$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \underset{(-1)^{1+1}}{a_1} \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} \overset{(-1)^{2+1}}{-} a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}$$

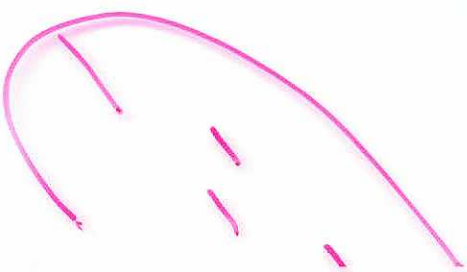
$$\underset{(-1)^{2+2}.}{+} a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$= \underset{(1\,2\,3)}{a_1 b_2 c_3} + \underset{(2\,3\,1)}{a_2 b_3 c_1} + \underset{(3\,1\,2)}{a_3 b_1 c_2}$$

$$- \underset{(1\,3\,2)}{a_1 b_3 c_2} - \underset{(2\,1\,3)}{a_2 b_1 c_3} - \underset{(3\,2\,1)}{a_3 b_2 c_1}$$

$$\pm\, a_i b_j c_k$$

$$i \neq j,\ j \neq k,\ k \neq i$$



(偶順列)

$(1\,2\,3)$

$(2\,3\,1) \longrightarrow (3\,2\,1) \longrightarrow (1\,2\,3)$

$(3\,1\,2) \longrightarrow (1\,3\,2) \longrightarrow (1\,2\,3)$

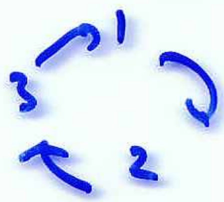
奇順列

$(1\,3\,2) \longrightarrow (1\,2\,3)$

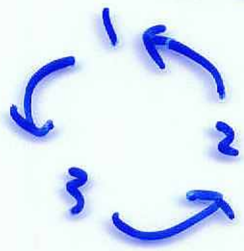
$(2\,1\,3) \longrightarrow (1\,2\,3)$

$(3\,2\,1) \longrightarrow (1\,2\,3)$

正回り



逆回り.



$\det(\vec{a}\ \vec{b}\ \vec{c})$

$$= \sum_{(ijk)} \underbrace{\sigma(ijk)}_{=\begin{cases} +1 & (ijk)\ \text{正回り} \\ -1 & (ijk)\ \text{逆回り.} \end{cases}} a_i b_j c_k.$$

$$
\begin{array}{cc}
+ & - \\
\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}
&
\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}
\end{array}
$$

$$
\begin{vmatrix} \alpha & * & * \\ 0 & & \\ 0 & B & \end{vmatrix} = \alpha\,|B|
$$

$$f : \mathbb{R}^n \longrightarrow \mathbb{R}$$

(b_j)の斉次1次式

$$
\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \longmapsto c_1 b_1 + c_2 b_2 + \cdots + c_n b_n = \sum_{j=1}^{n} c_j b_j
$$

$\cancel{f(\vec{\alpha})}$

$$
f(\lambda\vec{\alpha} + \mu\vec{\beta}) = \lambda f(\vec{\alpha}) + \mu f(\vec{\beta})
$$

$$
= \sum_{j=1}^{n} c_j (\lambda \alpha_j + \mu \beta_j)
$$

$$
= \lambda \sum_{j=1}^{n} c_j \alpha_j + \mu \sum_{j=1}^{n} c_j \beta_j
$$

$$|\vec{a}\ \vec{b}\ \vec{c}|$$

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$= -\left\{ a_1 \begin{vmatrix} c_2 & b_2 \\ c_3 & b_3 \end{vmatrix} - a_2 \begin{vmatrix} c_1 & b_1 \\ c_3 & b_3 \end{vmatrix} + a_3 \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} \right\}$$

$$= -|\vec{a}\ \vec{c}\ \vec{b}|$$

(IV) $|\vec{b}\ \vec{b}\ \vec{c}| = -|\vec{b}\ \vec{b}\ \vec{c}|$

$\rightsquigarrow |\vec{b}\ \vec{b}\ \vec{c}| = 0$

(VI) $|\vec{a}\ (\vec{b}+\lambda\vec{a})\ \vec{c}|$

$= |\vec{a}\ \vec{b}\ \vec{c}| + \lambda \underbrace{|\vec{a}\ \vec{a}\ \vec{c}|}_{=0}$

