復習

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

eigentlich.

ケーリー・ハミルトンの定理.

$$A^2 - (a+d)A + \det(A)\cdot I_2 = O_2$$

$$A = \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix}$$

φ, ϕ, Φ
phi.

は

$$A^2 - 2A - 24 I_2 = O_2.$$

$$\Phi_A(\lambda) = \lambda^2 - 2\lambda - 24$$ 特性多項式

$$= (\lambda - 6)(\lambda + 4)$$ 固有値.
eigenvalue

$$A^2 - (6 + (-4))A + 6\cdot(-4) I_2 = O_2$$

$$\begin{cases} A(A - 6I_2) = (-4)(A - 6I_2) \\ A(A + 4I_2) = 6(A + 4I_2) \end{cases}$$

$$\begin{aligned} A^2(A - 6I_2) &= A\cdot A(A - 6I_2) \\ &= A\cdot(-4)(A - 6I_2) \\ &= (-4)A(A - 6I_2) \\ &= (-4)^2(A - 6I_2) \end{aligned}$$

$$\begin{cases} A^n(A - 6I_2) = (-4)^n(A - 6I_2) \\ A^n(A + 4I_2) = 6^n(A + 4I_2) \end{cases}$$

$$\left\{\begin{array}{l} A^{n+1} - 6A^n = (-4)^n (A - 6I_2) \\ A^{n+1} + 4A^n = 6^n (A + 4I_2) \end{array}\right.$$

$$-10A^n = (-4)^n (A - 6I_2) - 6^n (A + 4I_2)$$

C-H 217 p. A^n : 218 p.

CT 223 page.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff \left\{\begin{array}{l} ax + by = 0 \\ cx + dy = 0 \end{array}\right.$$

法線方向

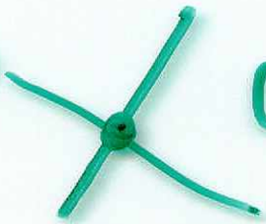
定理

(1) $\left(A\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \right)$

$\iff \det(A) = ad - bc \neq 0.$

(2) $\left(A\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0}\right.$ かつ $\begin{pmatrix} x \\ y \end{pmatrix} \neq \vec{0}$ となる $\left.\begin{pmatrix} x \\ y \end{pmatrix}\right.$ が存在する $\Big)$

$\iff \det(A) = ad - bc = 0.$

(1)  $\begin{pmatrix} a \\ b \end{pmatrix} \nparallel \begin{pmatrix} c \\ d \end{pmatrix}$

(2) $\begin{pmatrix} a \\ b \end{pmatrix} \parallel \begin{pmatrix} c \\ d \end{pmatrix}$

$A \neq O_2$ のとき.

$A = \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix}$ lambda.

「$A \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$ となる $\begin{pmatrix} x \\ y \end{pmatrix} \neq \vec{0}$ はあるか.」

$$A \begin{pmatrix} x \\ y \end{pmatrix} = \lambda I_2 \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\boxed{I_2 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}}$$

$$(\lambda I_2 - A) \begin{pmatrix} x \\ y \end{pmatrix} = \vec{0}$$

$$\Leftrightarrow \det(\lambda I_2 - A) = 0$$

$$= \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} \lambda - 5 & -3 \\ -3 & \lambda + 3 \end{pmatrix}$$

$$= \begin{vmatrix} \lambda - 5 & -3 \\ -3 & \lambda + 3 \end{vmatrix} = (\lambda - 5)(\lambda + 3) - 9$$

$$= \lambda^2 - 2\lambda - 24 = (\lambda - 6)(\lambda + 4)$$

$$\Phi_A(\lambda) = |\lambda I_2 - A| = \begin{vmatrix} \lambda - a & -b \\ -c & \lambda - d \end{vmatrix}$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= (\lambda - a)(\lambda - d) - bc$$

$$= \lambda^2 - (a + d)\lambda + (ad - bc)$$

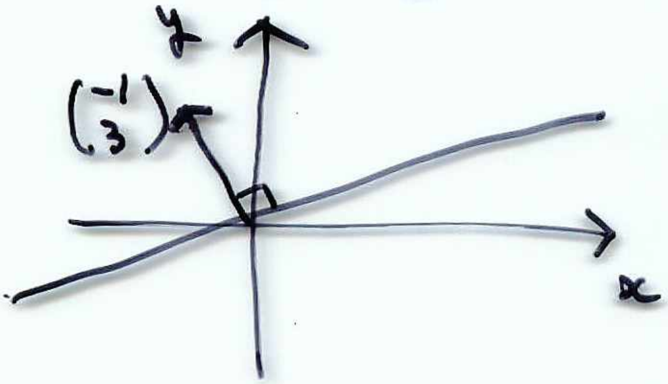
を A の ~~特性~~ 固有多項式

eigen-polynomial.

$\underline{\lambda = 6}$ a.e.t.

