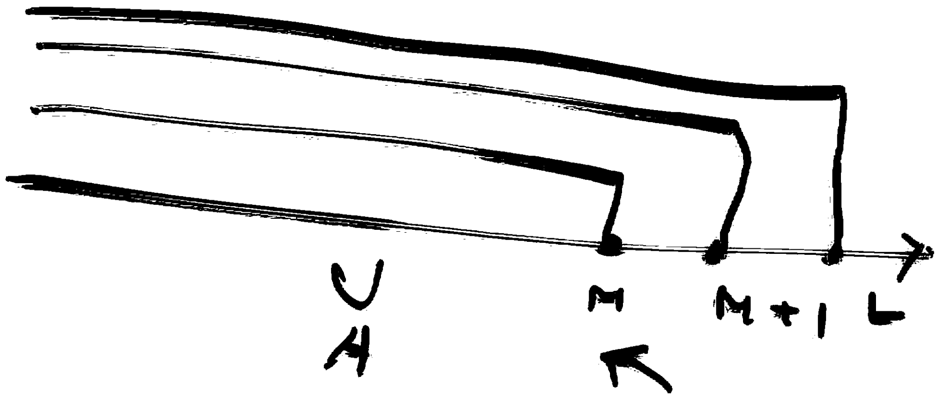
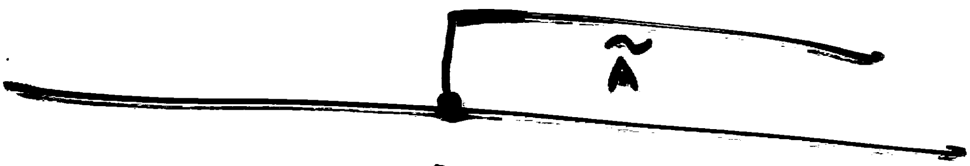



上界
upper bound

$A$: 上に有界

$M$: $A$の上界 $\Longrightarrow$ $M+1$: $A$の上界.

$\left.\begin{array}{l} M: A\text{の上界} \\ M < L \end{array}\right\} \Longrightarrow L: A$の上界.




$A$の上限
supremum.

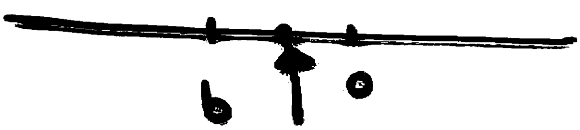
---

$A = \left\{ -\frac{1}{n} \,;\, n = 1, 2, 3 \cdots \right\}$ $\quad 0 \notin A$


i) $-\frac{1}{n} \leq 0$

上界.

ii) $b < 0$




$\rightsquigarrow$ $b$は上界でない…

$-\frac{1}{n}$ ($n$十分大)

$\rightsquigarrow$ 0が $\tilde{A}$の最小値 $\sup A = 0$

$A$: 上に有界

$\alpha$: $A$の上限 $\Longleftrightarrow$ i) $a \leq \alpha \quad (a \in A)$

$\alpha$は$A$の上界

ii) $b < \alpha \Longrightarrow \exists a \in A$ が存在して

$b < a \leq \alpha$ が成立

$\alpha$より小さい $b$ は $A$ の上界でない

( $\alpha$ が $\tilde{A}$ の最小値 )

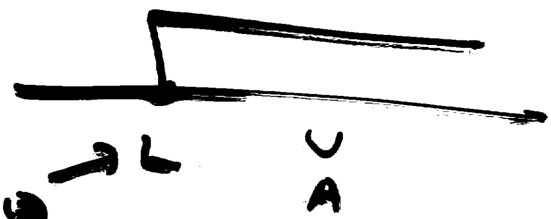
---

$A$が上に有界でないとき

$\sup A = +\infty$

---

$A$が下に有界 $\Longleftrightarrow$




下界 → $L$

lower bound

$\tilde{A} = \{ L : L$は$A$の下界$\}$

→ 最大値より

→ $\inf A = \max \tilde{A}$

$$a_n = \left(1+\frac{1}{n}\right)^n \quad \left.\begin{array}{l}\text{上に有界} \\ \text{単調増加}\end{array}\right\}$$

