確率変数

2項変数

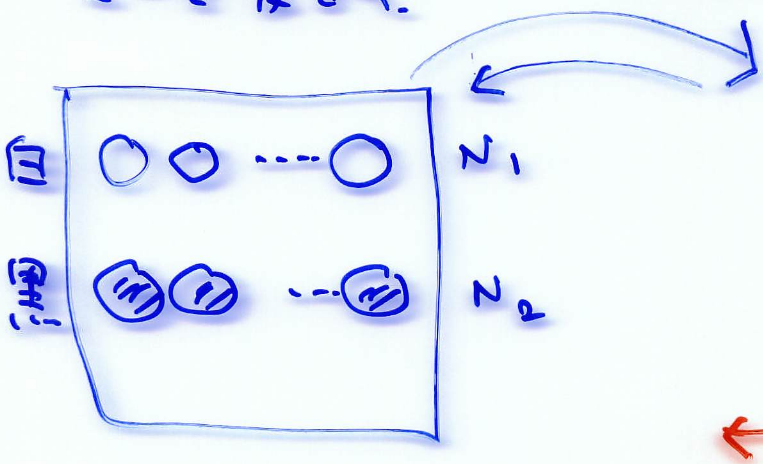
1個取ってもどす.

試行.

N回繰り返す.

復元抽出.

$\longleftrightarrow$ 非復元抽出.



$$p = \frac{N_1}{N_1+N_2}, \quad q = \frac{N_2}{N_1+N_2}.$$

$$\boxed{p+q=1.}$$

X: 白玉が出る個数.

$$P(X=k) = {}_N C_k \, p^k q^{N-k}.$$

$k = 0, 1, 2, \dots, N.$

$$\sum_{k=0}^{N} P(X=k) = \sum_{k=0}^{N} {}_N C_k \, p^k q^{N-k}$$

$$= (p+q)^N = 1^N = 1$$

↑ 2項定理

全確率 $= 1$

X の
期待値. expected value

$$E(X) = \sum_{k=0}^{N} P(X=k)\cdot k$$

$$= \sum_{k=0}^{N} k\,{}_N C_k\, p^k q^{N-k} = Np(p+q)^{N-1} = Np$$

$$(p+q)^N = \sum_{k=0}^{N} {}_N C_k\, p^k q^{N-k}$$

2項定理

$\frac{\partial}{\partial p}$:

$$N(p+q)^{N-1}\cdot 1 = \sum_{k=0}^{N} {}_N C_k\, k\, p^{k-1} q^{N-k}$$

$f(u)$ $u = u(t)$

$$\frac{d}{dt} f(u(t)) = \frac{df}{du}\cdot\frac{du}{dt}$$

$$\frac{d}{dt}\{u(t)\}^N = N u^{N-1}\cdot u'(t)$$

$$Np(p+q)^{N-1} = \sum_{k=0}^{N} k\,{}_N C_k\, p^k q^{N-k}$$

$$\lambda \sum_{k=0}^{N} x_k = \sum_{k=0}^{N} (\lambda x_k)$$

$$\lambda(x_0 + x_1 + \cdots + x_N)$$
$$= \lambda x_0 + \lambda x_1 + \cdots + \lambda x_N$$

E(x) のもう一つの計算法.

$$E(x) = \sum_{k=0}^{N} k \, {}_N C_k \, p^k q^{N-k}$$

$k=1$

$$\boxed{{}_N C_k = \frac{N!}{k!\,(N-k)!}}$$

$${}_8 C_3 = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} \qquad ○_1 \ ○_2 \ ○_3$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}{(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) \cdot 3 \cdot 2 \cdot 1}$$

