I $\quad y = \sqrt{t}\,(t-1) \qquad (t > 0)$

$$y' = (\sqrt{t})'\,(t-1) + \sqrt{t}\,(t-1)'$$

$$= \frac{1}{2}\frac{1}{\sqrt{t}}\,(t-1) + \sqrt{t} \qquad (\sqrt{t})' = \frac{1}{2}\frac{1}{\sqrt{t}}.$$

$$= \frac{(t-1) + \sqrt{t}\cdot 2\sqrt{t}}{2\sqrt{t}}$$

$$= \frac{3t-1}{2\sqrt{t}}$$

J'en ai du mal !

Ça suffit !

$3t-1$: $\quad - \quad \frac{1}{3} \quad +$

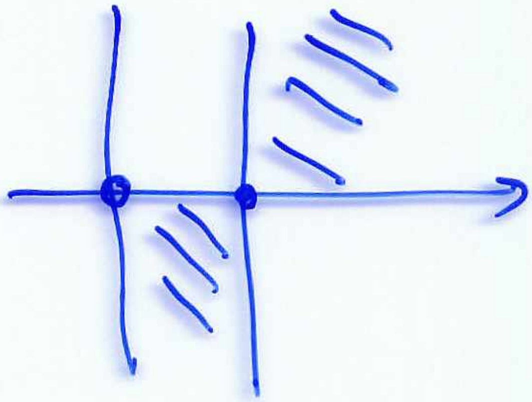
$$y'' = \frac{1}{2}\left\{-\frac{1}{2}\frac{1}{t\sqrt{t}}\,(3t-1) + \frac{1}{\sqrt{t}}\cdot 3\right\}$$

$$= \frac{1}{2}\cdot\frac{-(3t-1) + 3\cdot 2t}{2t\sqrt{t}} = \frac{3t+1}{4t\sqrt{t}} > 0$$

$y' \gtreqless 0 \iff t \gtreqless \frac{1}{3}$ に注意して

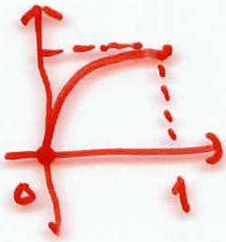
	(0)		$\frac{1}{3}$	
y'	/	$-$	0	$+$
y''	/	$+$	$+$	$+$
y	(0)	↘	$-\frac{2}{3}\cdot\frac{1}{\sqrt{3}}$	↗

$y = \underbrace{\sqrt{t}}_{\substack{\vee \\ 0}} (t-1) \begin{matrix} > \\ = \\ < \end{matrix} 0 \iff t \begin{matrix} > \\ = \\ < \end{matrix} 1$



$\lim\limits_{t \to +0} \sqrt{t} = 0$

$y = t^2$



$t = \sqrt{y}$ ← $y = 0$ で連続

$\lim\limits_{y \to +0} \sqrt{y} = 0$

逆関数.

$\sqrt{t}(t-1) \xrightarrow[t \to +0]{} 0 \times (-1) = 0$

$\sqrt{t}(t-1) = t\sqrt{t}\left(1 - \frac{1}{t}\right) \longrightarrow +\infty \cdot 1 = +\infty$