$= |\vec{a}\ \vec{b}\ \vec{c}|$

$$
{}^t\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}
$$

$$
3 \times 2 \xrightarrow[t]{} 2 \times 3
$$

$$
{}^t\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{pmatrix} = \begin{pmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \alpha_3 & \beta_3 \end{pmatrix}
$$

$$
\det\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} = a_1 b_2 - b_1 a_2
$$

$$
\parallel
$$

$$
\det\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = a_1 b_2 - a_2 b_1
$$

$B : 2 \times 2$

$$
\det({}^tB) = \det(B)
$$

$$f : \mathbb{R}_n \longrightarrow \mathbb{R}$$

$$(a_1, a_2 \cdots a_n) \longmapsto \sum_{i=1}^{n} c_i a_i$$

$$\Rightarrow f(\lambda \boldsymbol{a} + \mu \boldsymbol{b})$$

$$= \lambda f(\boldsymbol{a}) + \mu f(\boldsymbol{b})$$

列の証明.

$$\begin{vmatrix} \boldsymbol{a} \\ \lambda \alpha + \mu \beta \\ \boldsymbol{c} \end{vmatrix} = \left| {}^t\boldsymbol{a} \quad \overset{\lambda\,{}^t\alpha + \mu\,{}^t\beta}{\overset{\parallel}{\boxed{{}^t(\lambda \alpha + \mu \beta)}}} \quad {}^t\boldsymbol{c} \right|$$

$$= \lambda \left| {}^t\boldsymbol{a} \quad {}^t\alpha \quad {}^t\boldsymbol{c} \right| + \mu \left| {}^t\boldsymbol{a} \quad {}^t\beta \quad {}^t\boldsymbol{c} \right|$$

$$= \lambda \begin{vmatrix} \boldsymbol{a} \\ \alpha \\ \boldsymbol{c} \end{vmatrix} + \mu \begin{vmatrix} \boldsymbol{a} \\ \beta \\ \boldsymbol{c} \end{vmatrix}$$

$$\begin{vmatrix} \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \end{vmatrix} = \left| {}^t\boldsymbol{a} \quad {}^t\boldsymbol{b} \quad {}^t\boldsymbol{c} \right| = -\left| {}^t\boldsymbol{b} \quad {}^t\boldsymbol{a} \quad {}^t\boldsymbol{c} \right| = -\begin{vmatrix} \boldsymbol{b} \\ \boldsymbol{a} \\ \boldsymbol{c} \end{vmatrix}$$

$$\begin{vmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{vmatrix}$$

$$= c_1 \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - c_2 \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + c_3 \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= -\left\{ c_1 \begin{vmatrix} b_2 & b_3 \\ a_2 & a_3 \end{vmatrix} - c_2 \begin{vmatrix} b_1 & b_3 \\ a_1 & a_3 \end{vmatrix} + c_3 \begin{vmatrix} b_1 & b_2 \\ a_1 & a_2 \end{vmatrix} \right\}$$

$$= -\begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ \boxed{c_1 \quad c_2 \quad c_3} \end{vmatrix}$$

3ぎう の展開

$$\begin{vmatrix} \mathbf{a}_1 \\ \mathbf{c} \\ \mathbf{c} \end{vmatrix} = - \begin{vmatrix} \mathbf{a}_1 \\ \mathbf{c} \\ \mathbf{c} \end{vmatrix}$$

$$\longrightarrow \quad \begin{vmatrix} \mathbf{a}_1 \\ \mathbf{c} \\ \mathbf{c} \end{vmatrix} = 0$$

$$\begin{vmatrix} \mathbf{a}_1 \\ \mathbf{b} \\ \mathbf{c} \end{vmatrix} = \begin{vmatrix} \mathbf{a}_1 \\ \mathbf{b} \\ \mathbf{c} + \lambda \mathbf{a}_1 \end{vmatrix}$$

$$(\text{Th}) = \begin{vmatrix} \mathbf{a}_1 \\ \mathbf{b} \\ \mathbf{c} \end{vmatrix} + \lambda \underbrace{\begin{vmatrix} \mathbf{a}_1 \\ \mathbf{b} \\ \mathbf{a}_1 \end{vmatrix}}_{=0}$$

$$= \begin{vmatrix} \mathbf{a}_1 \\ \mathbf{b} \\ \mathbf{c} \end{vmatrix}$$

$$\begin{vmatrix} \lambda - 6 & 4 & -2 \\ -1 & \lambda - 2 & -1 \\ 1 & -2 & \lambda - 3 \end{vmatrix}$$