i\xi-\lambda} e^{(i\xi-\lambda)x} \right\}' = e^{(i\xi-\lambda)x}$$

$$\int_a^b G F' \, dx = \left[GF \right]_a^b - \int_a^b G' F \, dx.$$

$$\lim_{x \to +\infty} \frac{x^k}{e^x} = 0 \qquad x > 0$$

Taylor展開を用いる.

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{1}{(k+1)!} x^{k+1} + \cdots$$

$$> \frac{1}{(k+1)!} x^{k+1}$$

$$\rightsquigarrow \quad \frac{1}{e^x} < \frac{(k+1)!}{x^{k+1}}$$

$$0 < \frac{x^k}{e^x} < \frac{1}{x} (k+1)! \quad \frac{1}{x} \to 0$$

Heaviside 関数

$$f(x) = \lambda e^{-\lambda x} Y(x)$$

$$Y(x) = \begin{cases} 1 & (x \geq 0) \\ 0 & (x < 0) \end{cases}$$

$$\hat{f}(\xi) = \lambda \int_0^{+\infty} e^{ix\xi} e^{-\lambda x} dx$$

$$= \lambda \left[\frac{1}{i\xi - \lambda} e^{(i\xi-\lambda)x} \right]_0^{+\infty}$$

$$0 \le \left| e^{(i\xi - \lambda)x} \right| \le e^{-\lambda x} \searrow 0$$

$$\left| e^{i\xi x} \right| = 1$$

$$e^{(i\xi - \lambda)x} \longrightarrow 0 \quad (x \longrightarrow +\infty)$$

$$\hat{f}(\xi) = \lambda \left(0 - \frac{1}{i\xi - \lambda} \right)$$

$$= \frac{\lambda}{i\xi - \lambda}$$

定理 積分と微分の交換.

$$\int_a^b |F(x, \lambda)| \, dx < +\infty \quad \text{各 } \lambda.$$

$$\int_a^b \left| \frac{\partial}{\partial \lambda} F(x, \lambda) \right| dx < +\infty \quad \text{各 } \lambda$$

$$\Longrightarrow \frac{d}{d\lambda} \int_a^b F(x, \lambda) \, dx = \int_a^b \frac{\partial}{\partial \lambda} F(x, \lambda) \, dx$$

$$\left(e^{ix\xi} \right)' = i x e^{ix\xi}$$

wrT ξ

with respect to ~

par rapport à ...

$$\left(\frac{1}{\lambda - i\xi} \right)' = -\frac{1}{(\lambda - i\xi)^2} \cdot (-i)$$

$$\boxed{f(t) = Y(t)\cdot \lambda e^{-\lambda t}}$$

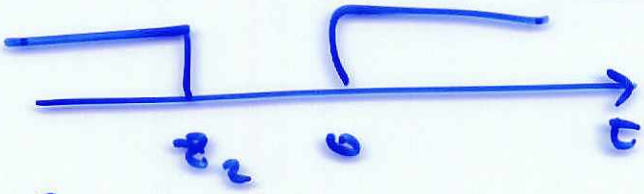
$$Y(t) = \begin{cases} 1 & (t \geqq 0) \\ 0 & (t < 0) \end{cases}$$

$$Y(z_2 - t) = \begin{cases} 1 & z_2 - t \geqq 0 \\ 0 & \text{Sinon} \end{cases} \rightarrow \text{i.e. } t \leqq z_2$$

