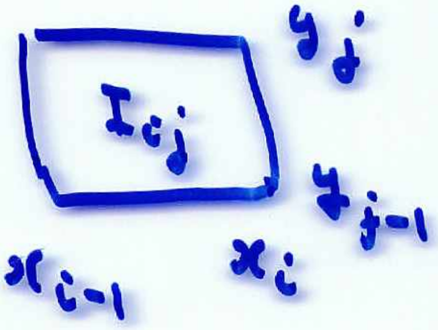
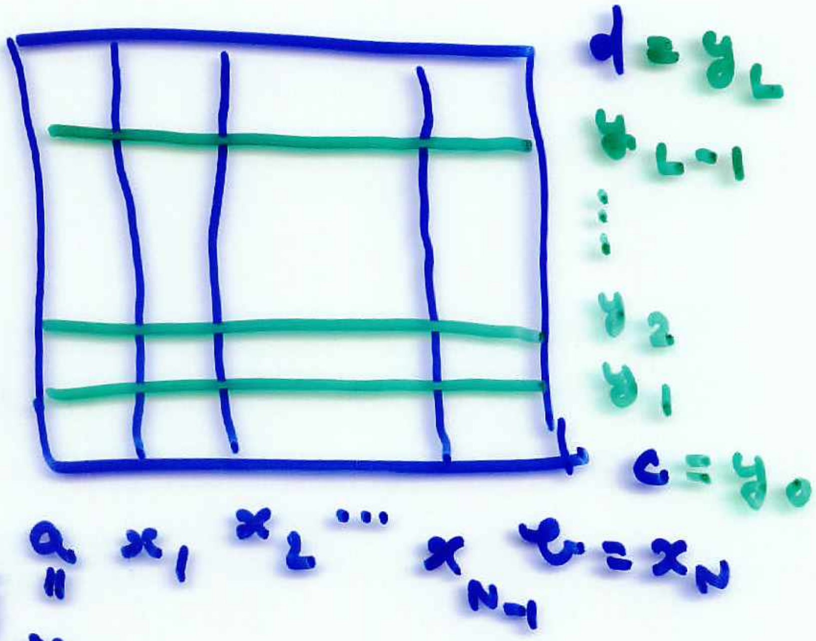
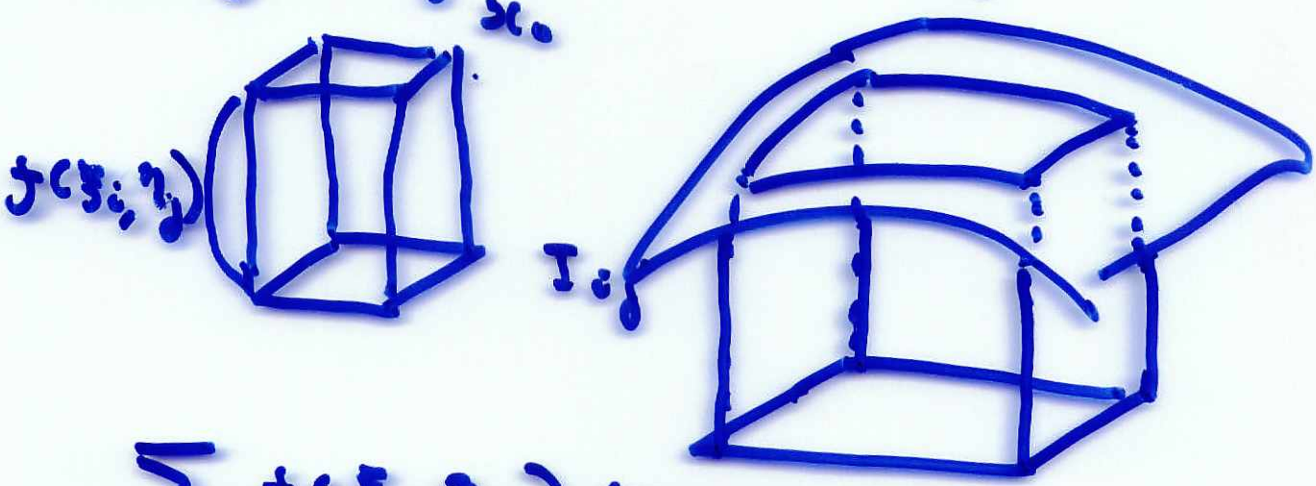
$f: [a,b]\times[c,d] \longrightarrow \mathbb{R}$





