$y = f(t) = \frac{1}{\sqrt{t}}$ に対して $f^{(n)}(t) = ?$.

$$(t^{\alpha})' = \alpha t^{\alpha-1}.$$

$$f'(t) = -\frac{1}{2}\cdot\frac{1}{t\sqrt{t}} = -\frac{1}{2}t^{-\frac{3}{2}}$$

$$f''(t) = \frac{1}{2}\cdot\frac{3}{2}\,\frac{1}{t^2\sqrt{t}} = \frac{3}{2^2}t^{-\frac{5}{2}}$$

$$f^{(3)} = -\frac{1\cdot 3\cdot 5}{2\cdot 2\cdot 2}\,\frac{1}{t^3\sqrt{t}} = -\frac{1\cdot 3\cdot 5}{2^3}t^{-\frac{7}{2}}$$

$$f^{(4)} = \frac{1\cdot 3\cdot 5\cdot 7}{2^4}\cdot\frac{1}{t^4\sqrt{t}} = \frac{1\cdot 3\cdot 5\cdot 7}{2^4}t^{-\frac{9}{2}}$$

$$f^{(n)}(t) = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^n}(-1)^{n-1}\frac{1}{t^n\sqrt{t}}$$

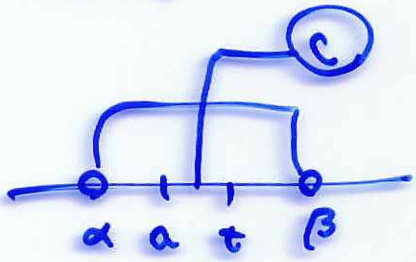
$$= t^{-\frac{2n+1}{2}}$$

と予想

$$f^{(n+1)} = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^n}\cdot(-1)^{n-1} \times\left(-\frac{2n+1}{2}\right)\frac{1}{t^{n+1}\sqrt{t}}$$

$$= \frac{1\cdot 3\cdots(2(n+1)-1)}{2^{n+1}}(-1)^{n+1-1}\frac{1}{t^{n+1}\sqrt{t}}$$

$f : (\alpha, \beta) \longrightarrow \mathbb{R}$ C^∞ 級



Taylor's theorem

$$f(t) = f(a) + f'(a)(t-a) + \frac{f''(a)}{2!}(t-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(t-a)^n + \boxed{\frac{f^{(n+1)}(c)}{(n+1)!}(t-a)^{n+1}}$$

ここで c は a と t の間にある。

剰余項

Remainder Term

項 term

因子 factor abc

IV $f(t) = \log(1+t)$

$$f'(t) = \frac{1}{1+t}, \quad f''(t) = -\frac{1}{(1+t)^2}$$

$$f^{(3)}(t) = \frac{2}{(1+t)^3}$$

$$f(t) = f(0) + f'(0)\cdot t + \frac{f''(0)}{2!}t^2 + \frac{f^{(3)}(c)}{3!}t^3$$

$$= 0 + 1\cdot t - \frac{1}{2!}t^2 + \frac{1}{3!}\cdot\frac{2}{(1+c)^3}t^3$$

$$= t - \frac{1}{2}t^2 + \frac{1}{3}\cdot\frac{2}{(1+c)^3}t^3$$

ただし c は 0 と t の間にある

$$e^t \geq 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} \quad (t \geq 0)$$

$$f(t) = e^t, \quad f'(t) = e^t, \quad \cdots \quad f^{(n)}(t) = e^t$$

$$e^t = f(0) + f'(0)t + \frac{1}{2!} f''(0) t^2$$

$$+ \frac{1}{3!} f^{(3)}(0) t^3 + \frac{1}{4!} f^{(4)}(c) t^4$$

$$= 1 + t + \frac{1}{2!} t^2 + \frac{1}{3!} t^3 + \underset{\substack{\vee\! \| \\ 0}}{\boxed{\frac{1}{4!} e^c t^4}}$$

$$\geq 1 + t + \frac{1}{2!} t^2 + \frac{1}{3!} t^3 .$$

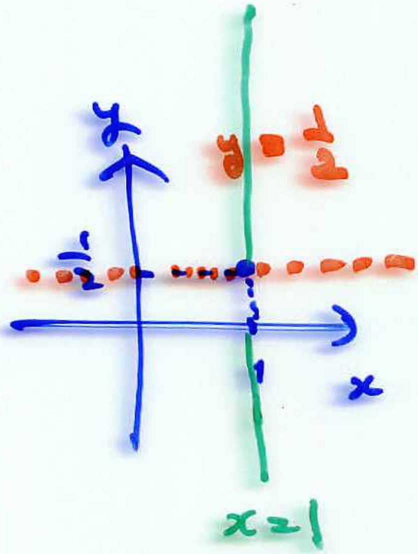
偏微分. CT 264 page.

