2階の差分方程式

$$a_{n+2} + p\,a_{n+1} + q\,a_n = 0$$

$$\lambda^2 + p\lambda + q = 0$$ 特性方程式

$p, q \in \mathbb{R}$

$D = p^2 - 4q$ 判別式

$D > 0$ OK.

$D = 0$ OK.

$D < 0$ の場合は?

C Text 15, 16 page.

$$\lambda = \frac{-p \pm \sqrt{p^2-4q}}{2}$$
$$= \frac{-p \pm i\sqrt{4q-p^2}}{2}$$

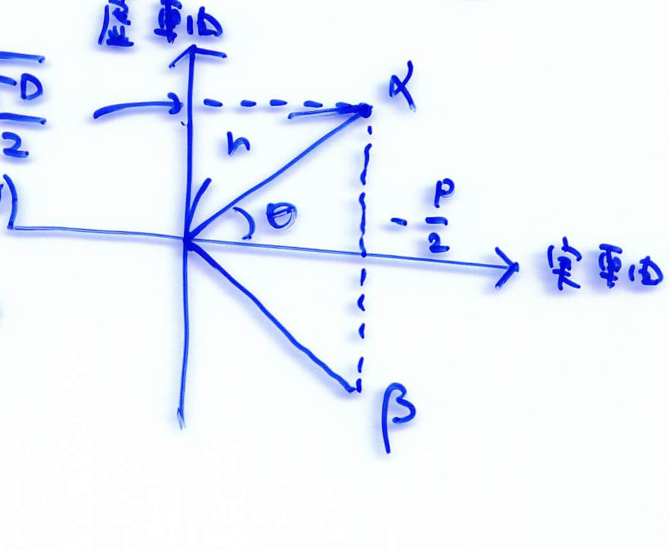
$z = a + ib \quad a, b \in \mathbb{R}$

$\bar{z} = a - ib$ z の共役複素数

$$\alpha = -\frac{p}{2} + i\frac{\sqrt{-D}}{2}, \quad \beta = -\frac{p}{2} - i\frac{\sqrt{-D}}{2}$$

$\alpha = r(\cos\theta + i\sin\theta)$ $\frac{\sqrt{-D}}{2}$

$\beta = r(\cos\theta - i\sin\theta)$



極表示.. Gauss 平面

複素平面

$$\alpha = r(\cos\theta + i\sin\theta)$$

$$\beta = r(\cos\theta - i\sin\theta)$$

$$= r(\cos(-\theta) + i\sin(-\theta))$$

一般的な公式

- $$(\cos\theta + i\sin\theta)(\cos\theta' + i\sin\theta')$$
$$= \cos(\theta+\theta') + i\sin(\theta+\theta')$$

- $$\frac{1}{\cos\theta + i\sin\theta} = \cos(-\theta) + i\sin(-\theta)$$

$$\alpha^n = r^n(\cos n\theta + i\sin n\theta)$$

$$\beta^n = r^n(\cos n\theta - i\sin n\theta)$$

$$a_{n+2} + p\, a_{n+1} + q\, a_n = 0$$

$$\boxed{\alpha+\beta = -q,\quad \alpha\beta = q.}$$

$$a_{n+2} - (\alpha+\beta)a_{n+1} + \alpha\beta\, a_n = 0$$

$$\begin{cases} a_{n+2} - \alpha\, a_{n+1} = \beta(a_{n+1} - \alpha a_n) \\ a_{n+2} - \beta\, a_{n+1} = \alpha(a_{n+1} - \beta a_n) \end{cases}$$

$$\begin{cases} a_{n+1} - \alpha\, a_n = \beta^n(a_1 - \alpha a_0) \\ a_{n+1} - \beta\, a_n = \alpha^n(a_1 - \beta a_0) \end{cases}$$

$-)$

$$(\beta - \alpha)a_n = \beta^n(a_1 - \alpha a_0) - \alpha^n(a_1 - \beta a_0)$$

$$a_n = \frac{\beta^n - \alpha^n}{\beta - \alpha}\, a_1 - \frac{\alpha\beta\left(\beta^{n-1} - \alpha^{n-1}\right)}{\beta - \alpha}\, a_0 \quad (3$$

$$= \cancel{-p} + q$$

$$\beta^n - \alpha^n = -2i \sin(n\theta) \cdot r^n$$

$$\beta - \alpha = -2i \sin\theta \cdot r$$

$$= r^{n-1} \frac{\sin n\theta}{\sin\theta}\, a_1 - \frac{\sin(n-1)\theta}{\sin\theta}\, r^{n-2}\, q\, a_0$$

$$|r|<1 \Longrightarrow r^n \to 0$$

はさみうちの定理

$$\left(\begin{array}{l} a_n \leqq b_n \leqq c_n \quad (\text{全ての } n) \\ a_n \to \alpha,\ c_n \to \alpha \end{array}\right.$$

