A : $n \times n$ 行列

A : 正則 $\Longleftrightarrow$ $\mathrm{rank}(A) = n$

$\Longleftrightarrow$ $\det(A) \neq 0$ $\Longleftrightarrow$ $\left(A\vec{x} = \vec{0} \Rightarrow \vec{x} = \vec{0} \right)$

$\boxed{\det(A) = 0 \Longleftrightarrow A\vec{x} = \vec{0} \text{ を満たす } \vec{x} \neq \vec{0} \text{ 有り}}$

A : 実 のとき

$\vec{x} \in \mathbb{R}^n$

A : 実対称.

A の固有値 $\in \mathbb{R}$.

$B = \lambda I_n - A$

$B\vec{x} = \vec{0}$ とする $B(A\vec{x}) = \lambda B\vec{x}$

$= \vec{0}$

A : 固有値 α.

$A\vec{v} = \alpha\vec{v} \Longrightarrow A(\lambda\vec{v}) = \alpha \cdot \lambda\vec{v}$

A : $m \times n$ 実. $\vec{x} \in \mathbb{R}^n$, $\vec{y} \in \mathbb{R}^m$

$$(\underbrace{A\vec{x}}_{\in \mathbb{R}^m}, \vec{y}) = (\vec{x}, {}^tA\vec{y})$$

$$A = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$$ ${}^tA = A$ Aは対称 symmetric

$$\left(A\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} \right) = ax^2 + 2cxy + by^2$$

bi-linear form

le biscuit

$$\Phi_A(\lambda) = \det(\lambda I_2 - A) = \begin{vmatrix} \lambda - a & -c \\ -c & \lambda - b \end{vmatrix}$$

$$= \lambda^2 - (a+b)\lambda + (ab - c^2) = 0$$

固有方程式

$$\boxed{A\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix} \text{ と対応}}$$

discriminant

$$D = (a+b)^2 - 4(ab - c^2)$$
$$= (a-b)^2 + 4c^2 \geqq 0$$

$$D = 0 \iff a = b,\ c = 0 \iff A = aI_2$$

以下 $D > 0$, 固有値 α, β とする $\underline{\alpha \neq \beta}$.

α の固有ベクトル $\vec{p_1}$

β の固有ベクトル $\vec{p_2}$

大きさを1にできる.

$${}^tA = A$$

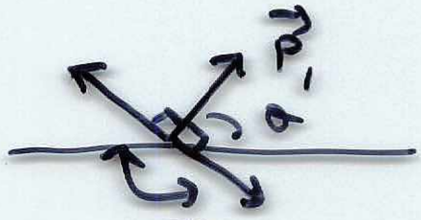
$$(A\vec{p_1}, \vec{p_2}) = (\vec{p_1}, A\vec{p_2}) = (\vec{p_1}, \beta\vec{p_2})$$

$$(A\vec{p_1}, \vec{p_2}) = (\alpha\vec{p_1}, \vec{p_2}) = \alpha(\vec{p_1}, \vec{p_2})$$

$$(\vec{p_1}, \beta\vec{p_2}) = \beta(\vec{p_1}, \vec{p_2})$$

$$\rightsquigarrow \underbrace{(\alpha - \beta)}_{\neq 0} (\vec{p_1}, \vec{p_2}) = 0$$

$$\rightsquigarrow (\vec{p_1}, \vec{p_2}) = 0$$



90°回転

$\vec{p}_1 \curvearrowright \vec{p}_2$

$\vec{p}_2$ は 2つの 可能性あり

$(\vec{p}_1, \vec{p}_2) = 0, \quad \|\vec{p}_2\| = 1$

$$P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad \vec{p}_1 = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

回転の行列.

$$ {}^tP = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = P^{-1}$$

直接計算.

$$\boxed{{}^tPP = P\,{}^tP = I_2}$$

が従う.

