$\alpha \in \mathbb{R}$ 定数 constant $\quad 0 \in (a, b)$

$$f'(x) = \alpha f(x) \qquad x \in (a, b)$$

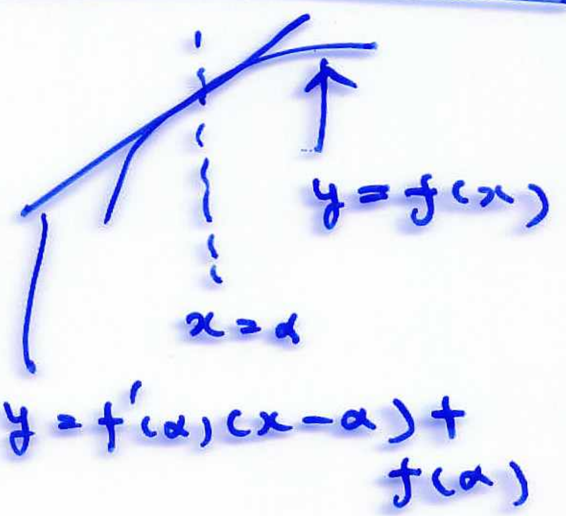
$$\Longrightarrow \quad f(x) = f(0)\, e^{\alpha x}$$

I $\quad \left.\begin{array}{l} f'(x) = 3 f(x) \\ f(0) = 5 \end{array}\right\} \rightarrow f(x) = 5 e^{3x}$

II.

$x \sim \alpha$ のとき

$$f(x) \doteqdot f'(\alpha)(x-\alpha) + f(\alpha)$$



(i) $f(x) = x^{\frac{1}{3}}$, $\alpha = 1 \qquad f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$

$$\sqrt[3]{0.95} = f(0.95) \doteqdot f(1) + f'(1)(0.95 - 1)$$

$$= 1 + \frac{1}{3} 1^{-\frac{2}{3}} \times (-0.05)$$

$$= 1 - \frac{1}{3} \cdot 0.05 \approx 0.983 \cdots$$

(2) $\log 0.95 \doteqdot ?$

$\log 1 = 0$, $f(x) = \log x$, $\alpha = 1$, $f'(x) = \frac{1}{x}$

$= f(0.95) \doteqdot f(1) + f'(1)(0.95 - 1)$

$= 0 + 1 \cdot (-0.05)$

$= -0.05$

III $y = t^2 e^{-t}$

$y' = (t^2)' e^{-t} + t^2 (e^{-t})'$

$= 2t\, e^{-t} + t^2 (-e^{-t})$

$= \underbrace{e^{-t}}_{>0} (2t - t^2)$ $\quad (e^{ct})' = c\, e^{ct}$

$=$

$y' \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0 \iff 2t - t^2 \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0 \iff$ $0 < t < 2$

$t = 0, 2$

$t < 0$ OR $t > 2$

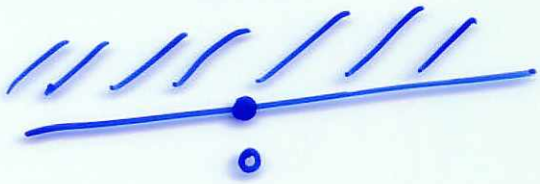
t		0		2	
y'	$-$	0	$+$	0	$-$
y	↘	0	↗	$\frac{4}{e^2}$	↘

