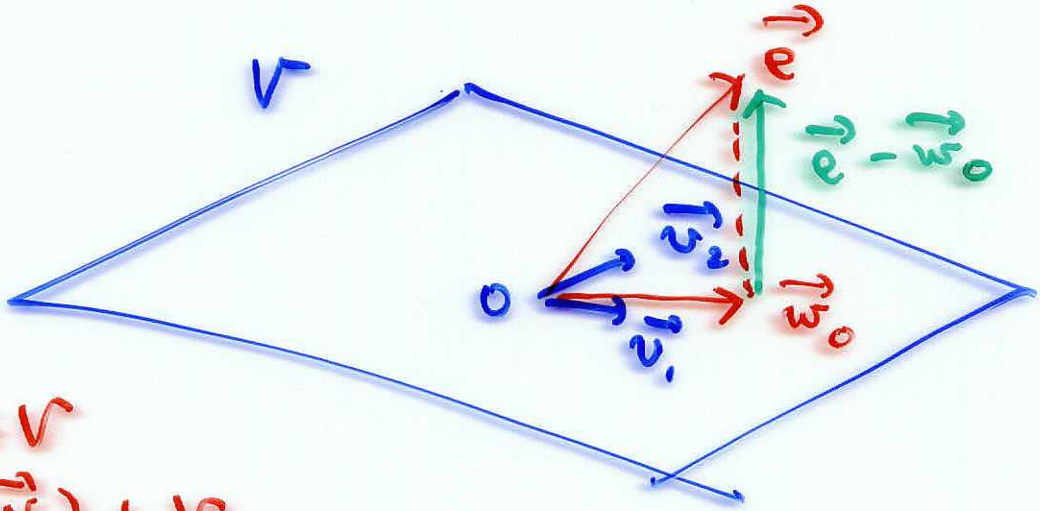
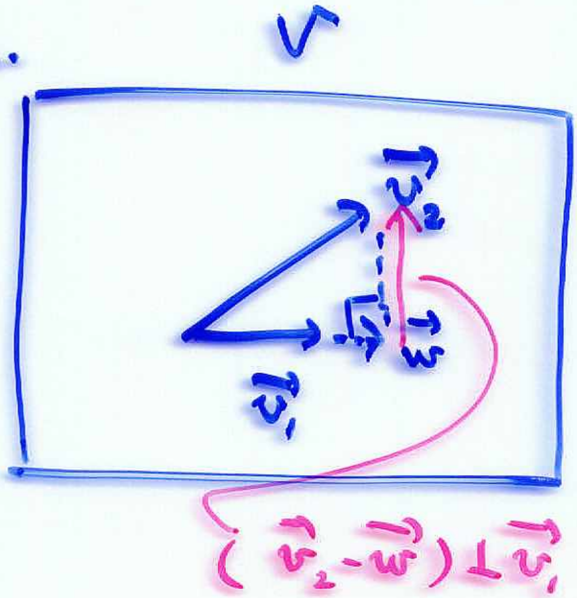


1) $\vec{w}_0 \in V$
2) $(\vec{e} - \vec{w}_0) \perp V$

$\vec{e}$ の V への正射影.

1) V の正規直交基底.

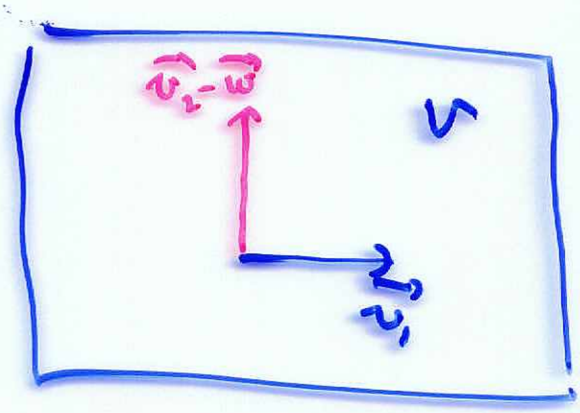


$$\vec{w} = \frac{(\vec{v}_1, \vec{v}_2)}{\|\vec{v}_1\|^2} \vec{v}_1$$

$$= \frac{4}{3} \vec{v}_1$$

$$\vec{v}_2 - \vec{w} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \frac{4}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$



$$\vec{p}_1 = \frac{1}{\|\vec{v}_1\|} \vec{v}_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{p}_2 = \frac{1}{\|\vec{v}_2 - w\|} (\vec{v}_2 - \vec{w})$$

