6月20日 出題のレポート 7/4(月) ⑭

7月4日 数題程度出題 7/27,28 提出 ⑮ ?

$$f''(t) - f'(t) - 2f(t) = 0$$

特性方程式 $\lambda^2 - \lambda - 2$

$$= (\lambda - 2)(\lambda + 1) = 0$$

特性根 characteristic roots

$$\lambda = 2, -1$$

$$\begin{cases} (f' - 2f)' = -(f' - 2f) \\ (f' + f)' = 2(f' + f) \end{cases}$$

$$\boxed{g'(t) = \alpha g(t) \Rightarrow g(t) = g(0) e^{\alpha t} \quad \alpha \in \mathbb{R}}$$

$$\begin{cases} f' - 2f = e^{-t}(f'(0) - 2f(0)) \\ f' + f = e^{2t}(f'(0) + f(0)) \end{cases}$$

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$$-3f(t) = e^{-t}(f'(0) - 2f(0)) - e^{2t}(f'(0) + f(0))$$

A Faire : $f'' + Af' + B = 0$

$\lambda^2 + A\lambda + B = 0$ Discriminant 判別式

$D = A^2 - 4B$

$D > 0$ ok $D = 0$, $D < 0$

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$$\vec{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \qquad \frac{d}{dt}\vec{x}(t) = \begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix}$$

$$\boxed{\frac{1}{t_1}\left(\vec{x}(t_0+t_1) - \vec{x}(t_0)\right) \longrightarrow \frac{d}{dt}\vec{x}(t_0)}$$

で説明することもできる.

$$A(t) = \begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix} = \left(\vec{a}(t) \ \ \vec{b}(t)\right) \qquad A'(t) = \begin{pmatrix} a'(t) & b'(t) \\ c'(t) & d'(t) \end{pmatrix} = \left(\frac{d}{dt}\vec{a}(t) \ \ \frac{d}{dt}\vec{b}(t)\right)$$

$$\boxed{\frac{d}{dt}\left(A(t)\vec{x}(t)\right) = A'(t)\vec{x}(t) + A(t)\frac{d}{dt}\vec{x}(t)}$$

を示す.

Leibnitz の公式

$$\begin{pmatrix} a(t)x(t) + b(t)y(t) \\ c(t)x(t) + d(t)y(t) \end{pmatrix}' = \cdots$$

$$(fg)' = f'g + fg'$$

$$\boxed{\begin{array}{l} A(t),\ B(t) \quad 2\times 2 \text{ 行列} \\ \dfrac{d}{dt}\left(A(t)B(t)\right) = A'(t)B(t) + A(t)B'(t) \end{array}}$$

プリント I ② でおく. $B(t) = \left(\vec{b}_1 \ \ \vec{b}_2\right)$

$$A(t)B(t) = \left(A(t)\vec{b}_1(t) \quad A(t)\vec{b}_2(t)\right)$$

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = A \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$ dynamical system

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$$ 力学系 / 動学系

$$\begin{cases} x_1'(t) = x_1(t) + 2x_2(t) \\ x_2'(t) = 4x_1(t) + 3x_2(t) \end{cases}$$

$$P = (\vec{v}_1\ \vec{v}_2) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

(-1)の固有ベクトル

Aの固有値 5, −1

5の固有ベクトル

$$AP = P\begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}$$ $$P^{-1}AP = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}$$

$\vec{y}(t) = P^{-1}\vec{x}(t)$ とおく $(P^{-1})' = O_2$

$$\frac{d}{dt}\vec{y}(t) = (P^{-1})'\vec{x}(t) + P^{-1}\underline{\frac{d}{dt}\vec{x}(t)}$$

$$= P^{-1}\underline{A\vec{x}(t)}$$

$$= P^{-1}A\underbrace{P\cdot P^{-1}}_{I_2}\vec{x}(t) = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}\vec{y}(t)$$

$P^{-1}\vec{x}(t) = \vec{y}(t)$

<u>まとめ</u> $$\frac{d}{dt}\vec{y}(t) = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}\vec{y}(t)$$

$$\begin{pmatrix} y_1'(t) \\ y_2'(t) \end{pmatrix} = \begin{pmatrix} 5y_1(t) \\ -y_2(t) \end{pmatrix}$$

$$\begin{cases} y_1'(t) = 5y_1(t) \\ y_2'(t) = -y_2(t) \end{cases} \rightsquigarrow \begin{cases} y_1(t) = e^{5t}y_1(0) \\ y_2(t) = e^{-t}y_2(0) \end{cases}$$

$$\vec{y}(t) = \begin{pmatrix} e^{5t} y_1(0) \\ e^{-t} y_2(0) \end{pmatrix}$$

$$\parallel$$

$$P^{-1}\vec{x}(t) = \begin{pmatrix} e^{5t} & 0 \\ 0 & e^{-t} \end{pmatrix} \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix}$$

$$\vec{x}(t) = P\vec{y}(t) = P \begin{pmatrix} e^{5t} & 0 \\ 0 & e^{-t} \end{pmatrix} \vec{y}(0)$$

