Open Office

$$\begin{vmatrix} \lambda-6 & 4 & -2 \\ -1 & \lambda-2 & -1 \\ 1 & -2 & \lambda-3 \end{vmatrix} = -\begin{vmatrix} 1 & -2 & \lambda-3 \\ -1 & \lambda-2 & -1 \\ \lambda-6 & 4 & -2 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & -2 & \lambda-3 \\ 0 & \lambda-4 & \lambda-4 \\ 0 & 2(\lambda-4) & -(\lambda-4)(\lambda-5) \end{vmatrix}$$

$$\begin{array}{rccc} & \lambda-6 & 4 & -2 \\ -) & \lambda-6 & -2(\lambda-6) & (\lambda-3)(\lambda-6) \\ \hline & 0 & 2\lambda-8 & -\lambda^2+9\lambda-20 \\ & & \| & \| \\ & & 2(\lambda-4) & -(\lambda^2-9\lambda+20) \\ & & & -(\lambda-4)(\lambda-5) \end{array}$$

$$(\lambda-4)\begin{pmatrix}1\\1\end{pmatrix}$$

$$= -\begin{vmatrix} \boxed{\begin{matrix}\lambda-4\\ \lambda-4\end{matrix}} & \boxed{\begin{matrix}\lambda-4\\ -(\lambda-4)(\lambda-5)\end{matrix}} \end{vmatrix}$$

$$(\lambda-4)\begin{pmatrix}1\\-(\lambda-5)\end{pmatrix}$$

$$= -(\lambda-4)^2 \begin{vmatrix} 1 & 1 \\ 1 & -(\lambda-5) \end{vmatrix}$$

$$= -(\lambda-4)^2\left(-(\lambda-5)-1\right)$$

$$= (\lambda-4)^2(\lambda-4) = (\lambda-4)^3$$

prosper Latex a style file.

$(i\,j\,k)$ $i \neq j,\ j \neq k,\ k \neq i$

$(1\,3\,2) \to (1\,2\,3)$ $\sigma(1\,3\,2) = -1$

$(2\,1\,3) \to (1\,2\,3)$ $\sigma(2\,1\,3) = -1$

$(2\,3\,1) \to (3\,2\,1)$ $\sigma(2\,3\,1) = 1$

$\downarrow$

$(1\,2\,3)$

$\det(\vec{x}_i\ \vec{x}_j\ \vec{x}_k) = \sigma(i\,j\,k) \det(\vec{x}_1\ \vec{x}_2\ \vec{x}_3)$

$(i\,j\,k)$ 111 213 4

$$(\vec{x}_1\ \vec{x}_2\ \vec{x}_3)\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = a_1\vec{x}_1 + a_2\vec{x}_2 + a_3\vec{x}_3$$

$$\cancel{(\vec{x}_1\ \vec{x}_2}\quad x(\vec{a}\ \vec{b}\ \vec{c}) = (x\vec{a}\ \ x\vec{b}\ \ x\vec{c})$$

\sum_{i=1}^{3} a_i\vec{x}_i

N.B. $\det(\vec{x}_1\ \vec{x}_1\ \vec{x}_3) = 0$

$\det(\vec{a}\ \vec{b}\ \vec{c})$

$$= \sum_{(i\,j\,k)} a_i\, b_j\, c_k\, \sigma(i\,j\,k)$$

定理 A, X : 3次正方行列

$$\det(AX) = \det(A)\det(X)$$

例　A : 正則 $\Rightarrow$ $\det(A) \neq 0$

$$\underset{\det(A)\det(A^{-1})}{\underset{\shortparallel}{\det(A \cdot A^{-1})}} = \det(I_3) = 1$$

N.B. = Nota Bene

$$x\vec{a} + y\vec{b} + z\vec{c} = \vec{\alpha}$$

$$\Downarrow$$

$$x = \frac{|\vec{\alpha}\ \vec{b}\ \vec{c}|}{|\vec{a}\ \vec{b}\ \vec{c}|}$$

$$y\vec{b} + x\vec{a} + z\vec{c} = \vec{\alpha}$$

$$\Downarrow$$

$$y = \frac{|\vec{\alpha}\ \vec{a}\ \vec{c}|}{|\vec{b}\ \vec{a}\ \vec{c}|} = \frac{|\vec{a}\ \vec{\alpha}\ \vec{c}|}{|\vec{a}\ \vec{b}\ \vec{c}|}$$

例

$$x = \frac{\left|\begin{matrix}0\\0\\0\end{matrix}\ \vec{b}\ \vec{c}\right|}{|\vec{a}\ \vec{b}\ \vec{c}|} = 0$$

$y = z = 0$ も同様.

★ ≠ 0

$$\begin{pmatrix} ★ & & \\ * & & \\ * & & \end{pmatrix} \text{ OR } \begin{pmatrix} 0 & & \\ ★ & & \\ * & & \end{pmatrix} \text{ OR } \begin{pmatrix} 0 & & \\ 0 & & \\ ★ & & \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} ★ & & \\ 0 & & \\ 0 & & \end{pmatrix}$$

det は (−1) 倍

$$\begin{pmatrix} ★ & & \\ 0 & & \\ * & & \end{pmatrix} \rightarrow \begin{pmatrix} ★ & & \\ 0 & & \\ 0 & & \end{pmatrix}$$

det は (−1) 倍

$$\begin{pmatrix} ★ & & \\ 0 & & \\ 0 & & \end{pmatrix}$$

$\vec{a} \neq \vec{0}$

$$b_{11}x + *_1 y + *_2 z = 0$$

$$A \to \cdots \to B = \left(\begin{array}{c|cc} b_{11} & *_1 & *_2 \\ \hline 0 & & \\ 0 & \multicolumn{2}{c}{C} \end{array}\right)$$

$b_{11} \neq 0$

$\det(C) = 0$

$$\boxed{A\vec{v} = \vec{0} \iff B\vec{v} = \vec{0}}$$

$C\begin{pmatrix} y_0 \\ z_0 \end{pmatrix} = \vec{0}$ $\quad \begin{pmatrix} y_0 \\ z_0 \end{pmatrix} \neq \vec{0}$

をみたす解あり.

$$x_0 = -\frac{1}{b_{11}}(*_1 y_0 + *_2 z_0)$$

$$\underbrace{B\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}}_{\neq \vec{0}} = \begin{pmatrix} x_0 b_{11} + *_1 y_0 + *_2 z_0 \\ C\begin{pmatrix} y_0 \\ z_0 \end{pmatrix} \end{pmatrix} = \vec{0}$$

Ce que l'on a montré.

$\det(A) = 0$

$\Rightarrow \exists \vec{v} \neq \vec{0}$ に対して

$A\vec{v} = \vec{0}$

<u>対偶</u> $\quad (A\vec{v} = \vec{0} \Rightarrow \vec{v} = \vec{0})$

$\Rightarrow \det(A) \neq 0.$

$\det(A) \neq 0 \Rightarrow (A\vec{v} = \vec{0} \Rightarrow \vec{v} = \vec{0})$

クラメールで示している.

$\Leftarrow$ ← クラメールの公式.

A : 正則 $\Longrightarrow$ $\det(A) \neq 0$

$$\Updownarrow$$

$$(A\vec{v} = \vec{0} \Rightarrow \vec{v} = \vec{0})$$

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 5 & 6 \end{pmatrix} \qquad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(1) $\det(\lambda I_3 - B)$ を計算

(2) $\det(\lambda I_3 - B) = 0$ を満たす λ に対して

$$(\lambda I_3 - B)\vec{v} = \vec{0}$$

を解け.