$\boxtimes \begin{cases} x - 3y = 0 \\ -3x + 9y = 0 \end{cases}$

$$\begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \iff x - 3y = 0$$



$$y = \frac{1}{3}x.$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3y \\ y \end{pmatrix} = y\begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (y \neq 0)$$

$\underline{\lambda = -4}$

$$\begin{pmatrix} -9 & -3 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff 3x + y = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -3x \end{pmatrix} = x\begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad (x \neq 0)$$

$$\vec{v}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$A\vec{v}_1 = 6\vec{v}_1, \quad A\vec{v}_2 = -4\vec{v}_2. \quad P = (\vec{v}_1\ \vec{v}_2)$$

$$A(\vec{v}_1\ \vec{v}_2) = (A\vec{v}_1\ A\vec{v}_2) = \begin{pmatrix} 3 & 1 \\ 1 & -3 \end{pmatrix}$$

$$= (6\vec{v}_1\ \ -4\vec{v}_2) = (\vec{v}_1\ \vec{v}_2)\begin{pmatrix} 6 & 0 \\ 0 & -4 \end{pmatrix}$$

$(ab)^{\lambda} = a^{\lambda} b^{\lambda}$
$\rightarrow$ easy
$\leftarrow$ not so easy.

$$(\vec{v}_1\ \vec{v}_2)\begin{pmatrix} p \\ q \end{pmatrix} = p\vec{v}_1 + q\vec{v}_2$$

まとめ

$P = (\vec{v}_1, \vec{v}_2)$

$$AP = P\begin{pmatrix} 6 & 0 \\ 0 & -4 \end{pmatrix}$$

逆行列が必要。 $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \boxed{\begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}}$$

$$\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \boxed{\begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}} = \det(A)\cdot I_2$$

$ad-bc \neq 0$ のとき.

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

A の逆行列.

$$A\cdot A^{-1} = A^{-1}\cdot A = I_2.$$

$$AP = P\begin{pmatrix} 6 & 0 \\ 0 & -4 \end{pmatrix}$$

$$\underbrace{AP\cdot P^{-1}}_{} = P\begin{pmatrix} 6 & 0 \\ 0 & -4 \end{pmatrix}P^{-1}$$

$$= A\cdot PP^{-1}$$

結合則

$$= A\cdot I_2 = A.$$

座標変換

(不半過)

$$P = \begin{pmatrix} 3 & 1 \\ 1 & -3 \end{pmatrix}$$

$$|P| = 3\cdot(-3) - 1\cdot 1 = -10 \neq 0.$$

$$(\vec{a}\ \vec{b})\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = (1\cdot\vec{a}+0\vec{b}\quad 0\vec{a}+1\vec{b})$$
$$= (\vec{a}\ \vec{b})$$

$\rightsquigarrow AI_2 = A$, $I_2A = A$.

$$I_2A = I_2(\vec{a}\ \vec{b}) = (I_2\vec{a}\ \ I_2\vec{b})$$
$$= (\vec{a}\ \vec{b})$$

まとめ　$A = P\begin{pmatrix}6 & 0\\ 0 & -4\end{pmatrix}P^{-1}$.

$$A^2 = P\begin{pmatrix}6 & 0\\ 0 & -4\end{pmatrix}\underbrace{P^{-1}P}_{=I_2}\begin{pmatrix}6 & 0\\ 0 & -4\end{pmatrix}P^{-1}$$

結合則

$$(AB)\cdot C = A(BC)$$

$$= P\begin{pmatrix}6 & 0\\ 0 & -4\end{pmatrix}\begin{pmatrix}6 & 0\\ 0 & -4\end{pmatrix}P^{-1}$$
$$= P\begin{pmatrix}6^2 & 0\\ 0 & (-4)^2\end{pmatrix}P^{-1}$$

$$\begin{pmatrix}\alpha & 0\\ 0 & \beta\end{pmatrix}\begin{pmatrix}a & 0\\ 0 & b\end{pmatrix} = \begin{pmatrix}\alpha a & 0\\ 0 & \beta b\end{pmatrix}$$

$$A^n = P\begin{pmatrix}6^n & 0\\ 0 & (-4)^n\end{pmatrix}P^{-1}$$

と計算できる.

$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ に対して A^n を計算する

$$\Phi_A(\lambda) = \lambda^2 - 4\lambda - 5 = (\lambda - 5)(\lambda + 1)$$

$$= \begin{vmatrix} \lambda - 1 & -2 \\ -4 & \lambda - 3 \end{vmatrix}$$

$$\lambda I_2 - A = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} \lambda - 1 & -2 \\ -4 & \lambda - 3 \end{pmatrix}$$

$\lambda = 5$

$$\begin{pmatrix} 4 & -2 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff 4x - 2y = 0$$

$$\iff 2x - y = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x \end{pmatrix} = x \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$