レポート問題修正.

$$x^2 - 4xy - 2y^2 + x - 2y - 1 = 0$$

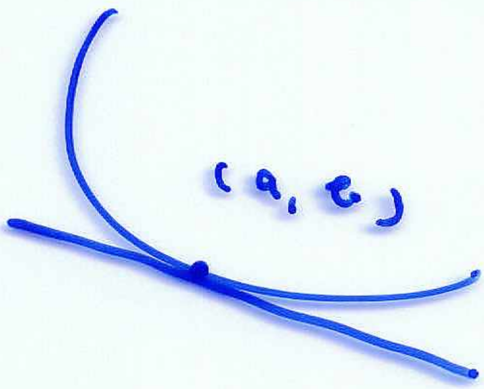
前回の復習.

$z = f(x, y)$

$(a, b, f(a, b))$ を通る接平面

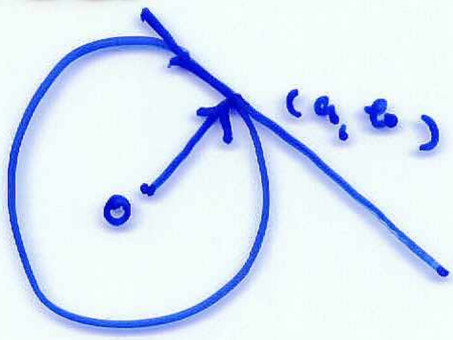
$$z = f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b)$$

$g(x, y) = 0$



例

$\underbrace{x^2 + y^2 - 1}_{= g(x, y)} = 0$



$g_x = 2x.$

$g_y = 2y$

$2a(x - a) + 2b(y - b) = 0$　　$ax + by = 1$

$ax + by$

$= a^2 + b^2$

$= 1$

$\left(a \cdot a + b \cdot b = 1 \right.$

$\downarrow$

$\left. (a, b) \text{ を通る.} \right)$

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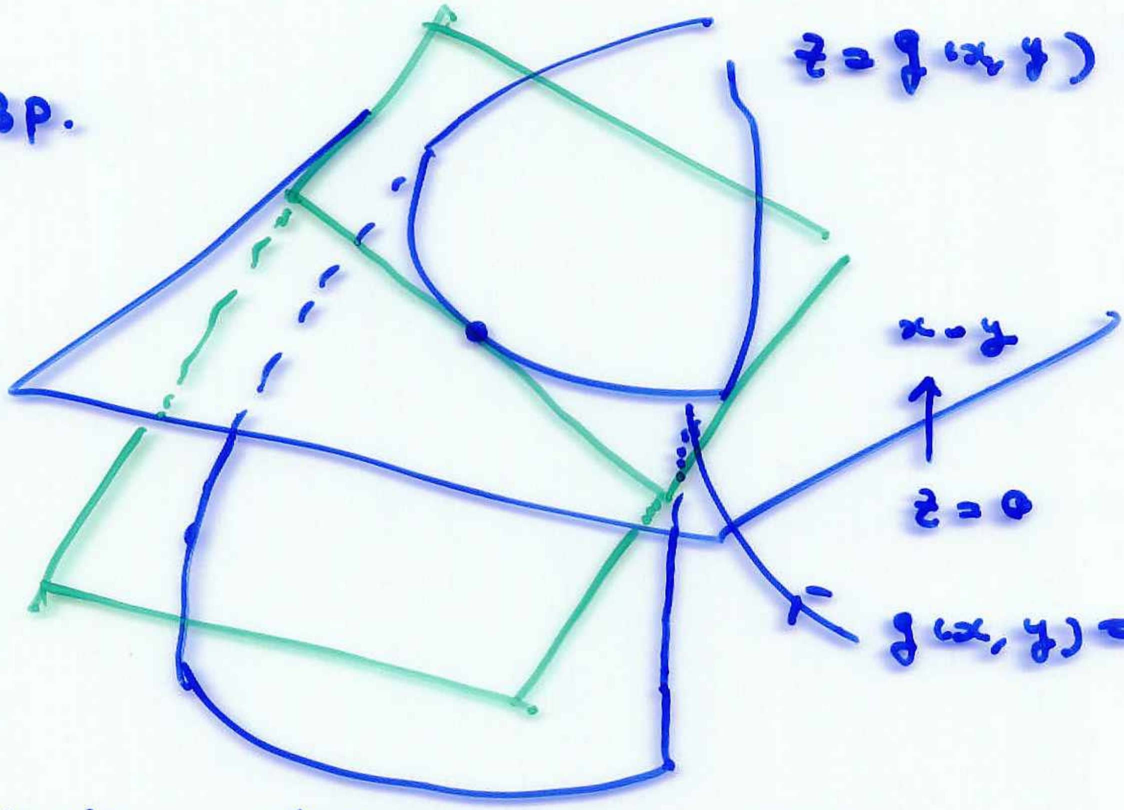


