ベクトル

CT 186 page.

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \in \mathbb{R}^N$$

$$\vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} \in \mathbb{R}^N$$

$$(\vec{x}, \vec{y}) = \sum_{i=1}^{N} x_i y_i$$

$$\|\vec{x}\|^2 = (\vec{x}, \vec{x})$$

$$= \sum_{i=1}^{N} x_i^2$$

1° $(\vec{x}+\vec{y}, \vec{z}) = (\vec{x}, \vec{z}) + (\vec{y}, \vec{z})$

2° $(\vec{x}, \vec{y}+\vec{z}) = (\vec{x}, \vec{y}) + (\vec{x}, \vec{z})$

3° $(\lambda\vec{x}, \vec{y}) = (\vec{x}, \lambda\vec{y}) = \lambda(\vec{x}, \vec{y})$

4° $(\vec{x}, \vec{y}) = (\vec{y}, \vec{z})$

公式 $\|\vec{x}+\vec{y}\|^2 = \|\vec{x}\|^2 + 2(\vec{x}, \vec{y}) + \|\vec{y}\|^2$

左 $= (\vec{x}+\vec{y}, \vec{x}+\vec{y})$

$= (\vec{x}, \vec{x}+\vec{y}) + (\vec{y}, \vec{x}+\vec{y})$

$= (\vec{x}, \vec{x}) + (\vec{x}, \vec{y}) + (\vec{y}, \vec{x}) + (\vec{y}, \vec{y})$

$= \|\vec{x}\|^2 + 2(\vec{x}, \vec{y}) + \|\vec{y}\|^2 =$ 右

the left hand side LHS

right RHS.

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x_1	y_1
x_2	y_2
$\vdots$	$\vdots$
x_N	y_N

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$\vec{x} = \frac{1}{\sqrt{N}}\begin{pmatrix} x_1 - \bar{x} \\ x_2 - \bar{x} \\ \vdots \\ x_N - \bar{x} \end{pmatrix}, \quad \vec{y} = \frac{1}{\sqrt{N}}\begin{pmatrix} y_1 - \bar{y} \\ y_2 - \bar{y} \\ \vdots \\ y_N - \bar{y} \end{pmatrix}$$

$$\|\vec{x}\|^2 = \sum_{i=1}^{N}\left\{\frac{1}{\sqrt{N}}(x_i - \bar{x})\right\}^2$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 = V(x)$$

$$(\vec{x}, \vec{y}) = \sum_{i=1}^{N} \frac{1}{\sqrt{N}}(x_i - \bar{x}) \cdot \frac{1}{\sqrt{N}}(y_i - \bar{y})$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})$$

$$= C_{xy} = \mathrm{Cov}(x, y)$$

共分散 (covariance)

$z = sx + ty$ $\quad s, t$ 定数 ③

$z_i = s x_i + t y_i \quad (i = 1, 2, \cdots, N)$

$$\bar{z} = \frac{1}{N} \sum_{i=1}^{N} (s x_i + t y_i)$$

$$= s \cdot \frac{1}{N} \sum_{i=1}^{N} x_i + t \cdot \frac{1}{N} \sum_{i=1}^{N} y_i$$

$$= s \bar{x} + t \bar{y}$$

<u>まとめ</u> $\bar{z} = s\bar{x} + t\bar{y}$.

第 i 成分.

$$\vec{z} = \left(\underbrace{\frac{1}{\sqrt{N}} (z_i - \bar{z})} \right)$$

$$= \frac{1}{\sqrt{N}} \left\{ (s x_i + t y_i) - (s \bar{x} + t \bar{y}) \right\}$$

$$= s \cdot \frac{1}{\sqrt{N}} (x_i - \bar{x}) + t \frac{1}{\sqrt{N}} (y_i - \bar{y})$$

$z = sx + ty \longleftrightarrow \vec{z} = s\vec{x} + t\vec{y}$

$(s, t \in \mathbb{R})$
定数

$$V(z) = \|\vec{z}\|^2 = \|s\vec{x} + t\vec{y}\|^2$$

$$= \|s\vec{x}\|^2 + 2(s\vec{x}, t\vec{y}) + \|t\vec{y}\|^2$$

$$= s^2 \|\vec{x}\|^2 + 2st(\vec{x}, \vec{y}) + t^2 \|\vec{y}\|^2$$

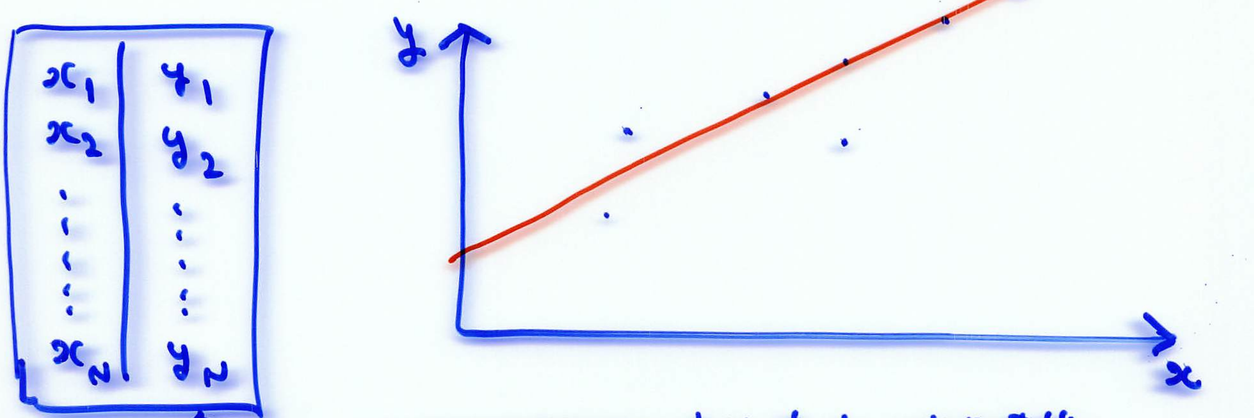
$$\boxed{\|t\vec{x}\| = |t| \, \|\vec{x}\|}$$