$$AP = P \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$$

$$\begin{aligned} A(\vec{p}_1\ \vec{p}_2) &= (A\vec{p}_1\ \ A\vec{p}_2) \\ &= (\boxed{\alpha\vec{p}_1}\ \ \beta\vec{p}_2) \\ &= (\vec{p}_1\ \vec{p}_2) \begin{pmatrix} \boxed{\begin{matrix}\alpha \\ 0\end{matrix}} & \begin{matrix}0 \\ \beta\end{matrix} \end{pmatrix} \end{aligned}$$

$$(\vec{a}_1 \cdots \vec{a}_d) \begin{pmatrix} c_1 \\ \vdots \\ c_d \end{pmatrix} = c_1 \vec{a}_1 + \cdots + c_d \vec{a}_d$$

$$AP = P\begin{pmatrix}\alpha & 0\\ 0 & \beta\end{pmatrix}$$

$$P = (\vec{p}_1\ \vec{p}_2)$$

$$\|\vec{p}_1\| = \|\vec{p}_2\| = 1$$

$$(\vec{p}_1, \vec{p}_2) = 0 \qquad \vec{p}_1 \xrightarrow{90^\circ} \vec{p}_2$$

^tP : (@ ~~I~~ 2_1.

det $(P) = 1$

2'' ok.

$$\left(\underbrace{{}^tP A}_{P\,{}^tP\ \|}\begin{pmatrix}x\\y\end{pmatrix}, {}^tP\begin{pmatrix}x\\y\end{pmatrix}\right) = \left(A\begin{pmatrix}x\\y\end{pmatrix}, \begin{pmatrix}x\\y\end{pmatrix}\right)$$

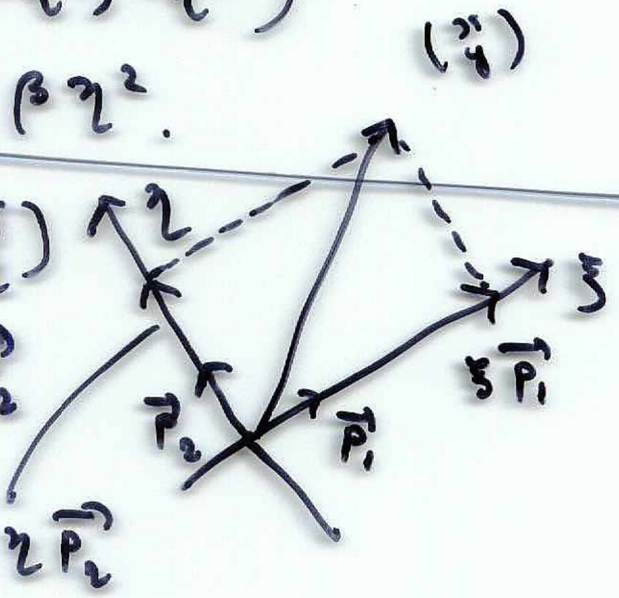
$${}^tP\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\xi\\ \eta\end{pmatrix}$$

$$= \left(\begin{pmatrix}\alpha & 0\\ 0 & \beta\end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix}, \begin{pmatrix}\xi\\ \eta\end{pmatrix}\right)$$

$$= \alpha\xi^2 + \beta\eta^2.$$

$$\begin{pmatrix}x\\y\end{pmatrix} = (\vec{p}_1\ \vec{p}_2)\begin{pmatrix}\xi\\ \eta\end{pmatrix}$$

$$= \xi\vec{p}_1 + \eta\vec{p}_2$$



$A = \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix}$ $\Phi_A(\lambda) = \left| \boxed{\begin{matrix} \lambda-5 & -3 \\ -3 & \lambda+3 \end{matrix}} \right|$