$a_n b_n \longrightarrow \begin{matrix} +\infty \\ -\infty \end{matrix}$

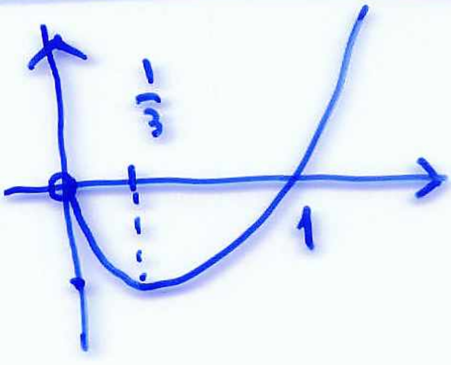
$a_n \longrightarrow +\infty$

$b_n \longrightarrow \begin{matrix} \alpha > 0 \\ \alpha < 0 \end{matrix}$

$b_n \to 0$

難しい

case by case



II. $y = te^{-t}$

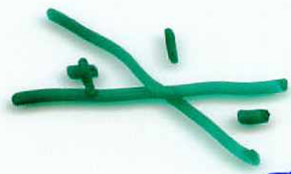
$y' = (t)' e^{-t} + t(e^{-t})'$

$(e^{ct})' = ce^{ct}$

c : 定数

$= e^{-t} + t(-e^{-t})$

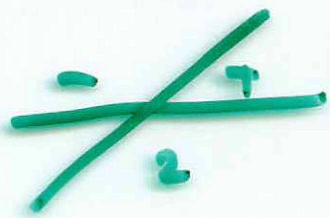
$= \underbrace{e^{-t}}_{>0}(1-t)$



$y'' = (e^{-t})'(1-t) + e^{-t}(1-t)'$

$= -e^{-t} \cdot (1-t) + e^{-t} \times (-1)$

$= \underbrace{e^{-t}}_{>0}(t-2)$



		1		2	
y'	+	0	−	−	−
y''	−	−	−	0	+
y	↗	$\frac{1}{e}$	↘	$\frac{2}{e^2}$	↘

$t \to +\infty \quad y = te^{-t} = \frac{t}{e^t} \to 0$

$\frac{t^n}{e^t} \to 0$

$(t \to +\infty)$

$n = 1, 2, \cdots$

[illegible].

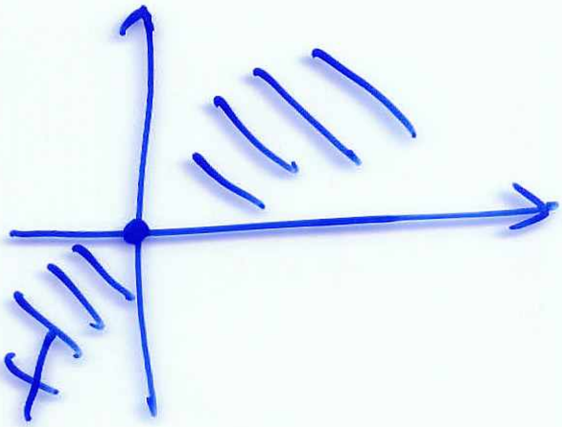
$t \to -\infty \quad e^{-t} \to +\infty$

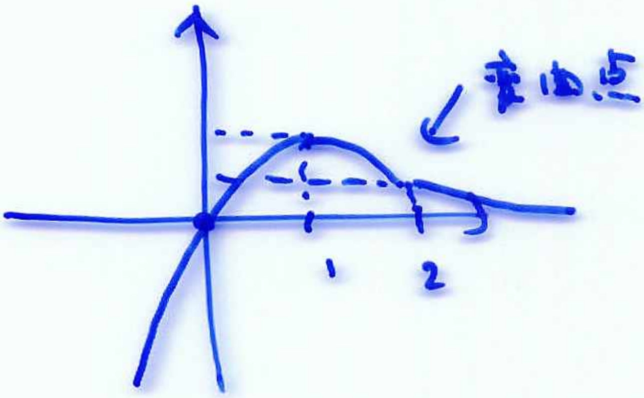
$t \to -\infty$

$\Rightarrow e^{-t} \quad t \to$

$$t\,e^{-t} \ \substack{> \\ = \\ <} \ 0$$

$$\iff t \ \substack{> \\ = \\ <} \ 0$$

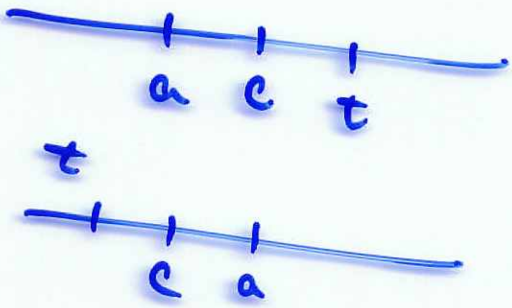




Taylor の定理

$$f(t) = f(a) + f'(a)(t-a) + \frac{1}{2!} f''(a)(t-a)^2 + \cdots + \frac{1}{n!} f^{(n)}(a)(t-a)^n + \underbrace{\frac{1}{(n+1)!} f^{(n+1)}(c)(t-a)^{n+1}}_{\text{剰余項}}$$

ここで c は t と a の間にある.