$\longrightarrow$ $\{a_n\}$ は収束

$$\lim_{n\to+\infty} a_n = e$$

Napier の数

---

$$b_n = \sup \underbrace{\{a_n, a_{n+1}, a_{n+2}, \cdots\}}_{= B_n}$$

$\searrow$ $\cdots \geq b_n \geq b_{n+1} \geq \cdots$

$\cdots \supset B_n \supset B_{n+1} \supset B_{n+2} \supset \cdots$

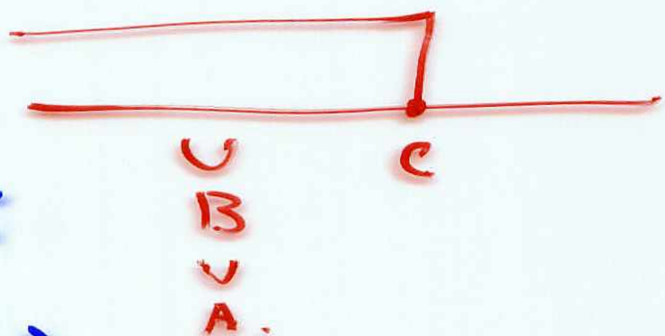
---

$A \subset B$ 上に有界

$\sup A \leq \sup B$.


$c$: $B$ の上界 $\Longrightarrow$ $c$: $A$ の上界

$\tilde{B} \subset \tilde{A}$




$\min \tilde{A} \leq \min \tilde{B}$

$(\sup A \leq \sup B)$

$\{b_n\}$ は単調減少

① $\{b_n\}$ : 下に有界 $\Longrightarrow$ $\displaystyle\lim_{n\to+\infty} b_n \in \mathbb{R}$

② $\{b_n\}$ : 下に有界でない..

$$\lim_{n\to+\infty} b_n = -\infty$$

(れい)

$a_n = -n$ $\quad b_n = -n \to -\infty$

---

$$\liminf a_n \leq \limsup a_n$$

$$c_n \leq b_n = \sup\{a_{n+1}, a_{n+2}, \cdots\}$$

$$c_n = \inf\{a_{n+1}, a_{n+2}, \cdots\}$$

---

$\{a_n\}$ の極限の存在

$$\Longleftrightarrow \limsup_{n\to+\infty} a_n = \liminf_{n\to+\infty} a_n.$$

---

$$s(\Omega) = \{\alpha_1, \cdots, \alpha_n\} \quad \alpha_i \neq \alpha_j$$

$$A_i = \{\omega \in \Omega;\ f(\omega) = \alpha_i\}$$

$$= f^{-1}(\{\alpha_i\})$$

$A \subset \Omega$

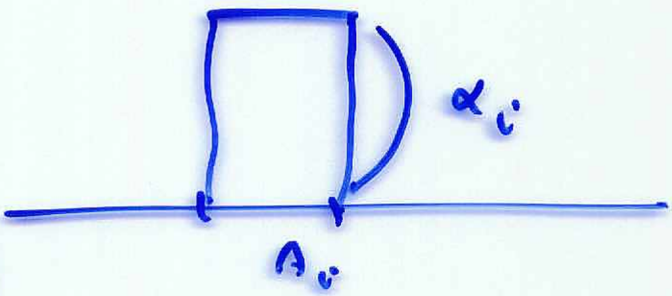
$$\chi_A(\omega) := \begin{cases} 1 & (\omega \in A) \\ 0 & (\omega \notin A) \end{cases}$$

Aの定義関数

$\mathbb{1}_A(\omega)$ とも言う.

$$s = \sum_{i=1}^{n} \alpha_i \chi_{A_i}$$

$$\int \alpha_i \chi_{A_i}$$




$$= \alpha_i P(A_i)$$

$$\rightsquigarrow \quad \int_\Omega s \, dP = \sum_{i=1}^{n} \alpha_i P(A_i)$$

---

$$\int_D f \, dP = \sup \Big\{ \int_\Omega \chi_D \cdot s \ ; \ s \leq f \text{ を満たす非負,可測な単関数} \Big\}$$

---

$s = \sum_{i=1}^{n} \alpha_i \chi_{A_i}$ のとき

$$\chi_D \cdot s = \sum_{i=1}^{n} \alpha_i \chi_{A_i \cap D}$$

$\chi_D \cdot s$ : 単関数.