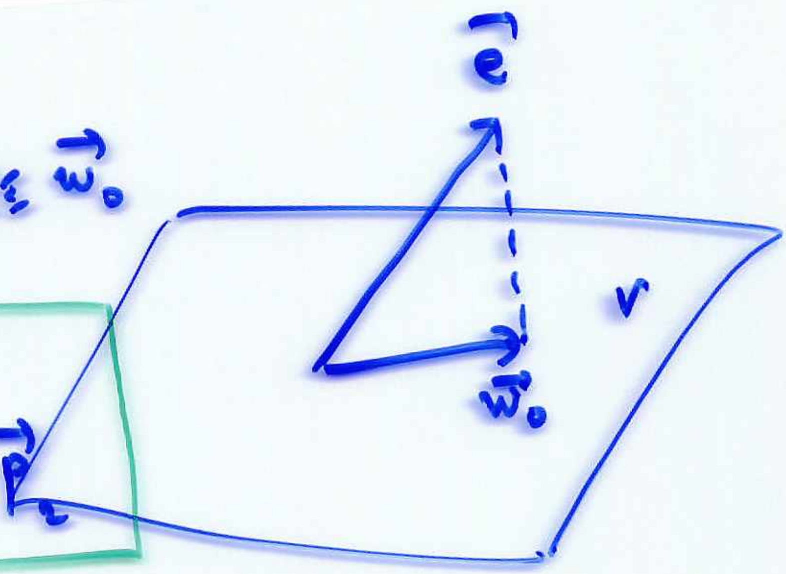
$$= \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

$$\rightsquigarrow \|\vec{p}_1\| = \|\vec{p}_2\| = 1$$

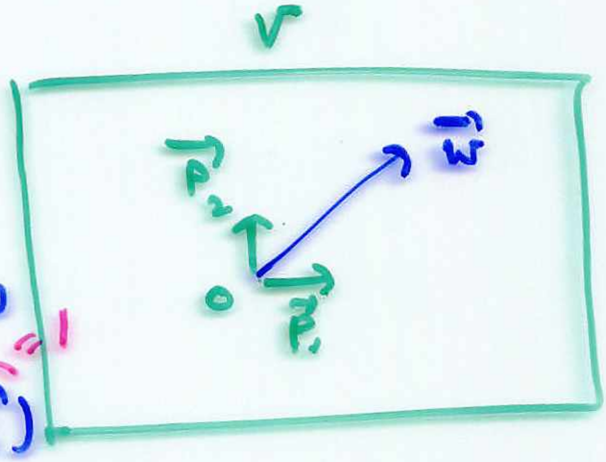
$$(\vec{p}_1, \vec{p}_2) = 0$$

V の正規直交基底

$\vec{e}$ の V への正射影を $\vec{w}_0$



$$\boxed{\vec{w}_0 = (\vec{p}_1,\vec{e})\,\vec{p}_1 + (\vec{p}_2,\vec{e})\,\vec{p}_2}$$



$$\boxed{\vec{w} = c_1\vec{p}_1 + c_2\vec{p}_2}$$

$$(\vec{e}-c_1\vec{p}_1-c_2\vec{p}_2,\ \vec{p}_1)=0$$

$$= (\vec{e},\vec{p}_1) - c_1 \overbrace{(\vec{p}_1,\vec{p}_1)}^{=1} - c_2\underbrace{(\vec{p}_2,\vec{p}_1)}_{=0}$$

$$\rightsquigarrow c_1 = (\vec{e},\vec{p}_1)$$

$$(\vec{e}-c_1\vec{p}_1-c_2\vec{p}_2,\ \vec{p}_2)=0 \rightsquigarrow c_2=(\vec{e},\vec{p}_2)$$