$n=2$, $a=1$, $f(t) = \sqrt{t}$.

$$f'(t) = \frac{1}{2}\frac{1}{\sqrt{t}}, \quad f''(t) = \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\frac{1}{t\sqrt{t}} = -\frac{1}{4}\frac{1}{t\sqrt{t}}$$

$$f^{(3)}(t) = \frac{1}{2}\cdot\frac{-1}{2}\cdot\frac{-3}{2}\,\frac{1}{t^2\sqrt{t}} = +\frac{3}{8}\cdot\frac{1}{t^2\sqrt{t}}$$

$\sqrt{t} = \cdots$

$$f(1) = 1, \quad f'(1) = \frac{1}{2}, \quad f''(1) = -\frac{1}{4}$$

$$\sqrt{t} = f(1) + f'(1)(t-1) + \frac{1}{2!} f''(1)(t-1)^2 + \frac{1}{3!}\cdot f^{(3)}(c)(t-1)^3$$

ここで c は t と 1 の間にある.

$$\sqrt{t} = 1 + \frac{1}{2}(t-1) - \frac{1}{8}(t-1)^2 + \underbrace{\frac{1}{3!}\cdot\frac{3}{8}\,\frac{1}{c^2\sqrt{c}}\,(t-1)^3}_{\text{剰余項}}$$

$t = 1 + h$ とおく.

$$\sqrt{1+h} \fallingdotseq 1 + \frac{1}{2}h - \frac{1}{8}h^2$$ 2次近似.

$$f(a+h) \fallingdotseq f(a) + f'(a)\,h + \frac{1}{2!}f''(a)\,h^2$$

2次近似.

IV $y = t^2 \log t$ の増減表.

$$y' = 2t\log t + t^2 \cdot \frac{1}{t} \qquad (\log t)' = \frac{1}{t}$$

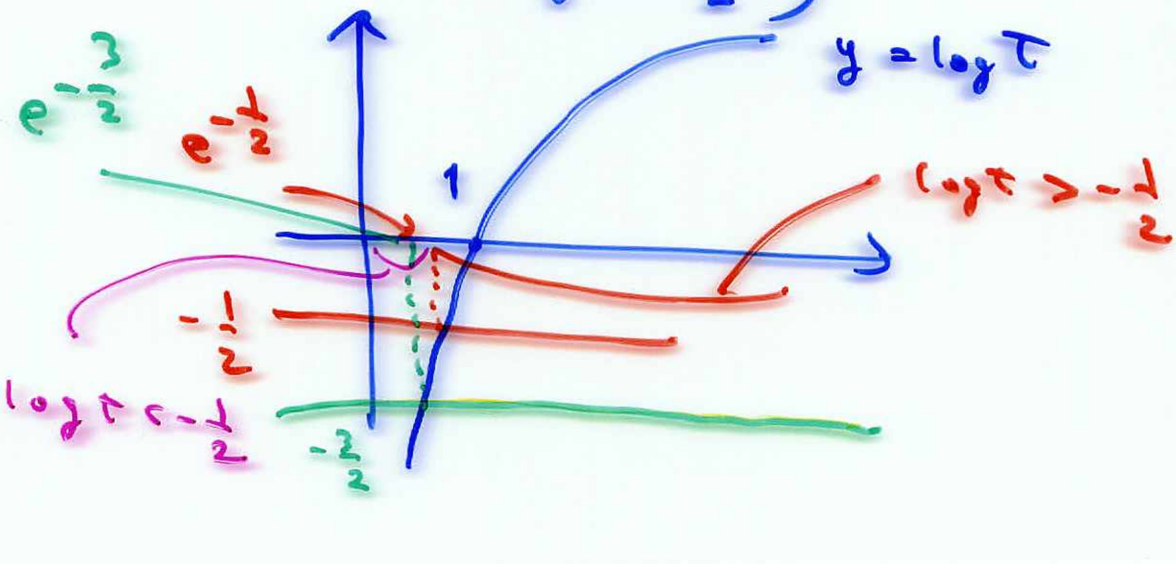
$$= 2t\log t + t$$

$0 <$ $$= 2t\left(\log t + \frac{1}{2}\right)$$

$$y'' = 2(t)'\left(\log t + \frac{1}{2}\right) + 2t\cdot\left(\log t + \frac{1}{2}\right)'$$

