5月16日 経数入門

x	y
x_1	y_1
x_2	y_2
$\vdots$	$\vdots$
x_N	y_N

モデル

$x \rightsquigarrow y$

説明変数　　目的変数

$$y = ax + b.$$

$$\varepsilon = y - (ax+b) \quad \text{誤差.}$$

↑ 実測値　　↑ 理論値

① $\bar{\varepsilon} = 0 \rightsquigarrow \boxed{\bar{y} = a\bar{x} + b}$.

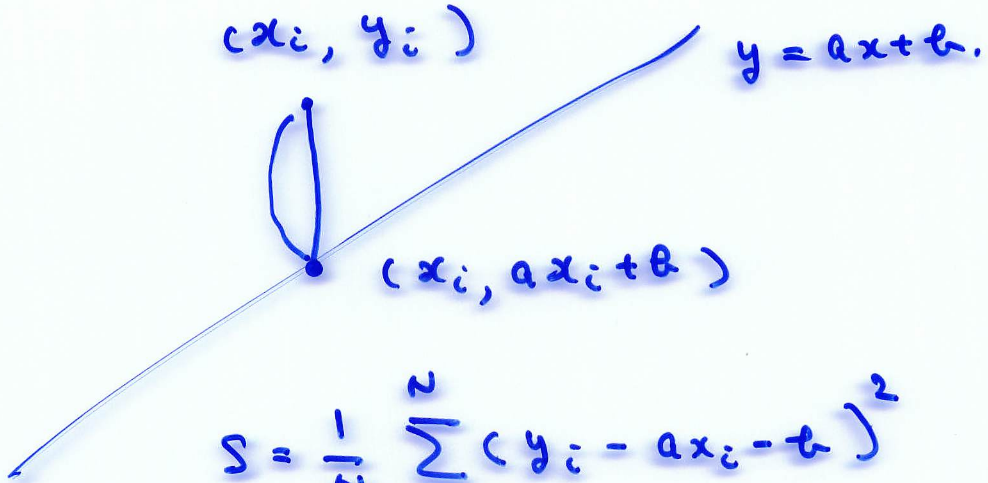
② $V(\varepsilon)$: 最小.

$$= \| \vec{y} - a\vec{x} \|^2$$

$$\rightsquigarrow = a^2 \|\vec{x}\|^2 - 2a(\vec{x}, \vec{y}) + \|\vec{y}\|^2$$

$$= \|\vec{x}\|^2 \left(a - \frac{(\vec{x}, \vec{y})}{\|\vec{x}\|^2} \right)^2 + \|\vec{y}\|^2 - \frac{(\vec{x}, \vec{y})^2}{\|\vec{x}\|^2}.$$

$$a = \frac{(\vec{x}, \vec{y})}{\|\vec{x}\|^2} = \frac{c_{xy}}{V(x)}$$



$$S = \frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i - b)^2$$

が最小にする a, b.

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aを止めてbで微分

$$\frac{\partial S}{\partial b} = \frac{1}{N}\sum_{i=1}^{N} 2(y_i - ax_i - b)\times(-1)$$

$$= 0$$

$$\boxed{\begin{array}{l}\dfrac{d\{x(t)\}^2}{dt} = 2x(t)x'(t) \\ \text{合成関数の微分.}\end{array}}$$

$$2\left(\frac{1}{N}\sum_{i=1}^{N} y_i - a\frac{1}{N}\sum_{i=1}^{N} x_i - b\underbrace{\frac{1}{N}\sum_{i=1}^{N} 1}_{=1}\right) = 0$$

$$\rightsquigarrow \cancel{2}\,\bar{y} - a\bar{x} - b = 0$$

代入

$$S = \frac{1}{N}\sum_{i=1}^{N}\{y_i - ax_i - (\bar{y} - a\bar{x})\}^2$$

$$= \frac{1}{N}\sum_{i=1}^{N}\{(y_i - \bar{y}) - a(x_i - \bar{x})\}^2$$

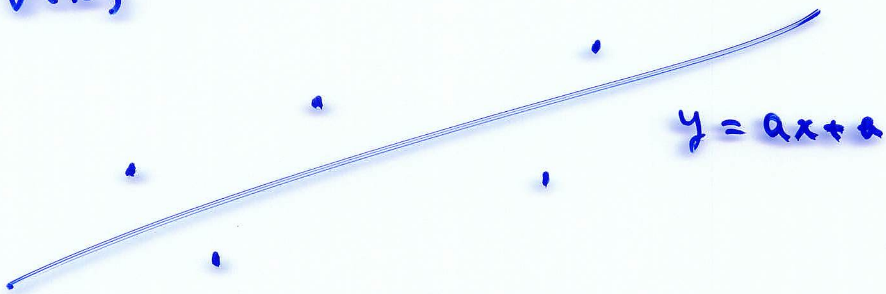
前回と同じ.

$$= \|\vec{y} - a\vec{x}\|^2 \rightsquigarrow \text{前回と同じ}$$

1) $b = \bar{y} - a\bar{x}$

2) $a = \dfrac{c_{xy}}{V(x)}$

回帰直線



$y = ax + b$: $x \rightsquigarrow y = ax + b$

モデルとして適したもの

$$V(\varepsilon) = \|\vec{y}\|^2 - \frac{(\vec{x}, \vec{y})^2}{\|\vec{x}\|^2}$$

回帰直線の場合.

$$\begin{aligned} V(\varepsilon) &\\ \| &\\ S &= \|\vec{y}\|^2 \left(1 - \frac{(\vec{x}, \vec{y})^2}{\|\vec{y}\|^2 \|\vec{x}\|^2}\right) \end{aligned}$$

rho

$$\boxed{\rho_{xy} = \frac{c_{xy}}{\sqrt{V(x)}\sqrt{V(y)}} = \frac{(\vec{x}, \vec{y})}{\|\vec{x}\| \, \|\vec{y}\|}}$$

xとyの相関係数

$$= V(y)\left(1 - \rho_{xy}^2\right) \geqq 0$$

- $S \geqq 0 \rightsquigarrow \underline{-1 \leqq \rho_{xy} \leqq 1}$

- $|\rho_{xy}| \longrightarrow 1 \Rightarrow S \longrightarrow 0$

回帰直線に（相関関係が強くなる.

$$\rho_{xy} = \frac{C_{xy}}{\sqrt{V(x)}\sqrt{V(y)}} = \frac{(\vec{x}, \vec{y})}{\|\vec{x}\| \|\vec{y}\|}$$

xとyの相関係数.