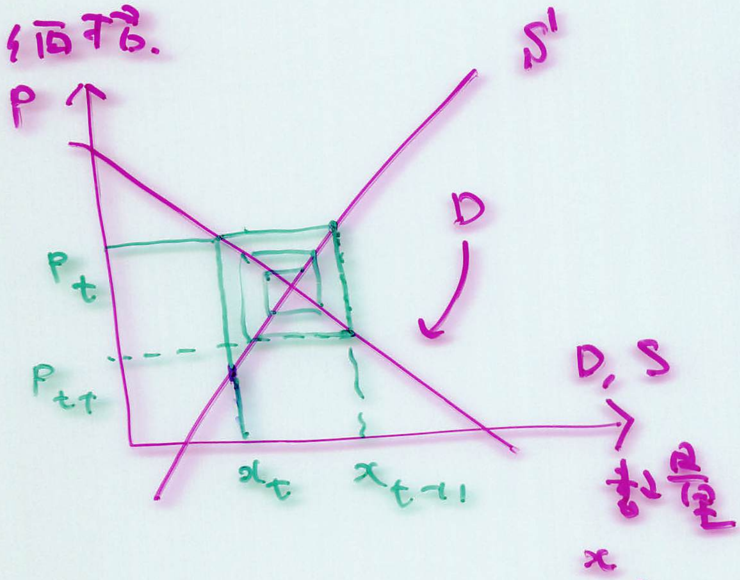
$$\Longrightarrow b_n \to \alpha.$$

$|r|<1$

$$r^n \sin n\theta \to 0$$

$$|r^n \sin n\theta| \leqq |r|^n$$

$$\underbrace{-|r|^n}_{\substack{\downarrow\\0}} \leqq \underbrace{r^n \sin n\theta}_{\substack{\downarrow\\0\ \text{結論}.}} \leqq \underbrace{|r|^n}_{\substack{\downarrow\\0}}$$



需要曲線

D: $p = aD + b$

供給曲線

S: $p = a'S + b'$

第 t 期の供給量 x_t

D を使う.

$$p_t = a x_t + b$$

第 (t+1) 期の供給量 x_{t+1}

S'

$$p_t = a' x_{t+1} + b'$$

$$x_{t+1} = \frac{1}{a'}(p_t - b')$$

$$x_{t+1} = \frac{1}{a'}(a x_t + b - b')$$

$$\boxed{x_{t+1} = \frac{a}{a'} x_t + \frac{(b - b')}{a'}}$$

各自

$$\lambda = \frac{a}{a'} \lambda + \frac{b - b'}{a'} \rightsquigarrow \lambda = \lambda_0 = \frac{b - b'}{a' - a}$$

$$x_{t+1} - \lambda_0 = \frac{a}{a'}(x_t - \lambda_0)$$

$$x_t - \lambda_0 = \left(\frac{a}{a'}\right)^t (x_0 - \lambda_0)$$

$$x_t = \lambda_0 + \left(\frac{a}{a'}\right)^t (x_0 - \lambda_0)$$

$\left|\frac{a}{a'}\right| < 1$ のとき $\quad x_t \longrightarrow \lambda_0$

くもの巣過程は安定.

$$p_t = a x_t + b$$

$$\longrightarrow a\lambda_0 + b.$$

$a_n \to \alpha,\ b_n \to \beta.$

1) $a_n \pm b_n \longrightarrow \alpha \pm \beta$

2) $a_n b_n \longrightarrow \alpha\beta.$

3) $a_n \neq 0,\ \alpha \neq 0 \quad \frac{1}{a_n} \to \frac{1}{\alpha}$

$$p = ax + b$$

$$-)\ p = a'x + b'.$$

$$0 = (a - a')x + (b - b')$$

$$x = \frac{b' - b}{a - a'} = \lambda_0$$

DとSの交点

$(\lambda_0,\ a\lambda_0 + b)$

ポートフォリオ理論入門.

I $r \neq 1$ とする

$$T_N = 1 + 2 \cdot r + 3r^2 + \cdots + N r^{N-1}$$

$$= ?$$

II. $$a_{n+1} = \frac{1}{4} a_n + 2$$

を満たして $\lim_{n \to +\infty} a_n$ を求めよ.