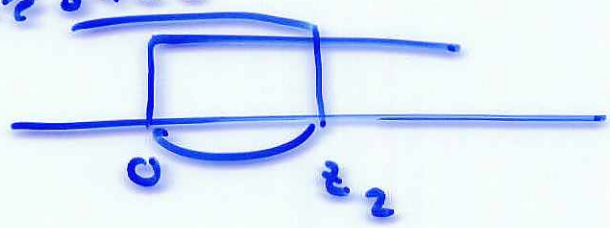
$z_2 < 0$ のとき

AND

$$\{t ;\ z_2 - t \geqq 0 \text{ and } t \geqq 0\} = \phi$$



$z_2 \geqq 0$ のとき



$$Z_3 = X_1 + X_2 + X_3 = Z_2 + X_3$$

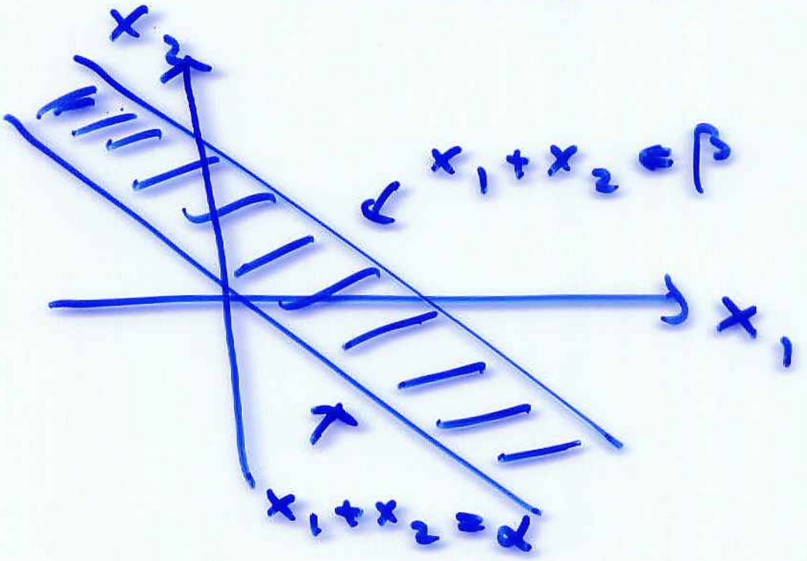
$$Z_2 = X_1 + X_2$$

- $\boxed{X_1, X_2, X_3 : \text{独立}} \Rightarrow z_2$ と X_3 は独立

$$P(a_1 \leq X_1 \leq b_1,\ a_2 \leq X_2 \leq b_2,\ a_3 \leq X_3 \leq b_3)$$
$$= P(a_1 \leq X_1 \leq b_1)\cdot P(a_2 \leq X_2 \leq b_2)\cdot P(a_3 \leq X_3 \leq b_3)$$

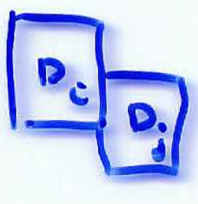
To be shown

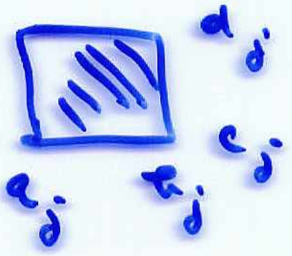
$$P(\alpha \le Z_2 \le \beta,\ a \le X_3 \le b)$$
$$= P(\alpha \le Z_2 \le \beta) \cdot P(a \le X_3 \le b)$$



$$D = \{(x_1, x_2);\ \alpha \le x_1 + x_2 \le \beta\}$$
$$= \bigcup_{j=1}^{+\infty} D_j \qquad D_j = [a_j, b_j] \times [c_j, d_j]$$

$i \ne j$
$$P(D_i \cap D_j) = 0$$





$$P(D \times \underbrace{[a, b]}_{x_3})$$
$$= \sum P(\bigcup D_j \times [a, b])$$
$$= \sum_j P(D_j \times [a, b])$$
$$= \sum_j P(D_j) \times P([a, b])$$
$$= P(D) \times P([a, b])$$

ごまかしあり

次回にノートを
配布する予定.

$D_j = I_j \times J_j$

I_j, J_j は閉とは
限らぬ区間
$D_j \cap D_i = \emptyset$ とする.

$$\int_0^{+\infty} \left| \lambda e^{ix\xi} e^{-\lambda x} \right| dx$$

$$= \lambda \int_0^{+\infty} e^{-\lambda x} dx < +\infty$$

$$\int_0^{+\infty} \left| \lambda i x e^{ix\xi} e^{-\lambda x} \right| dx$$

$$= \lambda \int_0^{+\infty} x e^{-\lambda x} dx < +\infty$$

だから交換できる

$$\left\{ \frac{1}{(\lambda - i\xi)^2} \right\}' = \frac{(-2)\cdot(-i)}{(\lambda - i\xi)^3} \times$$

$$= \frac{2i}{(\lambda - i\xi)^3}$$

メモ

$$\lambda \int_0^{+\infty} x^{n-1} e^{ix\xi} e^{-\lambda x}$$

$$= \frac{\lambda (n-1)!}{(\lambda - i\xi)^n}$$

$\xi = 0$ とすれば

$$\int_0^{+\infty} x^{n-1} \cdot \overbrace{\lambda e^{-\lambda x}}^{\text{pdf}} dx$$

$$= \frac{(n-1)!}{\lambda^{n-1}}$$

$$\rightsquigarrow \quad E[X^{n-1}] = \frac{(n-1)!}{\lambda^{n-1}}$$

$x_1, \dots, x_n$: 独立

$$x_i \sim f_i(x_i) \text{ pdf.}$$

$$z = x_1 + \cdots + x_n \sim h(z)$$

$$\hat{h}(\xi) = \hat{f}_1(\xi)\,\hat{f}_2(\xi) \cdots \hat{f}_n(\xi)$$

$$\int_0^{+\infty} e^{ix\xi} \frac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!}\, dx$$

$$= \left(\frac{\lambda}{\lambda - i\xi}\right)^n \Rightarrow \left(\frac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!} Y(x)\right)^{\wedge} \equiv \hat{h}(\xi)$$

$$\int_{\mathbb{R}} |f_1|\, dx < +\infty, \quad \int_{\mathbb{R}} |f_2|\, dx < +\infty$$

$$\hat{f}_1 \equiv \hat{f}_2 \Longrightarrow \int_a^b f_1\, dx = \int_a^b f_2\, dx$$

全ての a, b.