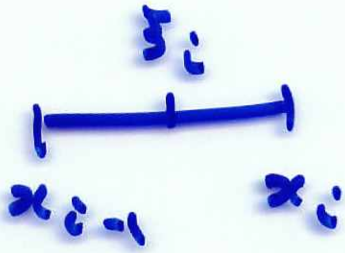
$(\xi_i, \eta_j) \in I_{ij}$



$$\sum_{i,j} f(\xi_i,\eta_j)\,(x_i - x_{i-1})\times(y_j - y_{j-1})$$

$$\longrightarrow \int_{[a,b]\times[c,d]} f(x,y)\,dx\,dy$$

$$\left(\begin{array}{l} \max_i (x_i - x_{i-1}) \longrightarrow 0 \\ \max_j (x_j - x_{j-1}) \longrightarrow 0 \end{array}\right.$$



$$\sum_i f(\xi_i)(x_i - x_{i-1}) \to \int_a^b f(x)\,dx$$

レポート $\int_0^{\frac{\pi}{2}} \sin x\,dx$ を計算する

n 等分

$F'(t) = f(t)$ である F を用いて

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

定理 f : 連続関数とする.
(Fubini)

$$\int_{[a,b]\times[c,d]} f(x,y)\,dx\,dy$$

$$= \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$$

$$= \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

X, Y : 連続的な確率変数.

確率密度 $f(x), g(y)$

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

$$f(x) \geqq 0, \quad \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

X, Y が独立

$$\Leftrightarrow P(a \leq X \leq b, c \leq Y \leq d)$$

$$= P(a \leq X \leq b) \cdot P(c \leq Y \leq d)$$

$D \subset \mathbb{R}^2$

$$P((X, Y) \in D) = \int_D f(x) g(y)\,dx\,dy.$$

D を区間で近似する.

$$D = \bigcup_{j=0}^{+\infty} I_j \quad \leftarrow \text{2次元の区間}$$

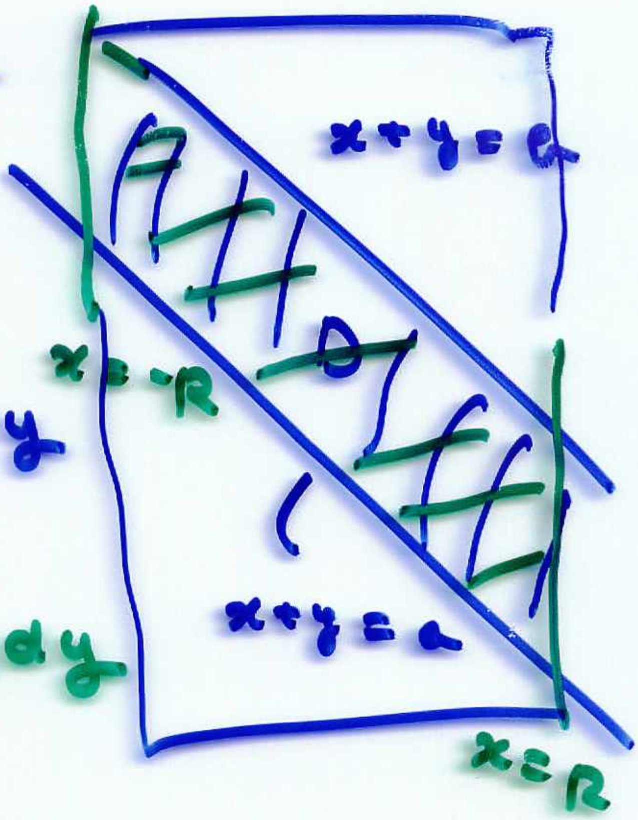
$$m(I_i \cap I_j) = 0 \qquad i \neq j.$$

$$P((X, Y) \in D) = \sum_j P((X, Y) \in I_j)$$

$$= \sum_j \int_{I_j} f(x) g(y)\,dx\,dy =$$

$z = x + y$ x, y 独立.