接線 = 接平面 ∩ x-y 平面.

$$\to z = g_x(a,b)(x-a) + g_y(a,b)\times(y-b)$$

$$\rightsquigarrow g_x(a,b)(x-a) + g_y(a,b)(y-b) = 0$$

$$\Downarrow$$

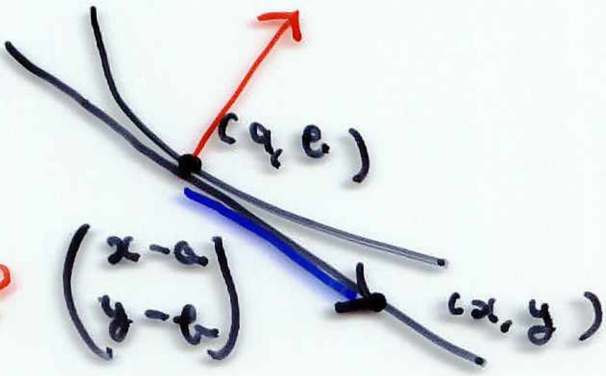
$$\begin{pmatrix} g_x(a,b) \\ g_y(a,b) \end{pmatrix} \cdot \begin{pmatrix} x-a \\ y-b \end{pmatrix} = 0$$

$\|$

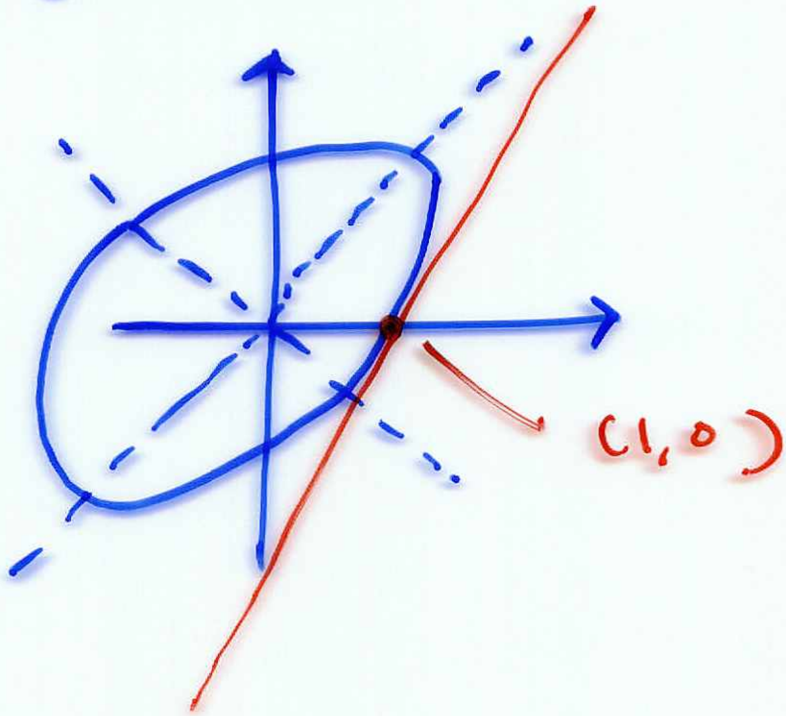
$\nabla(g)(a,b)$

勾配ベクトル

g の (a,b) における



$$x^2 - xy + y^2 - 1 = 0$$



$$g_x = 2x - y \,, \quad g_y = -x + 2y$$

$$g_x(1,0) = 2 \,, \quad g_y(1,0) = -1$$

$$2(x-1) + (-1)y = 0$$

$$\rightsquigarrow \quad y = 2(x-1)$$

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$$z = f(x, y)$$

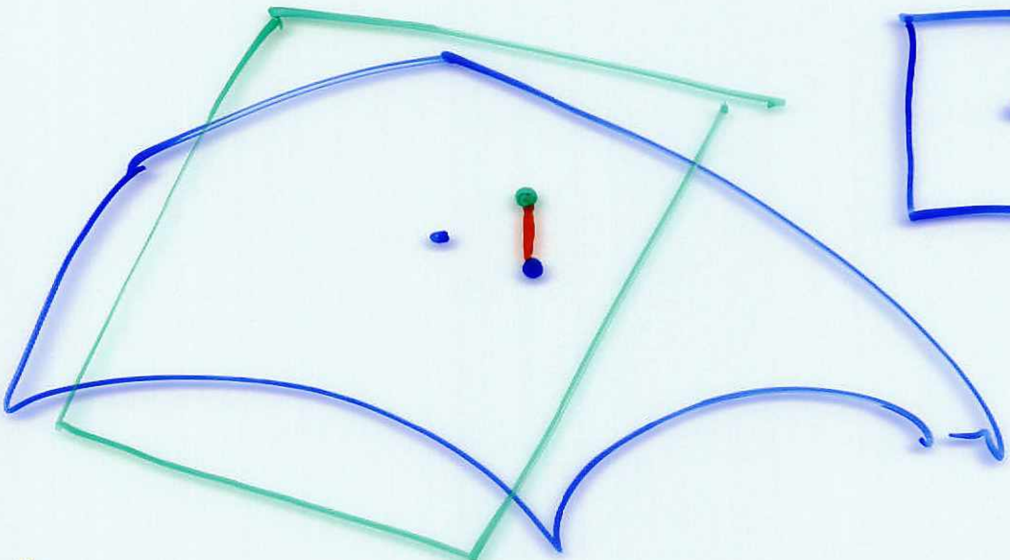
$$F(t) = f(x(t), y(t))$$

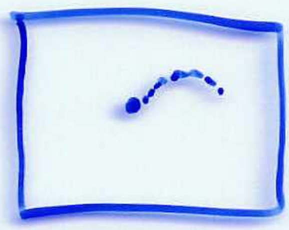
$$\rightsquigarrow F'(t) = ?$$

定義 8.5

$$\beta(x, y)$$

$$:= f(x, y) - \left\{ f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b) \right\}$$





f が (a, b) で全微分可能.

$$\frac{\beta(x, y)}{\sqrt{(x-a)^2 + (y-b)^2}} \longrightarrow 0 \quad ((x, y) \to (a, b))$$

f が (a, b) で全微分可能とする。

$$(x(0), y(0)) = (a, b)$$

$$F'(0) = ? \qquad \boxed{F(t) = f(x(t), y(t))}$$

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$$F(0) = f(x(0), y(0)) = f(a, b)$$

$$\frac{F(t) - F(0)}{t}$$

$x(0)$ $\quad$ $y(0)$

$$= \frac{1}{t}\Big\{ f_x(a, b)(x(t) - a) + f_y(a, b)(y(t) - b) + f(a, b) + \beta(x, y) - f(a, b) \Big\}$$