un exemple

例

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$$

$$(x, y) \longmapsto x^2 - y^2$$



$(1, \frac{1}{2})$

$$F(x) = f(x, \tfrac{1}{2})$$

$$= x^2 - \frac{1}{4}$$

$$F'(x) = 2x - 0 = 2x$$

$$F'(1) = \lim_{x \to 1} \frac{F(x) - F(1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{f(x, \frac{1}{2}) - f(1, \frac{1}{2})}{x - 1}$$

$$= 2$$

$$\rightsquigarrow \quad f_x(1, \tfrac{1}{2}) = F'(1) = 2.$$

$$= \frac{\partial f}{\partial x}(1, \tfrac{1}{2})$$

$$G(y) = f(1, y) = 1 - y^2$$

$$G'(y) = -2y$$

$$f_y(1, \tfrac{1}{2}) = G'(\tfrac{1}{2}) = -1$$

$$\shortparallel$$

$\frac{\partial f}{\partial y}(1, \frac{1}{2})$ y に関する偏微分係数

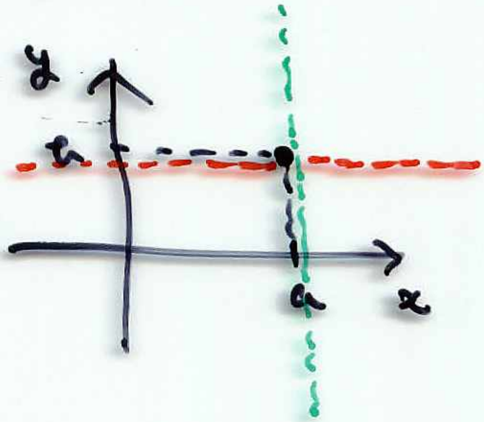
with respect to y

par rapport à y

134 8.10.

$$f(x, y) = x^3 + 2xy^2 + y^3$$

(a, b)



$$F(x) = x^3 + 2xb^2 + b^3$$

$$F'(x) = 3x^2 + 2b^2$$

$$f_x(a, b) = 3a^2 + 2b^2$$

$$G(y) = a^3 + 2ay^2 + y^3$$

$$G'(y) = 2a \cdot 2y + 3y^2$$

$$= 4ay + 3y^2$$

$$f_y(a, b) = 4ab + 3b^2$$

Exo 8.8 (1)

$$f(x,y) = x^3 + 2x^2y + 5y^4$$

$$f_x(x,y) = 3x^2 + 2y\cdot 2x + 0$$
$$= 3x^2 + 4xy.$$

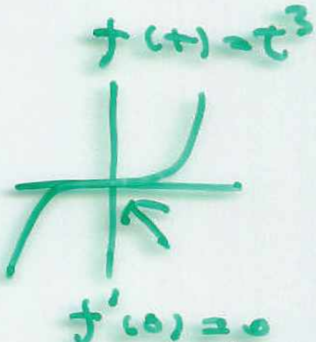
$$f_y(x,y) = 0 + 2x^2\cdot 1 + 5\cdot 4y^3$$
$$= 2x^2 + 20y^3.$$

1変数の話

$$f : (a,b) \longrightarrow \mathbb{R}$$

t_0 で極大(小)とする

$$\Longrightarrow f'(t_0) = 0$$

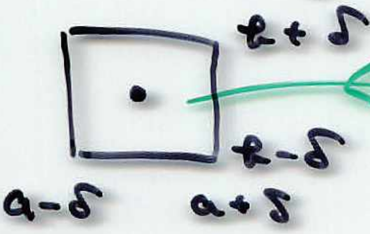


$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$$

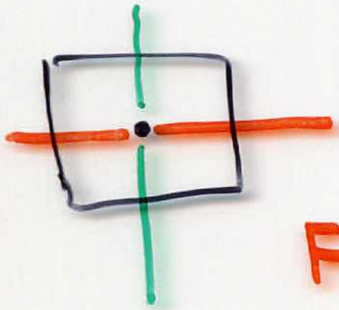
(a,b) で極小(大) とは 小さな $\delta > 0$ が存在し

$$f(x,y) \geqq (\leqq)\ f(a,b)$$

$$\left((x,y) \in [a-\delta, a+\delta] \times [b-\delta, b+\delta]\right)$$



ここに限ると
$f(a,b)$ が最小(大)値



$$F(x) = f(x, b)$$
$$G(y) = f(a, y)$$

$F(x)$ は $x = a$ で極小。

$G(y)$ は $y = b$ で極小。

$$\longrightarrow \quad F'(a) = 0, \quad G'(b) = 0$$

$$\rightsquigarrow \quad \frac{\partial f}{\partial x}(a, b) = \frac{\partial f}{\partial y}(a, b) = 0$$

Th 8.18 f が (a, b) で極大 OR 極小。

$$\Longrightarrow \quad \boxed{\frac{\partial f}{\partial x}(a, b) = \frac{\partial f}{\partial y}(a, b) = 0}$$

($\Leftarrow$ は成立せず)

停留点 stationary point

例 8.11 $z = f(x, y) = x^2 + 4xy + 2y^2 - 6x - 8y$

$$\begin{cases} f_x = 2x + 4y \cdot 1 + 0 - 6 - 0 \\ \quad = 2x + 4y - 6 = 0 \\ f_y = 0 + 4x \cdot 1 + 4y - 0 - 8 \\ \quad = 4x + 4y - 8 = 0 \end{cases}$$

$\rightsquigarrow$ $(x, y) = (1, 1)$ が極値をとる候補。

→ 注8.1で見ておく。

Ⅰ $z = f(x, y) = x^2 + xy + y^2 - 4x - 2y$

の停留点を求める.

Ⅱ $z = f(x, y) = x^3 + y^3 + 6xy$

の停留点を求める.

Ⅲ Taylorの定理を用いて

$$x - \frac{x^2}{2} \leqq \log(1+x) \leqq x - \frac{x^2}{2} + \frac{x^3}{3}$$

$(x > 0)$

を示せ.