graph , la graphe

$y = t^2 e^{-t}$

$t \neq 0 \quad y > 0$

$t = 0 \quad y = 0$



$t \to +\infty$

$\frac{t^2}{e^t} \to 0 \quad (t \to +\infty)$

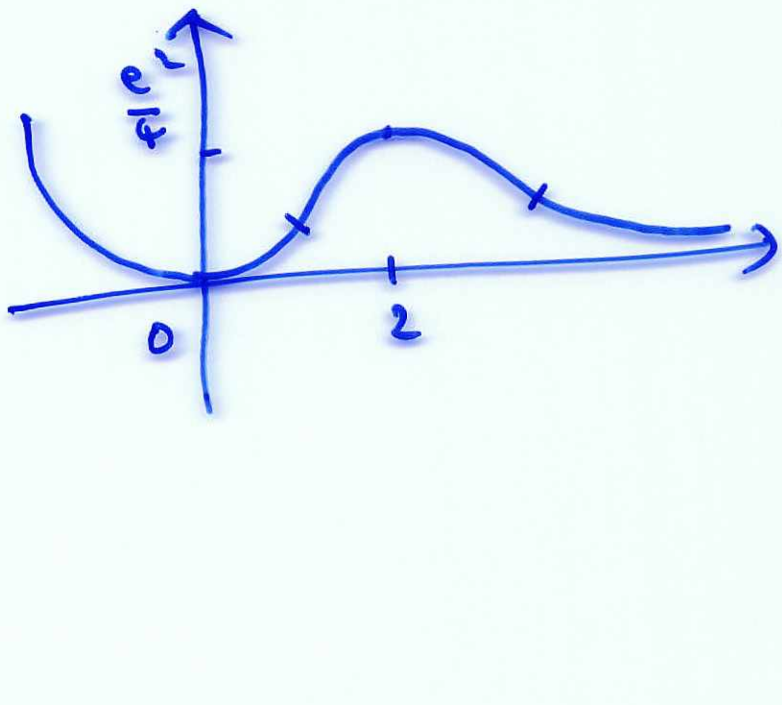
$\boxed{\frac{t^4}{e^t} \to 0}$

$t \to -\infty$ alors

$t^2 \to +\infty$

$-t \to +\infty \Rightarrow e^{-t} \to +\infty$

$\rightsquigarrow y = t^2 e^{-t} \to +\infty$



IV

$y = f(t) = \log(1+t)$

$u = 1+t$

$\frac{du}{dt} = 1$

$\frac{d\log u}{du} = \frac{1}{u}$

$\frac{d}{du}\left(\frac{1}{u}\right) = -\frac{1}{u^2}$

$\frac{d}{du}\left(\frac{1}{u^3}\right) = -\frac{3}{u^4}$

$y' = \frac{1}{1+t}$.

$y'' = -\frac{1}{(1+t)^2}$

$y''' = \frac{2}{(1+t)^3}$

$y^{(4)} = -\frac{\underbrace{2\cdot 3}_{=(4-1)!}}{(1+t)^4}$

$y^{(5)} = \frac{\underbrace{2\cdot 3\cdot 4}_{=(5-1)!}}{(1+t)^5}$.

予想 $y^{(n)} = \frac{(n-1)!}{(1+t)^n} \times (-1)^{n-1}$

$n = 1, 2, \ldots$

予想が正しいとき

$y^{(n+1)} = (-1)^{n-1}(n-1)!\,\frac{(-n)}{(1+t)^{n+1}}$

$= (-1)^n\, n!\,\frac{1}{(1+t)^{n+1}}$

$= (-1)^{(n+1-1)}\cdot(n+1-1)!$

$\times \frac{1}{(1+t)^{n+1}}$

$f : (a, b) \longrightarrow \mathbb{R}$.

$C^1(a, b) \ni f$ 　fは 各点で連続, 微分可能
f'が 各点で連続.

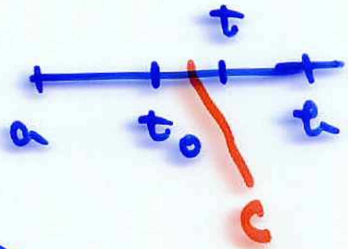
$C^2(a, b) \ni f$, 　$f \in C^1(a, b)$, $f' \in C^1(a, b)$
(f, f', f'' が あって 連続)

$C^n(a, b) \ni f$, 　$f, f', \cdots, f^{n-1} \in C^1(a, b)$
($f, f', \cdots, f^n$: 存在して 連続)

$C^\infty(a, b) \ni f$ 　全ての n に対して
$f \in C^n(a, b)$

Taylor の定理

$f \in C^{\infty}(a,b)$



$$
\begin{aligned}
f(t) = f(t_0) &+ f'(t_0)(t-t_0) \\
&+ \frac{1}{2!} f''(t_0)(t-t_0)^2 + \frac{1}{3!} f^{(3)}(t_0)(t-t_0)^3 \\
&+ \cdots + \frac{1}{n!} f^{(n)}(t_0)(t-t_0)^n \\
&+ \frac{1}{(n+1)!} f^{(n+1)}(c)(t-t_0)^{n+1}
\end{aligned}
$$

つじつま合わせ.