$$= f_x(a, b)\frac{x(t) - x(0)}{t} + f_y(a, b)\frac{y(t) - y(0)}{t} + \frac{\beta(x, y)}{t}$$

$$\frac{\beta(x, y)}{t} \to 0 \quad (t \to 0)$$

$$\longrightarrow f_x(a, b)\, x'(0) + f_y(a, b)\, y'(0) \quad (t \to 0)$$

$$\boxed{F'(0) = f_x(a, b)\, x'(0) + f_y(a, b)\, y'(0)}$$

$$\left|\frac{\beta(x(t),y(t))}{t}\right| = \left|\frac{\beta(x(t),y(t))}{\sqrt{(x(t)-a)^2+(y(t)-b)^2}}\right| \times \sqrt{\left(\frac{x(t)-x(0)}{t}\right)^2+\left(\frac{y(t)-y(0)}{t}\right)^2} \to 0$$

$\left|\frac{\beta(x(t),y(t))}{\sqrt{(x(t)-a)^2+(y(t)-b)^2}}\right| \to 0$ ← 全微分可能 $(x(t),y(t))\to(a,b)$

$\sqrt{\left(\frac{x(t)-x(0)}{t}\right)^2+\left(\frac{y(t)-y(0)}{t}\right)^2} \to \sqrt{x'(0)^2+y'(0)^2}$

$x(t),\ y(t)$ 微分可能 $\longrightarrow$ $x(t),\ y(t)$

$t=0$ で連続

$$\left.\begin{aligned} x(t) &\to x(0)=a \\ y(t) &\to y(0)=b \end{aligned}\right\} \longrightarrow (x(t),y(t)) \to (a,b) \quad (t\to 0)$$

示す

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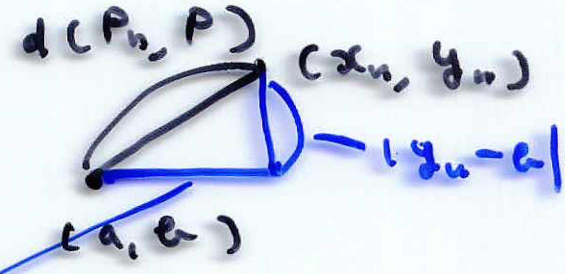
$P_n(x_n, y_n)$ $\quad$ $P_n \to \underset{(a,b)}{P}$ $(n \to +\infty)$ $\in \mathbb{R}^2$

$(n = 0, 1, 2, \cdots)$

$$d(P_n, P) = \sqrt{(x_n - a)^2 + (y_n - b)^2}$$

distance $\searrow 0$



$|x_n - a|$

$$0 \le \left\{ \begin{array}{l} \overbrace{|x_n - a|}^{\to 0} \\ \underbrace{|y_n - b|}_{\to 0} \end{array} \right\} \le \boxed{d(P_n, P)} \to 0$$

$$d(P_n, P) \le \overbrace{|x_n - a| + |y_n - b|}^{\to 0}, \quad |x_n - a| \to 0, \quad |y_n - b| \to 0$$

$P_n \to P \quad \rightsquigarrow \quad \left. \begin{array}{l} x_n \to a \\ y_n \to b \end{array} \right\}$

$\begin{array}{l} x_n \to a \\ y_n \to b \end{array} \longrightarrow P_n \to P.$

$$P_n \to P \iff (x_n \to a, \ y_n \to b)$$

定理 $f: U \longrightarrow \mathbb{R}$ (開)

が U に C^1 級 とする.

1) U の各点で f_x, f_y あり

2) f_x, f_y, f が連続.

$\Longrightarrow$ f は U の各点で全微分可能.

Ⅰ $z = e^{2x-3y}$ の $(0, 0, 1)$ における接平面を求めよ.

Ⅱ $x^2 - y^2 = -1$ の $(1, \sqrt{2})$ における接線の方程式を求めよ.