$= (\lambda-5)(\lambda+3) - 9$ $\lambda I_2 - A$

$= \lambda^2 - 2 - 24$

$= (\lambda - 6)(\lambda + 4)$

$\lambda = -4,\ 6$

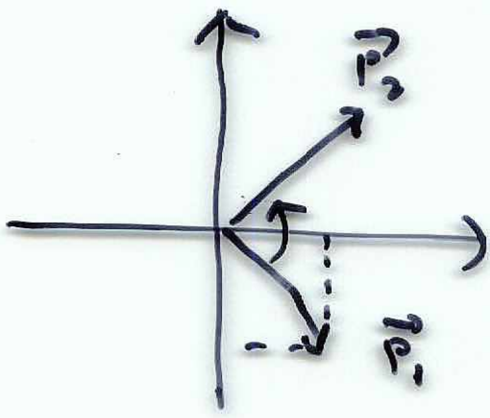
$\underline{\lambda = -4}$ $\begin{pmatrix} -9 & -3 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff 3x + y = 0$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -3x \end{pmatrix} = x\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\vec{p}_1 = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\underline{\lambda = 6}$ $\begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \iff x - 3y = 0$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3y \\ y \end{pmatrix} = y\begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\vec{p}_2 = \frac{1}{\sqrt{10}}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$



$P = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3 \\ -3 & 1 \end{pmatrix}$ 回転.

$AP = P\begin{pmatrix} -4 & 0 \\ 0 & 6 \end{pmatrix}$

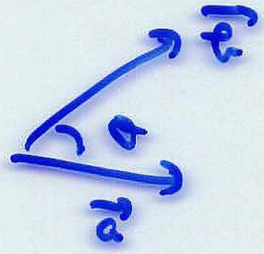
P: 回転

$(P\vec{x}, P\vec{y}) = (\vec{x}, \vec{y})$

$$(P\vec{x}, P\vec{y}) = (\vec{x}, {}^tPP\vec{y}) = (\vec{x}, \vec{y})$$

$${}^tPP = I_2$$

$$(\vec{a}, \vec{b}) = \|\vec{a}\| \cdot \|\vec{b}\| \cos\theta$$



P: n次実正方

$${}^tPP = P\,{}^tP = I_n$$

を満たす行列 $\longrightarrow$ 直交行列.

$\left(A\begin{pmatrix}x\\y\end{pmatrix}, \begin{pmatrix}x\\y\end{pmatrix}\right)$ が正定値

$\overset{\Longleftrightarrow}{\text{def.}}$ $\left(A\begin{pmatrix}x\\y\end{pmatrix}, \begin{pmatrix}x\\y\end{pmatrix}\right) > 0 \quad \begin{pmatrix}x\\y\end{pmatrix} \neq \vec{0}$

$\Longleftrightarrow$ $\alpha, \beta > 0$ $\Longleftrightarrow$ $a > 0, \det(A) > 0$

$z = f(x, y)$ 1) $f_x(a, b) = f_y(a, b) = 0$

2) $f_{xx}(a, b) > 0$

$f_{xx} f_{yy} - f_{xy}^2 > 0$

at (a, b)

$\Longrightarrow$ $f(a, b)$ 極小値

$\alpha, \beta > 0$ とする. $\xrightarrow{\text{十分}}$ 正定値

$\underbrace{\begin{pmatrix}\xi\\\eta\end{pmatrix} \neq \vec{0}}$ $\Longrightarrow$ $\alpha \xi^2 + \beta \eta^2 > 0$

$\Updownarrow$ $\begin{pmatrix}x\\y\end{pmatrix} \neq \vec{0}$

$\alpha \leq 0$ ($\beta \leq 0$ でも同様)

$(A\vec{P_1}, \vec{P_1}) = (\alpha \vec{P_1}, \vec{P_1})$

$= \alpha \|\vec{P}\|^2 = \alpha \leq 0$

$\alpha, \beta > 0$ が必要.

$\alpha, \beta > 0 \iff a > 0$, $\det(A)$

は読んでおく.

$$\Phi_A(\lambda) = (\lambda - \alpha)(\lambda - \beta) = \lambda^2 - (a+b)\lambda + \det(A)$$

$$\alpha\beta = \det(A)$$

$$\alpha + \beta = a + b$$