をみたす c が t と t_0 の間にある.

剰余項.

$$f(t)=\frac{1}{1-t},\quad f'(t)=-\frac{1}{(1-t)^2}\cdot(-1)=\frac{1}{(1-t)^2}$$

$$f''(t)=-\frac{2}{(1-t)^3}\cdot(-1)=\frac{2}{(1-t)^3}$$

$$f^{(3)}(t)=-\frac{2\cdot 3}{(1-t)^4}\cdot(-1)=\frac{3!}{(1-t)^4}$$

$$f^{(n)}(t)=\frac{n!}{(1-t)^{n+1}}.$$

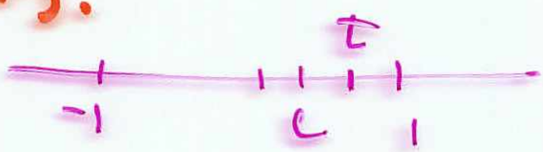
$t=0$ $\quad f^{(n)}(0)=\dfrac{n!}{(1-1)^{n+1}}=n!.$

$$\begin{aligned}
f(t) &= 1+(t-0)+\frac{2!}{2!}(t-0)^2+\frac{3!}{3!}(t-0)^3 \\
&\quad +\cdots+\frac{n!}{n!}(t-0)^n \\
&\quad +\frac{(n+1)!}{(n+1)!}\cdot\frac{1}{(1-c)^{n+2}}(t-0)^{n+1} \\
&= 1+t+t^2+\cdots+t^n+\frac{1}{(1-c)^{n+2}}t^{n+1}.
\end{aligned}$$

$$f(t)=\frac{1}{1-t}$$

c は 0 と t の間にあり.

$|t|<1$ とする



2) $f(t)=e^t,\quad t_0=0$

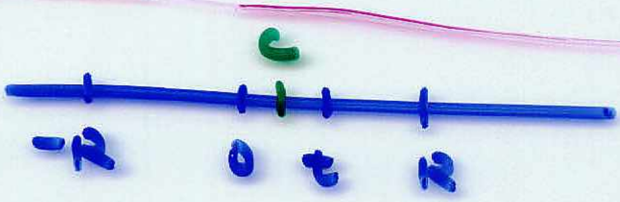
$f'(t)=e^t,\quad f''(t)=e^t,\quad f^{(n)}(t)=e^t$

$f^{(n)}(0)=1.$

$$f(t)=e^t = 1+1\cdot(t-0)^2+\frac{1}{2!}\cdot 1\cdot(t-0)^2+\cdots$$

$$\cdots+\frac{1}{n!}\,1\cdot(t-0)^n + \overset{R_n}{\overset{\parallel}{\boxed{\frac{1}{(n+1)!}\cdot e^c\ t^{n+1}}}}$$

ここで c は 0 と t の間にあり.



$|t|<R$

剰余項

$$0\le |R_n| \le \underbrace{\frac{1}{(n+1)!}\,e^R\cdot R^{n+1}}_{\longrightarrow\, 0}$$

shown later

$R_n \to 0$

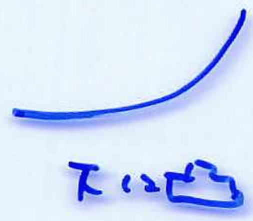
$$e^t = 1+t+\frac{1}{2!}t^2+\frac{1}{3!}t^3+\cdots\cdots$$

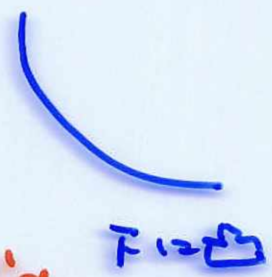
$$=\sum_{n=0}^{+\infty}\frac{1}{n!}t^n$$

Poisson分布.

y''の応用.

