

6月13日演習解答.

I $y = \sqrt{t}(t-1)$ の増減表.

$$y' = \frac{1}{2} \frac{1}{\sqrt{t}} (t-1) + \sqrt{t} \cdot 1 = \frac{1}{2\sqrt{t}} \{ (t-1) + \sqrt{t} \cdot 2\sqrt{t} \}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{t}} (3t-1)$$

$$y'' = \frac{1}{2} \cdot \left\{ -\frac{1}{2} \cdot \frac{1}{t\sqrt{t}} (3t-1) + \frac{1}{\sqrt{t}} \cdot 3 \right\}$$

$$= \frac{1}{4} \frac{1}{t\sqrt{t}} \{ -(3t-1) + 6t \}$$

$$= \frac{1}{4} \cdot \frac{1}{t\sqrt{t}} (3t+1) > 0$$

$$y' \underset{<}{\geq} 0 \Leftrightarrow 3t-1 \underset{<}{\geq} 0$$

∴ 以下の増減表を得る

	(0)		$\frac{1}{3}$	
y'	/	-	0	+
y''	/	+	+	+
y	(0)	↘	↗	↗

$-\frac{2}{3} \frac{1}{\sqrt{3}}$

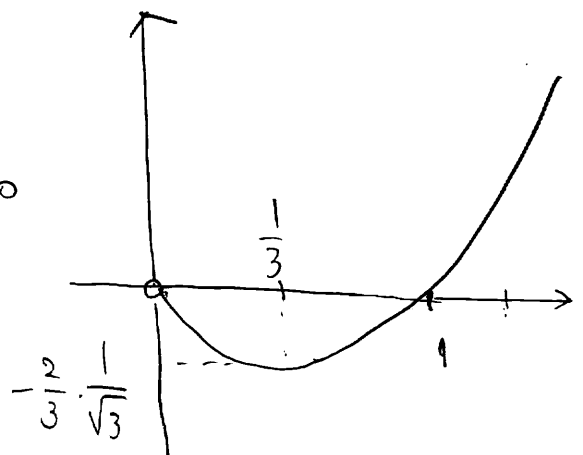
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$$y \underset{<}{\geq} 0 \Leftrightarrow t \underset{<}{\geq} 0$$

$$\lim_{t \rightarrow +0} \sqrt{t}(t-1) = 0 \cdot (-1) = 0$$

$$\lim_{t \rightarrow +\infty} \sqrt{t} \cdot (t-1) = +\infty$$

∴ $2 \leq \frac{1}{2} \leq 12$ かつ $7 \leq 12$



II $y = t e^{-t}$ の増減表.

$$y' = e^{-t} + t(-e^{-t}) = (1-t)e^{-t}$$

$$y'' = (-1)e^{-t} + (1-t)(-e^{-t}) \\ = (t-2)e^{-t}$$

から増減表は右図. t

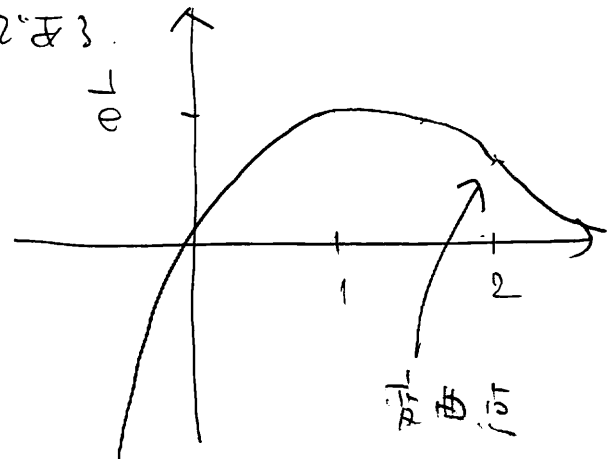
	1	2	
y'	+	-	-
y''	-	0	+
y	$\nearrow$	$\searrow$	$\nearrow$
	$\frac{1}{e}$	$\frac{2}{e^2}$	

$$t e^{-t} \rightarrow 0 \quad (t \rightarrow +\infty)$$

$$t e^{-t} \rightarrow -\infty \quad (t \rightarrow -\infty)$$

$$y \geq 0 \Leftrightarrow t \geq 0$$

に注意すると右図がグラフである.



$$\text{III} \quad f(t) = \sqrt{t}, \quad f'(t) = \frac{1}{2\sqrt{t}}, \quad f''(t) = -\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{t\sqrt{t}}$$

$$f^{(3)}(t) = \frac{3}{8} \cdot \frac{1}{t^2\sqrt{t}} = -\frac{1}{4} \cdot \frac{1}{t\sqrt{t}}$$

Σ 133.

$$f(t) = f(1) + f'(1)(t-1) + \frac{1}{2!} f''(1)(t-1)^2 + \frac{1}{3!} f^{(3)}(1)(t-1)^3$$

か) 3

$$\sqrt{t} = 1 + \frac{1}{2}(t-1) - \frac{1}{8}(t-1)^2 + \frac{1}{3!} \cdot \frac{3}{8} \frac{1}{t\sqrt{t}} (t-1)^3$$

Σ 473 c d n 1 と $(t-1)^2$

t の 間 隙 に 存 在 可 3 = 2 か 合 可 3.

IV $y = t^2 \log t$ の 増 減 表

$$y' = 2t \log t + t^2 \cdot \frac{1}{t} = 2t \log t + t = 2t \left(\log t + \frac{1}{2} \right)$$

$$y'' = 2 \log t + 2t \cdot \frac{1}{t} + 1 = 2 \left(\log t + \frac{3}{2} \right)$$

か) 増 減 表 は 下 図 の 如 く.

	(0)		$e^{-\frac{3}{2}}$		$e^{-\frac{1}{2}}$	
y'		-	-	-	0	+
y''		-	0	+		
y	(0)	↘		↗		↗
			$-\frac{3}{2e^3}$		$-\frac{1}{2e}$	

$$t^2 \log t \rightarrow +\infty \quad (t \rightarrow +\infty)$$

$$t^2 \log t \rightarrow 0 \quad (t \rightarrow t_0)$$

$$y \geq 0 \Leftrightarrow \log t \geq 0 \Leftrightarrow t \geq 1$$

0.5 1.5 1.7 1.8 2.0 2.2 2.4 2.6 2.8 3.0 3.2 3.4 3.6 3.8 4.0 4.2 4.4 4.6 4.8 5.0 5.2 5.4 5.6 5.8 6.0 6.2 6.4 6.6 6.8 7.0 7.2 7.4 7.6 7.8 8.0 8.2 8.4 8.6 8.8 9.0 9.2 9.4 9.6 9.8 10.0