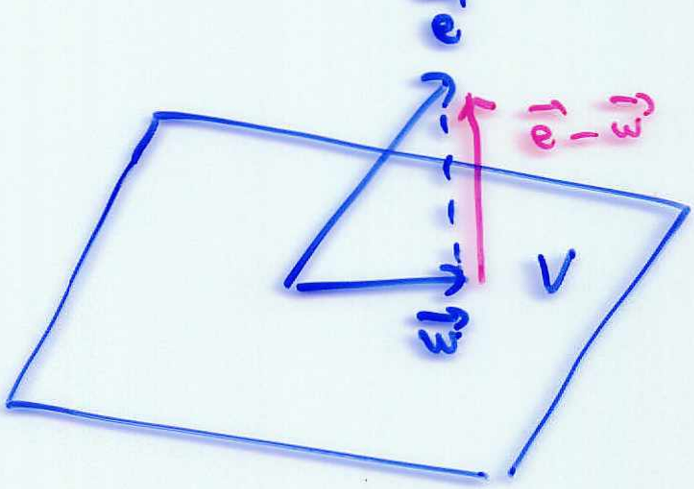
$$\vec{w} = (\vec{e},\vec{p}_1)\,\vec{p}_1 + (\vec{e},\vec{p}_2)\,\vec{p}_2$$

$\vec{e} = \begin{pmatrix}1\\0\\0\end{pmatrix}$ のとき

$$\vec{w} = \left(\begin{pmatrix}1\\0\\0\end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}\right)\cdot\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix} + \left(\begin{pmatrix}1\\0\\0\end{pmatrix}, \frac{1}{\sqrt{6}}\begin{pmatrix}2\\-1\\-1\end{pmatrix}\right)\cdot\frac{1}{\sqrt{6}}\begin{pmatrix}2\\-1\\-1\end{pmatrix}$$

$$= \frac{1}{3}\cdot 1\begin{pmatrix}1\\1\\1\end{pmatrix} + \frac{1}{6}\cdot 2\begin{pmatrix}2\\-1\\-1\end{pmatrix} = \frac{1}{3}\begin{pmatrix}3\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$(\vec{e} - \vec{w}) \perp V$



$\vec{e} - \vec{w}$

$= \begin{pmatrix}1\\0\\0\end{pmatrix} - \begin{pmatrix}1\\0\\0\end{pmatrix}$。

$= \vec{0}$

$\begin{pmatrix}1\\0\\0\end{pmatrix} = -\begin{pmatrix}1\\1\\1\end{pmatrix} + \begin{pmatrix}2\\1\\1\end{pmatrix}$

$\in V$

$\vec{e} = \begin{pmatrix}0\\1\\0\end{pmatrix}$

$$\vec{w}_0 = \left(\begin{pmatrix}0\\1\\0\end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}\right) \cdot \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$+ \left(\begin{pmatrix}0\\1\\0\end{pmatrix}, \frac{1}{\sqrt{6}}\begin{pmatrix}2\\-1\\-1\end{pmatrix}\right) \cdot \frac{1}{\sqrt{6}}\begin{pmatrix}2\\-1\\-1\end{pmatrix}$$

$$= \frac{1}{3} \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} - \frac{1}{6} \cdot \begin{pmatrix}2\\-1\\-1\end{pmatrix}$$

$$= \frac{1}{6}\left\{\begin{pmatrix}2\\2\\2\end{pmatrix} - \begin{pmatrix}2\\-1\\-1\end{pmatrix}\right\} = \frac{1}{6}\begin{pmatrix}0\\3\\3\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$\vec{e} - \vec{w} = \begin{pmatrix}0\\1\\0\end{pmatrix} - \frac{1}{2}\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\ \frac{1}{2}\\ -\frac{1}{2}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