$P(a \le z \le t)$



$$= \int_D f(x)\,g(y)\,dx\,dy$$

$$= \lim_{R \to +\infty} \int_{D_R} f(x)\,g(y)\,dy$$

$$\int_{D_R} f(x)\,g(y)\,dy$$

$y = t - x$

$$= \int_{-R}^{R} \left(\int_{a-x}^{t-x} g(y)\,dy \right) f(x)\,dx$$

x

$y = a - x$

↑ Fubini

$y = t - \underbrace{x}_{\text{定まる}}$

$$= \int_{-R}^{R} \left(\int_{a}^{t} g(\overset{t-x}{\cancel{y}})\,d\overset{\tau}{\cancel{y}} \right) \times f(x)\,dx.$$

$dy = d\tau$

$R \to +\infty$

$$\longrightarrow \int_{-\infty}^{+\infty} \left(\int_{a}^{t} g(\overset{t-x}{\tau})\,d\underset{t}{\tau} \right) f(x)\,dx$$

Fubini

$$= \int_{a}^{t} \left(\int_{-\infty}^{+\infty} g(t - x)\,f(x)\,dx \right) d\tau$$

$z \geq 0$

$$h(z) = \int_0^z f(x)\, f(z-x)\, dx$$

$$= \int_0^z \frac{1}{\lambda} e^{-\lambda x} \cdot \frac{1}{\lambda} e^{-\lambda (z-x)}\, dx$$

$$= \frac{e^{-\lambda z}}{\lambda^2} \cdot \int_0^z dx$$

$$= \frac{z\, e^{-\lambda z}}{\lambda^2}.$$

$$h(z) = \begin{cases} \frac{1}{\lambda^2} z e^{-\lambda z} & (z \geq 0) \\ 0 & (z < 0) \end{cases}$$

Γ分布となる.

X, Y : 独立

$X \sim f(x)$, $Y \sim g(y)$

$$Z = X + Y \sim h(z)$$

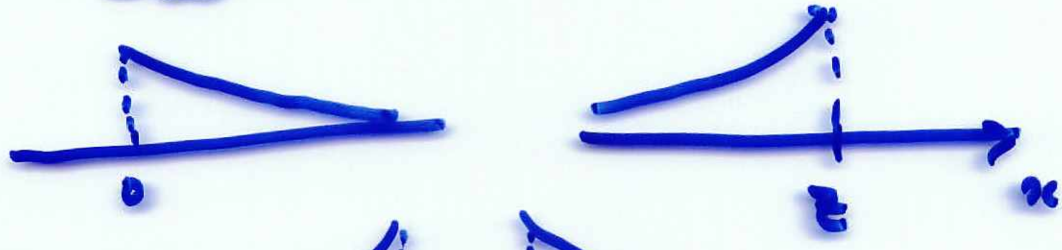
$$= \int_{-\infty}^{+\infty} f(x)\, g(z-x)\, dx$$

例 X, Y. 独立な 指数分布

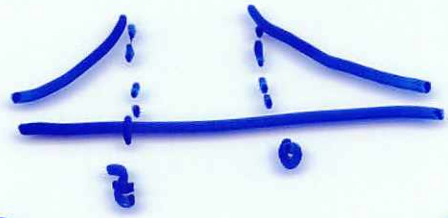
パラメータ λ

$$X \sim f(x) = \begin{cases} \frac{1}{\lambda} e^{-\lambda x} & (x \geq 0) \\ 0 & (x < 0) \end{cases}$$

$$h(z) = \int_{-\infty}^{+\infty} f(x)\, g(z-x)\, dx$$



$z < 0$



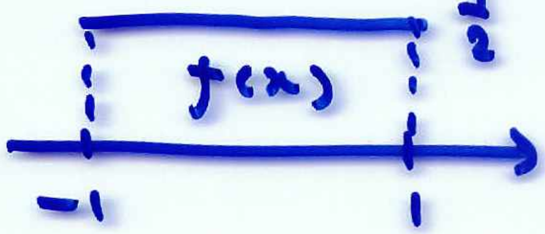
$\to h(z) = 0$

$z \geq 0$

X, Y : 一様分布, 独立.
同一の

$$X \sim f(x) = \begin{cases} \frac{1}{2} & (x \in [-1, 1]) \\ 0 & (\text{otherwise}) \end{cases}$$

$Z = X + Y$ の確率密度を計算.



$f(t-x)$

$t - x = 1$

$x = t - 1$

$t - x = -1$

$x = t + 1$