$$= s^2 V(x) + 2st\, C_{xy} + t^2 V(y)$$

回帰分析

CT193 p.

「相関関係がある」



実験家が測定する量を出す.

実験家が可以的得(決)

理論

$x \to y$

説明変数　目的変数 objective var.

exlanetary var.

$$y = ax + b$$

y_i：実測値

$ax_i + b$：理論値.

$\varepsilon = y - ax - b$.　誤差.

① 平均 0.
② $V(\varepsilon)$ 最小.

a, b を決める.

① $\bar{\varepsilon} = \bar{y} - a\bar{x} - b = 0$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i = \frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i - b)$$

$$= \frac{1}{N}\sum_i y_i - \frac{a}{N}\sum_i x_i - \frac{b}{N}\underbrace{\sum 1}_{=N} = \bar{y} - a\bar{x} - b.$$

② ① $\boxed{b = \bar{y} - a\bar{x}}$ $\quad \bar{y} \quad \varepsilon = y - ax - b$

$$V(\varepsilon) = \frac{1}{N}\sum_{i=1}^{N}\left\{(y_i - ax_i - b) - (\bar{y} - a\bar{x} - b)\right\}^2$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left\{(y_i - \bar{y}) - a(x_i - \bar{x})\right\}^2$$

$$= \|\vec{y} - a\vec{x}\|^2$$

$$= \|\vec{y}\|^2 - \underline{\underline{2}}(\vec{y}, a\vec{x}) + a^2\|\vec{x}\|^2$$

$$= a\,\|\vec{x}\|^2 - \underline{\underline{2}}a(\vec{x}, \vec{y}) + \|\vec{y}\|^2$$

$\|\vec{x}\| = 0 \iff \vec{x} = \vec{0}$

$\iff x_1 = \bar{x},\ x_2 = \bar{x},\ \cdots,\ x_N = \bar{x}$

データ

以下 $\|\vec{x}\|^2 > 0$ と考える.

$$= \|\vec{x}\|^2\left(a^2 - \frac{2(\vec{x}, \vec{y})}{\|\vec{x}\|^2}a + \frac{\|\vec{y}\|^2}{\|\vec{x}\|^2}\right)$$

$$= \|\vec{x}\|^2\left(a - \frac{(\vec{x}, \vec{y})}{\|\vec{x}\|^2}\right)^2 + \boxed{\|\vec{y}\|^2 - \frac{(\vec{x}, \vec{y})^2}{\|\vec{x}\|^2}}$$

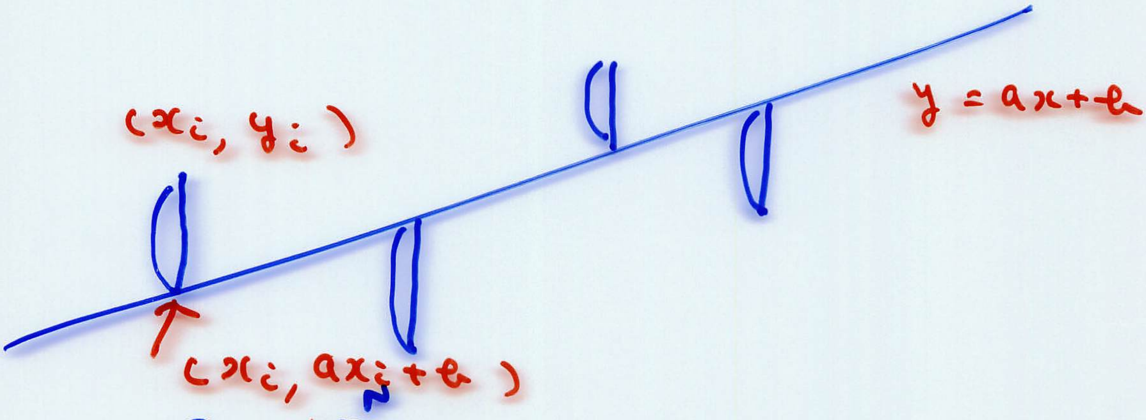
[illegible]使う.

① $\bar{y} = a\bar{x} + b$ $b = \bar{y} - a\bar{x}$

② $V(\varepsilon)$: 問2.4.

$$a = \frac{(\vec{x}, \vec{y})}{\|\vec{x}\|^2} = \frac{C_{xy}}{V(x)}$$

$y = ax + b$ 回帰直線



$$S = \frac{1}{N}\sum_{i=1}^{N} (y_i - a x_i - b)^2$$ 5.11 問2.11.12

残差平方. ですよ. → 平均.

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$f(t) = \| t\vec{a} - \vec{b} \|^2$ を最小にする

t を求めよ.