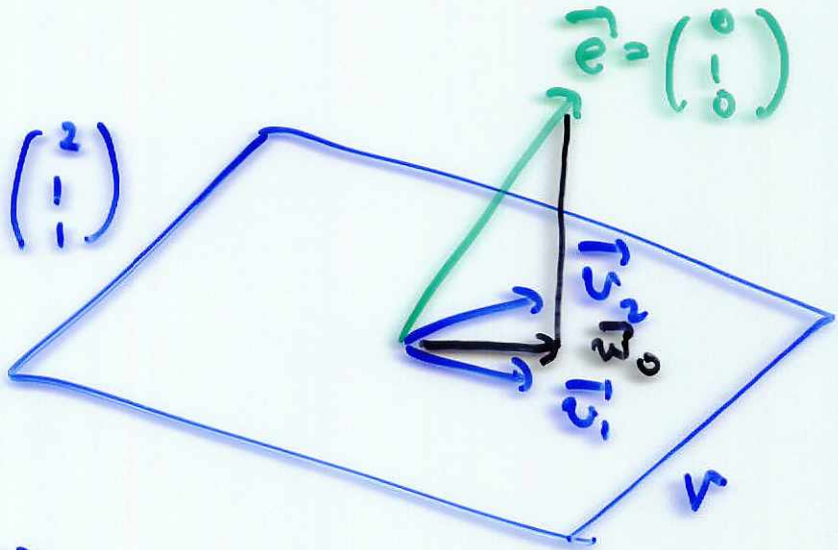
$$\vec{p}_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\|\vec{p}_1\| = \|\vec{p}_2\| = \|\vec{p}_3\| = 1$$

$$(\vec{p}_i, \vec{p}_j) = 0 \quad (i \neq j)$$

レポートVIIのヒント.

最小自乗法.



$$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$A = (\vec{v}_1 \ \vec{v}_2)$$

$$V = \mathrm{Im}(A) = \{ c_1\vec{v}_1 + c_2\vec{v}_2 \,;\, c_1, c_2 \in \mathbb{R} \}$$

$$\vec{w}_0 = c_1\vec{v}_1 + c_2\vec{v}_2 = (\vec{v}_1\ \vec{v}_2)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = A\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$(\vec{e} - \vec{w}_0, A\vec{y}) = 0 \quad (全ての\ \vec{y} \in \mathbb{R}^2)$$

$$\iff \left(\vec{e} - A\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, A\vec{y}\right) = 0 \quad (全ての\ \vec{y})$$

$$\iff \left({}^tA\left(\vec{e} - A\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}\right), \vec{y}\right) = 0 \quad (\ 〃\ )$$

$$\iff {}^tAA\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = {}^tA\vec{e}$$

正規方程式!

$A : m \times n \quad \vec{x} \in \mathbb{R}^n \rightsquigarrow A\vec{x} \in \mathbb{R}^m$

$$(A\vec{x}, \vec{y}) = (\vec{x}, {}^tA\vec{y})$$

$$(y \in \mathbb{R}^m)$$

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$${}^tAA = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 4 & 6 \end{pmatrix}$$

$${}^tA\vec{e} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 4 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 & -4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$ad - bc \neq 0$ のとき

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$\vec{w}_0 = \vec{v}_1 - \frac{1}{2}\vec{v}_2$ と書ける.

回帰分析

x	y	z
x_1	y_1	z_1
x_2	y_2	z_2
$\vdots$	$\vdots$	$\vdots$
x_N	y_N	z_N

z : 目的変数

モデル $z = ax + by + c$

x, y : 説明変数.