$$= \frac{8!}{(8-3)!\ 3!}$$

$$k \, {}_N C_k = \frac{k \ N!}{k!\,(N-k)!} = \frac{N!}{(k-1)!\,(N-k)!}$$

$$= \frac{N \cdot (N-1)!}{(k-1)!\ \boxed{(N-k)}!}$$

$= \{(N-1)-(k-1)\}$

$$= N \cdot {}_{N-1}C_{k-1}.$$

__まとめ__ $k \, {}_N C_k = N \, {}_{N-1}C_{k-1} \quad (k \geqq 1)$

$$E(X) = \sum_{k=1}^{N} k \, {}_N C_k \, p^k q^{N-k}$$

$$= \sum_{k=1}^{N} N \, {}_{N-1} C_{k-1} \, p^k q^{N-k}$$

変数変換

$$= N \sum_{k=1}^{N} {}_{N-1} C_{k-1} \, p^k q^{N-k}$$

$\boxed{l = k-1}$

$$= N \sum_{l=0}^{N-1} {}_{N-1} C_l \, p^{l+1} q^{N-l-1}$$

$$= Np \sum_{l=0}^{N-1} {}_{N-1} C_l \, p^l q^{(N-1)-l}$$

$$= Np (p+q)^{N-1} = Np.$$

$$p_k = P(X=k) = {}_N C_k \, p^k q^{N-k}.$$

とおく.

<u>分散</u> variance $\mu = E(X) = Np$

$$V(X) := E((X-\mu)^2)$$

$$E(f(X)) = \sum_{k=0}^{N} f(k) P(X=k)$$

$f(X)$ の期待値.

$$V(X) = \sum_{k=0}^{N} (k-\mu)^2 p_k . \quad \mu = E(X)$$

$$= \sum_{k=0}^{N} (k^2 - 2\mu k + \mu^2) p_k$$

$$= \sum_{k=0}^{N} k^2 p_k - 2\mu \underbrace{\sum_{k=0}^{N} k p_k}_{= E(X) = \mu} + \mu^2 \underbrace{\sum_{k=0}^{N} p_k}_{=1}$$

$p_k = P(X=k)$

$$= E(X^2) - \mu^2$$

$$= E(X^2) - (E(X))^2$$

一般に

$V(X) =$ ← 定義 $\mu = E(X)$

$$E((X-\mu)^2)$$

$$= E(X^2) - (E(X))^2$$

$E(X^2) = ?$ 計算する.

$$E(x^2) = \sum_{k=0}^{N} k^2 \, {}_N C_k \, p^k q^{N-k}$$

$2 \geq \xi \ a\xi - x \geq t$

$$k^2 = \{k(k-1) + k\}$$

$$= \sum_{k=0}^{N} k(k-1) \, {}_N C_k \, p^k q^{N-k}$$

$k=2$

$$+ \boxed{\sum_{k=0}^{N} k \, {}_N C_k \, p^k q^{N-k}}$$

$$= E(x) = Np.$$

- $(p^k)'' = k(k-1)p^{k-2}.$

$$N(p+q)^{N-1} = \sum_{k=0}^{N} k \, {}_N C_k \, p^{k-1} q^{N-k}$$

$$N(N-1)(p+q)^{N-2} = \sum_{k=0}^{N} k(k-1) \, {}_N C_k \, p^{k-2} q^{N-k}$$

$$N(N-1)p^2(p+q)^{N-2} = \sum_{k=0}^{N} k(k-1) \, {}_N C_k \, p^k q^{N-k}$$

$$E(X^2) = N(N-1)p^2(p+q)^{N-2}$$

$$+ Np$$

$$= N(N-1)p^2 + Np$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= N(N-1)p^2 + Np - N^2p^2$$

$$= Np\{(N-1)p + 1 - Np\}$$

$$= Np(1-p) = Npq.$$

まとめ. 2項 変数 X $Bi(N, p)$

Binomial Distribution

$$E(X) = Np.$$

$$V(X) = Npq \quad (q = 1-p)$$

(8

$k \geqq 2 \quad N \geqq 2$ のとき.

$$k(k-1)\,{}_N C_k = N(N-1)\,{}_{N-2}C_{k-2}$$

を示せ.

レポート

$$\sum_{k=2}^{N} k(k-1)\,{}_N C_k\, p^k q^{N-k}$$

$= \cdots$

↑ 上の公式を使って