$0 \le s \le f$ 는 單純函數 可測이고 [illegible]

$\Rightarrow \quad 0 \le s \le g$ ———

$$\left\{ \int_\Omega \chi_D \, s \; ; \; \text{———} \; f \sim f \right\}$$

$$\cap$$

$$\left\{ \int_\Omega \chi_D \, s \; ; \; \text{———} \; f \sim g \right\}$$

$$\xrightarrow[\sup]{} \quad \int_D f \, d\mu \le \int_D g \, d\mu$$

---

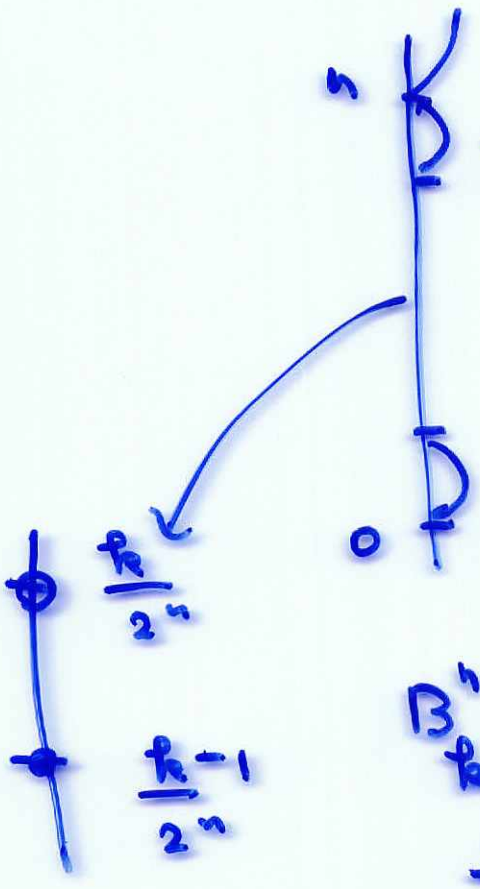
$A \subset \mathbb{R}$ 는 有界, $c > 0$

$A' = \{ ac \; ; \; a \in A \}$
$\quad \shortparallel$
$cA$

$\sup A' = c \sup A$.

$-A = \{ -a \; ; \; a \in A \}$

$\sup(-A) = -\inf A$




$\frac{1}{2^n}$ 幅で分割

$\frac{1}{2^n}$ の幅で分割.

$$B_k^n = \left\{ \omega \in \Omega ; \frac{k-1}{2^n} \leq f(\omega) < \frac{k}{2^n} \right\}$$

$$\omega \in B_k^n \Rightarrow \varphi_n(\omega) = \frac{k-1}{2^n}$$

---

$$B_\infty^n = \{ \omega \in \Omega ; f(\omega) \geq n \}$$

$$\begin{array}{c} \cup \! \! \mid \\ \omega \end{array}$$

$$\varphi_n(\omega) = n$$

---

$$\int_D f \, dP := \lim_{n \to +\infty} \int \chi_D \cdot \varphi_n$$

実は $0 \leq \varphi_1 \leq \varphi_2 \leq \cdots \leq f$

$$\lim_{n \to \infty} \varphi_n(\omega) = f(\omega)$$

を満たす単調増加（下から増加）ならばOK.

positive part of f.

$$f_+(w) = \begin{cases} f(w) & (f(w) \geq 0) \\ 0 & (f(w) < 0) \end{cases}$$

$$f_-(w) = \begin{cases} -f(w) & (f(w) \leq 0) \\ 0 & (f(w) > 0) \end{cases}$$

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レポート.

1, 3, 4, 5, 6 ※. 5問.