$$= 2\left(\log t + \frac{1}{2}\right) + 2t\cdot\frac{1}{t}$$

$$= 2\left(\log t + \frac{3}{2}\right)$$



$\log t = -\frac{3}{2}$ $\qquad$ $\log t = -\frac{1}{2}$

(0)		$e^{-\frac{3}{2}}$		$e^{-\frac{1}{2}}$	
	$-$	$-$	$-$	0	$+$
	$-$	0	$+$	$+$	$+$
(0)	$\searrow$		$\searrow$		$\nearrow$

à expliquer

$$y = e^{-1} \cdot \left(-\frac{1}{2}\right) = \frac{1}{2e}$$

$$y = t^2 \log t = e^{-3} \cdot \left(-\frac{3}{2}\right) = -\frac{3}{2e^3}$$

$t \to +\infty$

$\log t \to +\infty$, $\quad t^2 \log t \to +\infty$

$t \to +0$

$\log t \to -\infty$

$$t^2 \log t = e^{-2s} \cdot (-s) = \frac{-s}{e^{2s}}$$

$-\log t = s \to +\infty$

$t = e^{-s}$

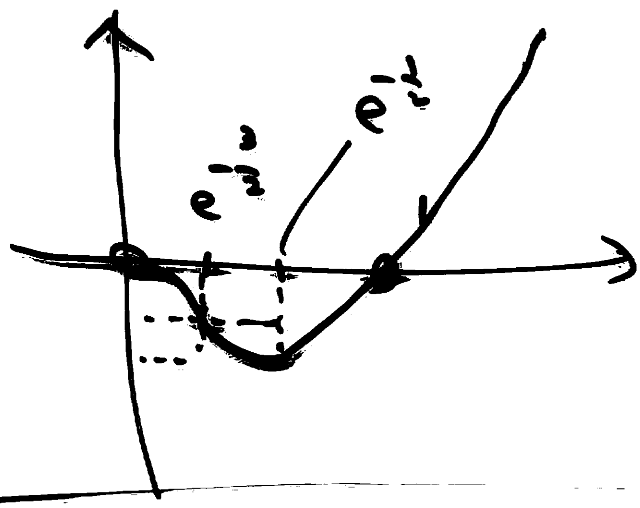
$$= -\frac{2s}{e^{2s}} \times \frac{1}{2} \longrightarrow \cancel{\pm\infty}$$

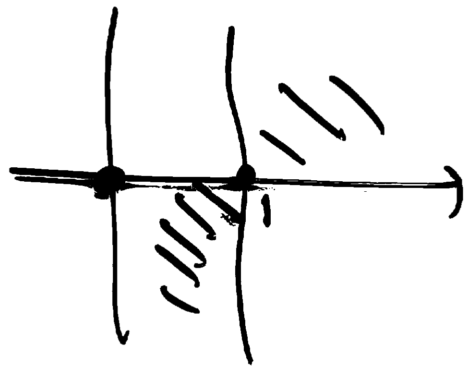
$0 \times \frac{1}{2} = 0$

$(2s \to +\infty)$

$y = t^2 \log t \gtreqless 0 \iff \log t \gtreqless 0 \iff t \gtreqless 1$

$t^2 > 0$





I　$y = \dfrac{1}{\sqrt{t}} = f(t)$ に対して

$$f^{(n)}(t) = ?$$

II　$y = e^{-t^2}$ の増減表

III　$y = t\,e^{-t^2}$ の増減表.

IV　$f(t) = \log(1+t)$ に対して

Taylorの定理を作う

$a = 0,\ n = 2.$

V.　Taylorの定理を用いて

$$e^t \geqq 1 + t + \frac{1}{2!}t^2 + \frac{1}{3!}t^3 \quad (t \geqq 0)$$

を示せ

$a = 0,\ n = 3$