等差数列

$$a_{n+1} - a_n = \alpha \rightsquigarrow a_n = a_0 + n\alpha$$

α：公差

等比数列

$$a_{n+1} = \alpha\, a_n \qquad \alpha：公比$$

$$\rightsquigarrow a_n = \alpha^n a_0$$

1階の差分方程式.

$$\begin{array}{rl} & a_{n+1} = 5\, a_n + 1 \\ -) & \alpha = 5\alpha + 1 \end{array} \quad \longleftrightarrow \quad \alpha = -\frac{1}{4}$$

$\boxed{a_n = -\frac{1}{4} \quad (n = 0, 1, 2, \cdots) \text{ が解.}}$

$$a_{n+1} - \alpha = 5(a_n - \alpha)$$

$$\rightsquigarrow a_n - \alpha = 5^n (a_0 - \alpha)$$

$$a_n = -\frac{1}{4} + 5^n \left(a_0 + \frac{1}{4}\right)$$

CT 9ページ.

0年度期首 借金 M　　$b_0 = M$

毎年度末に T 払う. 利息の計算をする.
100r %.

$$b_1 = (b_0 - T)(1+r)$$

↑ 第1年度期首の残高.　　b_n：第n年度の期首の残高

⋮

$$b_{n+1} = (b_n - T)(1+r)$$

$$-)\quad \alpha = (\alpha - T)(1+r)$$

$(1+r)T = r\alpha.$

$$b_{n+1} - \alpha = (b_n - \alpha)(1+r) \qquad \boxed{\alpha = \frac{1+r}{r}T}$$

$$(b_n - \alpha) = (1+r)^n (b_0 - \alpha)$$

$$b_n = \alpha + (1+r)^n (b_0 - \alpha)$$

$r = 0.01 \quad M = b_0 = 600.$

$b_{100} = 0$ となる T はいくらか?

$$a_{n+2} + p\, a_{n+1} + q\, a_n = 0$$

$$a_{n+2} - 5\, a_{n+1} + 6\, a_n = 0$$

$$a_{n+2} - (3+2)\, a_{n+1} + 3 \cdot 2\, a_n = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$ 特性方程式.

$$(\lambda - 3)(\lambda - 2) = 0$$

$$\begin{cases} a_{n+2} - 3\, a_{n+1} = 2(a_{n+1} - 3a_n) \\ a_{n+2} - 2\, a_{n+1} = 3(a_{n+1} - 2a_n) \end{cases}$$

$$\begin{array}{rl} & a_{n+1} - 3a_n = 2^n(a_1 - 3a_0) \\ -) & a_{n+1} - 2a_n = 3^n(a_1 - 2a_0) \\ \hline & -a_n = 2^n(a_1 - 3a_0) - 3^n(a_1 - 2a_0) \end{array}$$

$$a_{n+2} + p a_{n+1} + q a_n = 0 \quad (p, q \in \mathbb{R})$$

$\lambda^2 + p\lambda + q = 0$ の2解 α, β.

$$\boxed{-p = \alpha + \beta, \quad q = \alpha\beta}$$

解と係数の関係..

$$a_{n+2} - (\alpha+\beta) a_{n+1} + \alpha\beta\, a_n = 0$$

$$\begin{cases} a_{n+2} - \alpha a_{n+1} = \beta (a_{n+1} - \alpha a_n) \\ a_{n+2} - \beta a_{n+1} = \alpha (a_{n+1} - \cancel{\alpha}\beta a_n) \end{cases}$$

$$\begin{cases} a_{n+1} - \alpha a_n = \beta^n (a_1 - \alpha a_0) \\ a_{n+1} - \beta a_n = \alpha^n (a_1 - \beta a_0) \end{cases}$$

$\alpha \neq \beta$

$$(\beta - \alpha) a_n = \beta^n (a_1 - \alpha a_0) - \alpha^n (a_1 - \beta a_0)$$

$$a_n = \frac{\beta^n - \alpha^n}{\beta - \alpha} a_1 - \alpha\beta \frac{\beta^{n-1} - \alpha^{n-1}}{\beta - \alpha} a_0$$

$D = p^2 - 4q = 0$ のとき $\alpha = \beta$.

$$a_{n+1} - \alpha a_n = \alpha^n (a_1 - \alpha a_0)$$

α^{n+1} で割る.