$$= P \begin{pmatrix} e^{5t} & 0 \\ 0 & e^{-t} \end{pmatrix} P^{-1} \vec{x}(0)$$

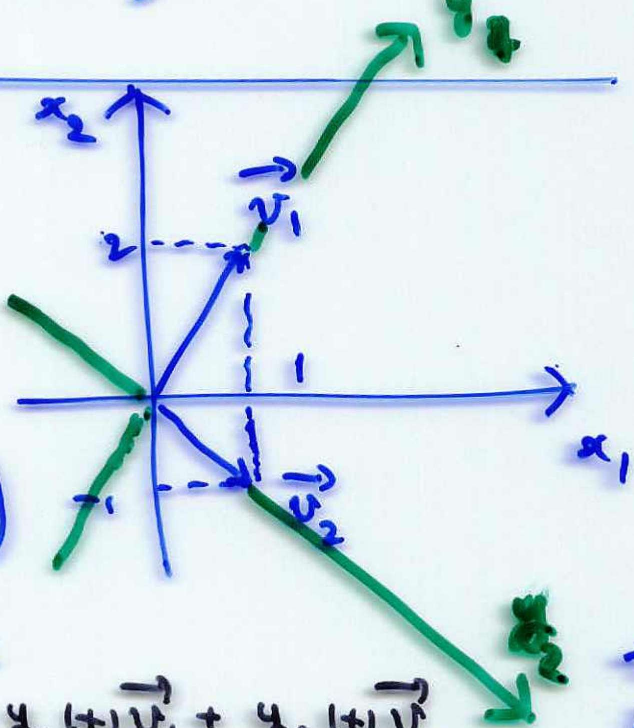
$$P = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$\vec{y} = P^{-1}\vec{x}$$

$$\vec{x} = P\vec{y}$$

$$= (\vec{v}_1\ \vec{v}_2)\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$= y_1\vec{v}_1 + y_2\vec{v}_2$$



$$\vec{x}(t) = P\vec{y}_0(t) = y_1(t)\vec{v}_1 + y_2(t)\vec{v}_2$$

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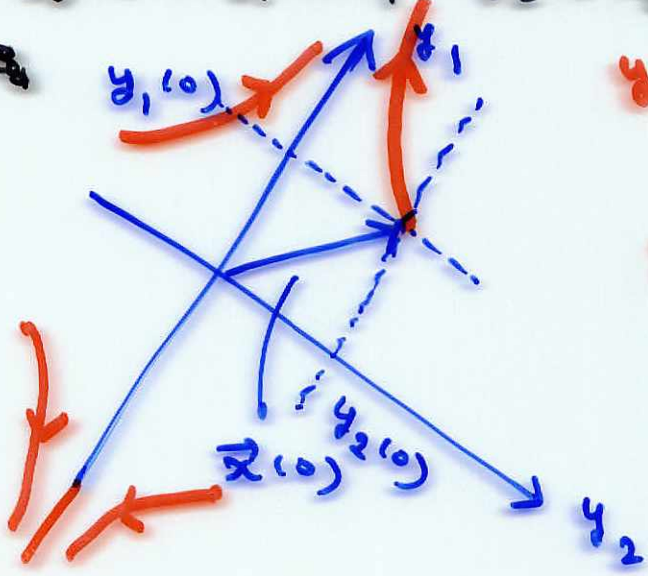
位相図



$$y_1(t) = y_1(0)e^{5t} \to +\infty$$

$$y_2(t) = y_2(0)e^{-t} \to 0$$

A: 2×2 行列　　A の指数関数.

$$e^{tA} := I_2 + tA + \frac{1}{2!}t^2A^2 + \frac{1}{3!}t^3A^3 + \frac{1}{4!}t^4A^4 + \cdots$$

$$e^s = 1 + s + \frac{1}{2!}s^2 + \frac{1}{3!}s^3 + \cdots$$

$$\frac{d}{dt}e^{tA} = O_2 + A + tA^2 + \frac{1}{2!}t^2A^3 + \frac{1}{3!}t^3A^4 + \cdots$$

$$= A\left(I_2 + tA + \frac{1}{2!}t^2A^2 + \frac{1}{3!}t^3A^3 + \cdots\right)$$

$$= \left(I_2 + tA + \frac{1}{2!}t^2A^2 + \cdots\right)A$$

<u>まとめ</u>　$\frac{d}{dt}e^{tA} = Ae^{tA} = e^{tA}A.$

$$\left(e^{tA}\vec{x}(0)\right)\Big|_{t=0} = I_2\vec{x}(0) = \vec{x}(0)$$

$$\frac{d}{dt}\left(e^{tA}\vec{x}(0)\right) = \left(e^{tA}\right)'\vec{x}(0) + e^{tA}\left(\vec{x}(0)\right)'$$

$$= Ae^{tA}\vec{x}(0)$$

$\vec{v}$

$\shortparallel$

<u>まとめ</u>　$\vec{x}(t) = e^{tA}\vec{x}(0)$ は

$$\begin{cases} \frac{d}{dt}\vec{x}(t) = A\vec{x}(t) \\ \vec{x}(0) = \vec{v} \end{cases}$$ の解.

<u>実は</u>　これは唯一つの解である.

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 実2次正方行列

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$$

$$\Phi_A(\lambda) = \begin{vmatrix} \lambda - a & -b \\ -c & \lambda - d \end{vmatrix} = \lambda^2 - (a+d)\lambda + (ad - bc)$$

$D > 0$　ok.

$\left.\begin{array}{l} D = 0 \\ D < 0 \end{array}\right\}$ 来学期までに..

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① $A = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$ に対し

A を対角化せよ　　$\lambda = -5, 7$ が固有値.

② $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$ を解け.