置換積分

$$\frac{d}{dx}F(x) = f(x)$$

$$\frac{d}{dt}F(x(t)) = F'(x(t))\,x'(t)$$
$$= f(x(t))\,x'(t)$$

$$\int_a^b f(x(t))\,x'(t)\,dt$$
$$= \Big[F(x(t))\Big]_a^b$$
$$= F(\underbrace{x(b)}_{=B}) - F(\underbrace{x(a)}_{=A})$$

$$\int_0^1 \frac{t}{1+t^2}\,dt \qquad (1+t^2)' = 2t$$

$$= \int_0^1 \frac{1}{1+t^2}\cdot\frac{1}{2}(1+t^2)'\,dt$$

$$= \frac{1}{2}\int_0^1 \frac{1}{1+t^2}\cdot(1+t^2)'\,dt$$

$$= \frac{1}{2}\Big[\log(1+t^2)\Big]_0^1$$

$f(x) = \frac{1}{x}$

$x(t) = 1+t^2$

$F(x) = \log x$

$$\int_a^b e^{-\frac{t^2}{2}} \cdot t \, dt \qquad e^{-x} = f(x)$$

$$x(t) = \frac{t^2}{2}$$

$$= \int_a^b e^{-\frac{t^2}{2}} \cdot t \, dt \qquad x'(t) = t$$

$$= \left[-e^{-\frac{t^2}{2}} \right]_a^b \qquad F(x) = -e^{-x}$$

$$\boxed{(e^{ct})' = c\,e^{ct}}$$

$A = x(a)$, $B = x(b)$

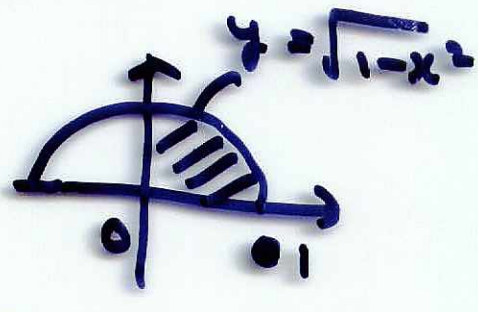
$$\int_A^B f(x)\,dx = \left[F(x)\right]_A^B$$

$$= F(B) - F(A)$$

公式

$$\int_A^B f(x)\,dx = \int_a^b f(x(t))\, x'(t)\,dt$$

$$\int_0^1 \sqrt{1-x^2}\,dx$$



$$= \int_0^{\frac{\pi}{2}} \cos\theta \cdot \cos\theta \, d\theta \qquad x = \sin\theta$$

$$= (*) \qquad x'(\theta) = \cos\theta$$

$\sin 0 = 0$, $\sin\frac{\pi}{2} = 1$

$$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \cos\theta$$

$$(*) = \int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\frac{\pi}{2}} \qquad (\sin 2\theta)' = \cos 2\theta \cdot 2$$

$$= \frac{\pi}{4}$$

$$(fg)' = f'g + fg'$$ integration by part

$$\int f'g\,dx = \int (fg)'\,dx - \int fg'\,dx$$
$$= fg - \int fg'\,dx$$

$$\int \log t\,dt = \int (t)'\log t\,dt$$
$$= t\log t - \int t\cdot\frac{1}{t}\,dt$$
$$= t\log t - t + C$$

$$\int_a^b f'g\,dx = [fg]_a^b - \int_a^b fg'\,dx.$$

連続に値をとる確率変数.

$f : \mathbb{R} \longrightarrow \mathbb{R}$ 各点で連続

$$\int_{-\infty}^{+\infty} f(t)\,dt = \lim_{\substack{M \to +\infty \\ L \to -\infty}} \int_{L}^{M} f(t)\,dt$$

X : $\mathbb{R}$に値をとる確率変数.

確率密度が $f(x)$

$f : \mathbb{R} \longrightarrow \mathbb{R}$

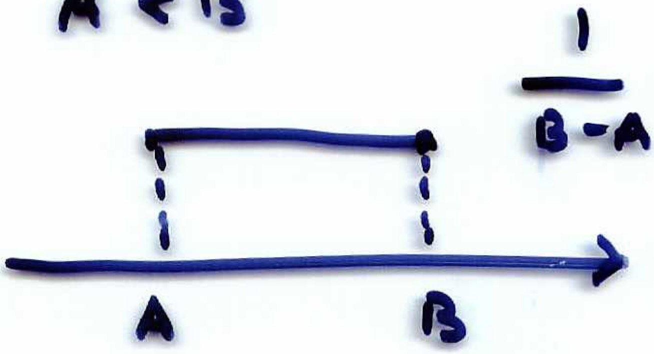
1) $f(t) \geqq 0$

2) $\displaystyle\int_{-\infty}^{+\infty} f(t)\,dt = 1$

3) $\displaystyle P(a \leqq X \leqq b) = \int_{a}^{b} f(t)\,dt.$

一様分布

$A < B$



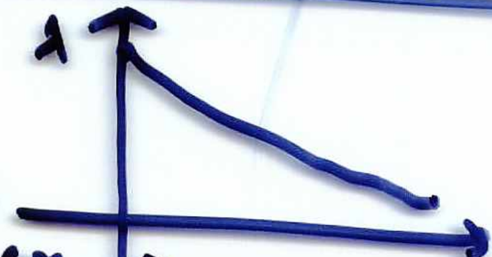
X:

確率密度

$$f(x) = \begin{cases} \dfrac{1}{B-A} & x \in [A, B] \\ 0 & (\text{otherwise}) \end{cases}$$