公差.

$$\frac{a_{n+1}}{\alpha^{n+1}} - \frac{a_n}{\alpha^n} = \alpha^{-1}(a_1 - \alpha a_0)$$

$t_n = \dfrac{a_n}{\alpha^n}$ とおくと $\{t_n\}$ は等差数列.

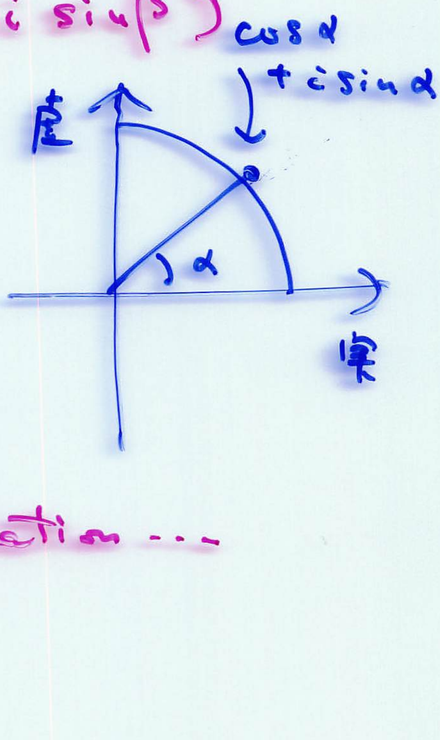
$$\frac{a_n}{\alpha^n} = \frac{a_0}{\alpha^0} + n\alpha^{-1}(a_1 - \alpha a_0)$$

$$a_n = \alpha^n a_0 + n\alpha^{n-1}(a_1 - \alpha a_0)$$

練習

① $(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)$
$= ?$

② $\dfrac{1}{\cos\alpha + i\sin\alpha} = ?$



$D<0$ の場合を示す.

景気 circulation …

$|r|<1 \qquad r^n \longrightarrow 0 \quad (n \to +\infty)$

<u>$r \neq 1$</u>

$$
\begin{array}{rl}
S_n = & 1 + r + r^2 + r^3 + \cdots + r^n \\
-)\ \ r S_n = & \quad\ \ r + r^2 + r^3 + \cdots + r^n + r^{n+1} \\
\hline
(1-r) S_n = & 1 - r^{n+1}
\end{array}
$$

$|r|<1$ とする.

$$S_n = \frac{1}{1-r} - \frac{1}{1-r} \times r^{n+1} \quad (r^{n+1} \longrightarrow 0)$$

$$\longrightarrow \frac{1}{1-r}$$

$|r|<1$

$$1 + r + r^2 + \cdots\cdots = \frac{1}{1-r}.$$

乗数効果
信用創造.

銀行に預金 d 　預金準備率 x

$$d \begin{cases} xd & \text{中央銀行に準備金} \\ (1-x)d & \text{貸出し.} \end{cases}$$

還 還流

$$(1-x)d \begin{cases} (1-x)xd & \text{準備金} \\ (1-x)^2 d & \text{貸し出し} \end{cases}$$

還流

貸し出しの総額.　　$0<x<1$

$$(1-x)d+(1-x)^2d+(1-x)^3d+\cdots$$

$$=(1-x)d\left\{1+(1-x)+(1-x)^2+\cdots\right\}$$

$$=(1-x)d\ \frac{1}{1-(1-x)}=\frac{1-x}{x}d.$$

預金dが生む信用創造.

- 応用について.
- D<0 の場合.
- 数列の拡張.
  (行列の話)

Ⅰ $a_{n+1} = \frac{1}{3} a_n + 2$ の解 を a_0 と n で表せ

Ⅱ $a_{n+2} + a_{n+1} - 6 a_n = 0$ を解け.