$\varepsilon = z - (ax + by + c)$ 誤差

・ $E(\varepsilon) = 0$ $\quad \bar{z} - a\bar{x} - b\bar{y} - c = 0$

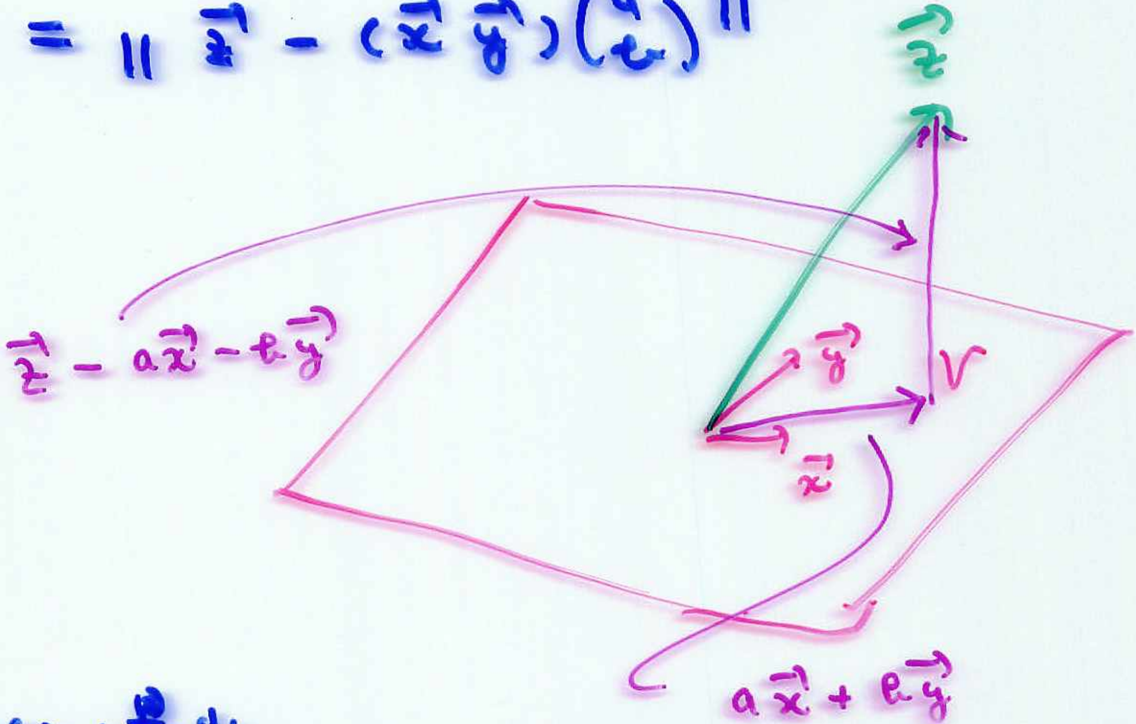
・ $\varepsilon = (z - \bar{z}) - a(x - \bar{x}) - b(y - \bar{y})$

$V(\varepsilon)$ が最小になるように a, b を定める.

$$\vec{x} = \frac{1}{\sqrt{N}} \begin{pmatrix} x_1 - \bar{x} \\ \vdots \\ x_N - \bar{x} \end{pmatrix} \qquad \boxed{\vec{\varepsilon} = \vec{z} - a\vec{x} - b\vec{y}}$$

$\vec{\varepsilon}, \vec{z}, \vec{x}, \vec{y}$ 同様.

$$V(\varepsilon) = \|\vec{\varepsilon}\|^2$$
$$= \|\vec{z} - a\vec{x} - b\vec{y}\|^2$$
$$= \|\vec{z} - (\vec{x}\ \vec{y})\begin{pmatrix} a \\ b \end{pmatrix}\|^2$$



$V(\varepsilon)$：最小

$\Longleftrightarrow (\vec{z} - a\vec{x} - b\vec{y}) \perp V$

$a\vec{x} + b\vec{y}$ は $\vec{z}$ の V への正射影

$A = (\vec{x}\ \vec{y})$

$$\boxed{{}^t\!AA \begin{pmatrix} a \\ b \end{pmatrix} = {}^t\!A\vec{z}.}$$ 正規方程式.

$${}^t\!AA = \begin{pmatrix} {}^t\vec{x} \\ {}^t\vec{y} \end{pmatrix} (\vec{x}\ \vec{y})$$
$$= \begin{pmatrix} \|\vec{x}\|^2 & (\vec{x}, \vec{y}) \\ (\vec{x}, \vec{y}) & \|\vec{y}\|^2 \end{pmatrix}$$
$$= \begin{pmatrix} V(x) & \sigma_{xy} \\ \sigma_{xy} & V(y) \end{pmatrix}$$ 分散行列.

$$V = \mathbb{R}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, + \mathbb{R}\begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix}$$ とする.

$\vec{e} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ の V への正射影を

正規方程式によって求めよ.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 2 \\ -1 & 1 \end{pmatrix}$$

2月 2日, 3日 に提出.
(木) (金) Box ①

今日までに出来なかった問題を.
提出してよい.