指数分布

$\lambda > 0$



$$f(x) = \begin{cases} \lambda e^{-\lambda x} & (x \geq 0) \\ 0 & (x < 0) \end{cases}$$

$$\int_0^{+\infty} f(x)\,dx = \lim_{N \to +\infty} \left[-e^{-\lambda x} \right]_0^N$$

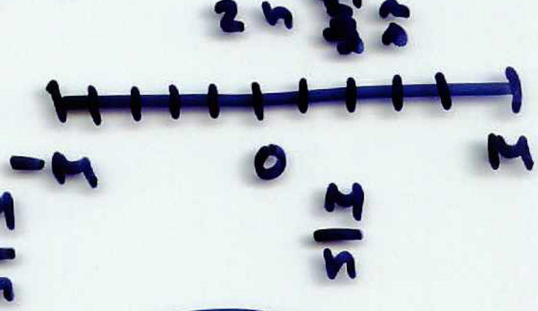
$$\left(-e^{-\lambda x}\right)' = \lambda e^{-\lambda x}$$

$$= \lim_{N \to \infty} \left[\underbrace{-e^{\overset{\to -\infty}{-\lambda N}}}_{\to 0} + 1 \right] = 1.$$

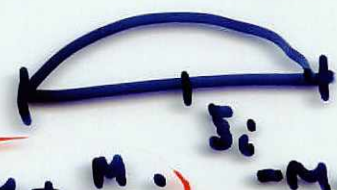
$$E(X) = \int_{-\infty}^{+\infty} x\, f(x)\, dx$$

$f(x)$: 確率変数 X の確率密度.

$$\fallingdotseq \int_{-M}^{M} x\, f(x)\, dx$$



$$\fallingdotseq \sum_i \xi_i\, f(\xi_i) \cdot \frac{M}{n}$$

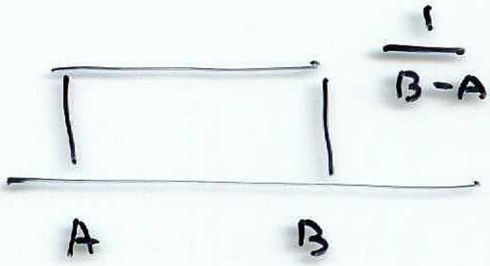


$*$ = $-M + \frac{M}{n} i$, $-M + \frac{M}{n}(i+1)$ = $**$

$$\fallingdotseq \sum_i \xi_i \times \boxed{\int_{-M+\frac{M}{n} i}^{-M+\frac{M}{n}(i+1)} f(x)\, dx}$$

$$= P(* \leqq X \leqq **)$$

一様分布.

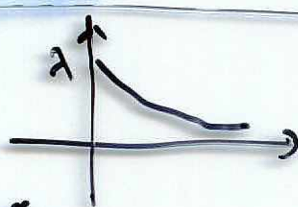


$$\int_{-\infty}^{+\infty} x\, f(x)\, dx$$

$$= \int_A^B x \frac{1}{B-A}\, dx = \frac{1}{B-A}\left[\frac{x^2}{2}\right]_A^B$$

$$= \frac{1}{B-A}\cdot\left(\frac{B^2}{2} - \frac{A^2}{2}\right) = \frac{1}{2}(B+A)$$

指数分布.



$$E(x) = \int_0^{+\infty} x\cdot\lambda e^{-\lambda x}\, dx$$

$$= \int_0^{+\infty} x\left(-e^{-\lambda x}\right)' dx$$

$$= \left[-x e^{-\lambda x}\right]_0^{+\infty} + \int_0^{+\infty} e^{-\lambda x}\, dx$$

$$= \int_0^{+\infty} e^{-\lambda x}\, dx = \left[-\frac{1}{\lambda} e^{-\lambda x}\right]_0^{+\infty}$$

$\lambda > 0$

$$\frac{x}{e^{\lambda x}} = \frac{1}{\lambda}\cdot\frac{\lambda x}{e^{\lambda x}} \to 0$$

$\lambda x \to +\infty$

$$= \frac{1}{\lambda}.$$

$$E(x) = \frac{1}{\lambda}.$$

$$\lim_{x \to +\infty} \frac{x^n}{e^x} = 0 \qquad (n = 1, 2, \cdots)$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \cdots + \frac{1}{n!}x^n + \cdots$$

$x > 0$

$$e^x > \frac{1}{(n+1)!}x^{n+1}$$

$$0 < \frac{x^n}{e^x} < x^n \cdot (n+1)!$$

$$= (n+1)! \frac{1}{x} \xrightarrow[x \to +\infty]{} 0$$

$\downarrow \qquad \downarrow$

$0 \qquad 0$

by はさみ打ち.

$$E(g(x)) = \int_{-\infty}^{+\infty} g(x)\, f(x)\, dx.$$

$$E(x^2) = \int_{-\infty}^{+\infty} x^2 f(x)\, dx.$$

2次のモーメント

$$V(x) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\, dx$$

$\mu = E(x)$ とおいて　x の分散

$$= \int_{-\infty}^{+\infty} (x^2 - 2\mu x + \mu^2) f(x)\, dx$$

$$= \int_{-\infty}^{+\infty} x^2 f(x)\, dx - 2\mu \int_{-\infty}^{+\infty} x f\, dx$$

$$+ \mu^2 \overset{=1}{\boxed{\int_{-\infty}^{+\infty} f(x)\, dx}}$$

$$= E(x^2) - 2\mu \overset{=\mu}{E(x)} + \mu^2$$

$$= E(x^2) - (E(x))^2$$

<u>まとめ</u>　$V(x) = E(x^2) - (E(x))^2$

一様分布に対して

1) $E(X^2)$ を求める

2) $V(X) = ?$

(例)

指数分布に対して $V(X)$ を求める.

$E(X^2)$,