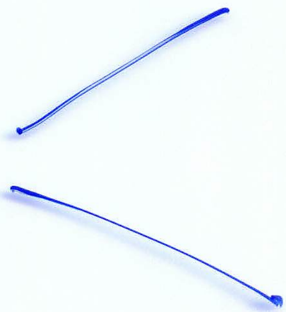
- $|\rho_{xy}| \leq 1$
- $|\rho_{xy}| \longrightarrow 1 \Rightarrow S = \sum_{i=1}^{N} (y_i - ax_i - b) \longrightarrow 0$ (回帰直線のS)

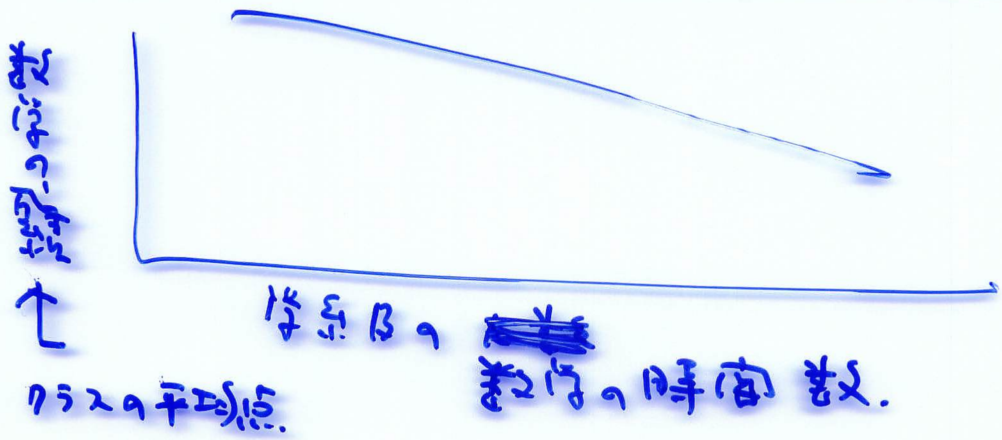
データがより直線上に分布.

$$a = \frac{C_{xy}}{V(x)} \gtreqless 0 \iff \rho_{xy} = \frac{C_{xy}}{\sqrt{V(x)}\sqrt{V(y)}} \gtreqless 0$$

$\rho_{xy} > 0$ 正の相関関係

< 0 負の ————





2×2

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{matrix} \leftarrow \text{1행 row} \\ \leftarrow \text{2행 row} \end{matrix}$$

$\uparrow$ $\uparrow$

1열 2열. column

1st row

‖

1열.

열 [illegible]

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

a_{12} — 1행 2열

a_{22} 2행 2열

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$A \pm B = \begin{pmatrix} a_{11} \pm b_{11} & a_{12} \pm b_{12} \\ a_{21} \pm b_{21} & a_{22} \pm b_{22} \end{pmatrix}$$

$$\lambda A = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} \\ \lambda a_{21} & \lambda a_{22} \end{pmatrix}$$

$$A = \begin{pmatrix} \boxed{\begin{matrix} a \\ c \end{matrix}} & \begin{matrix} b \\ d \end{matrix} \end{pmatrix}$$

$\vec{a}_1$ $\quad$ $\vec{a}_2$

$$A \begin{pmatrix} x \\ y \end{pmatrix} = x \begin{pmatrix} a \\ c \end{pmatrix} + y \begin{pmatrix} b \\ d \end{pmatrix} = \begin{pmatrix} xa + yb \\ xc + yd \end{pmatrix}$$

$$A = (\vec{a}_1 \ \vec{a}_2)$$

$$A \begin{pmatrix} x \\ y \end{pmatrix} = (\vec{a}_1 \ \vec{a}_2) \begin{pmatrix} x \\ y \end{pmatrix} = x\vec{a}_1 + y\vec{a}_2 .$$

$$\mathbb{R}^2 \ni \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto A\begin{pmatrix} x \\ y \end{pmatrix} = x\vec{a}_1 + y\vec{a}_2 \in \mathbb{R}^2$$

$$A = (\vec{a}_1 \ \vec{a}_2)$$

$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

単位行列

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = x\begin{pmatrix} 1 \\ 0 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\boxed{I_2 \vec{x} = \vec{x}}$$

identity.

A : 2×2行列　$B = (\vec{b}_1 \ \vec{b}_2)$ 2×2行列.

$AB = (A\vec{b}_1 \ \ A\vec{b}_2)$ 2×2行列.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & \boxed{a_{11}b_{12} + a_{12}b_{22}} \\ a_{21}b_{11} + a_{22}b_{21} & \boxed{a_{21}b_{12} + a_{22}b_{22}} \end{pmatrix}$$

I

(1) $\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(2) $\begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(3) $\begin{pmatrix} \lambda & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(4) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

II $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ に対して

$A^2 - (a+d)A$

を計算する.

III. $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$