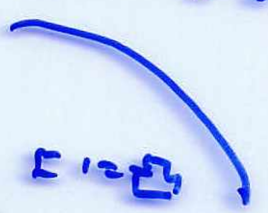
$y' > 0$, $\boxed{y'' > 0} \to$ y' 増加, $y' < 0$ $y'' > 0$





$y' > 0$, $\boxed{y'' < 0} \to$ y' 減少

$y' < 0$, $y'' < 0$



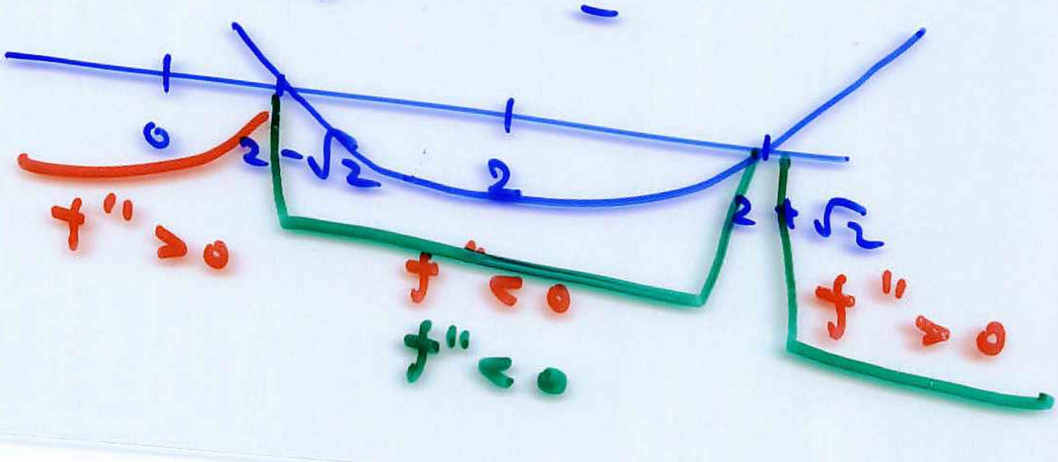
III. $y = t^2 e^{-t}$, $y' = (2t - t^2)e^{-t}$

$$
\begin{aligned}
y'' &= (2t - t^2)' e^{-t} + (2t - t^2)(e^{-t})' \\
&= (2 - 2t)e^{-t} + (2t - t^2)(-e^{-t}) \\
&= e^{-t}(t^2 - 4t + 2) \\
&= e^{-t}\{t - (2+\sqrt{2})\}\{t - (2-\sqrt{2})\}
\end{aligned}
$$

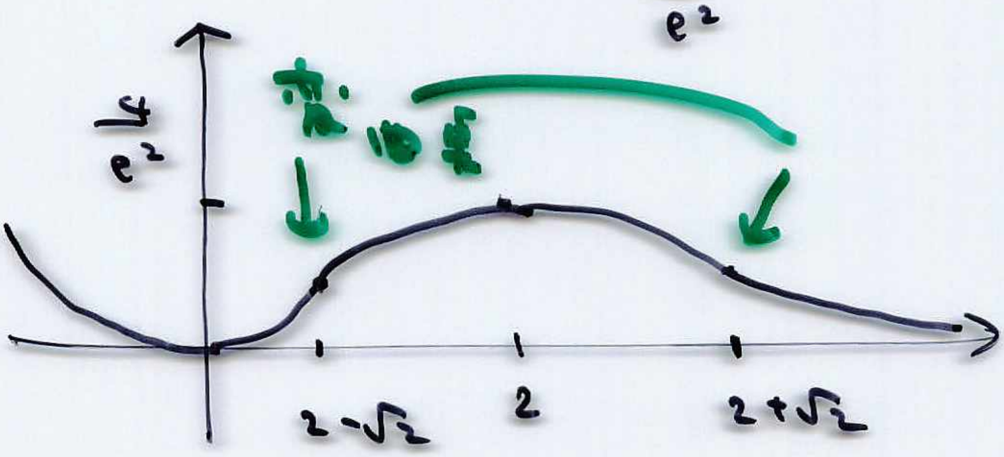
$t^2 - 4t + 2 = (t-2)^2 - 2$

$e^{-t} > 0$

f' $-$ $+$ $-$



t		0		$2-\sqrt{2}$		2		$2+\sqrt{2}$	
y'	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
y	$\searrow$	0	$\nearrow$		$\nearrow$	$\frac{4}{e^2}$	$\searrow$		$\searrow$



I $y = \sqrt{t}\,(t-1)$ の増減表を描く.
$(t > 0)$

II. $y = t e^{-t}$ ―――――――――

III. $f(t) = \sqrt{t}$ に対して Taylor の定理を適用.
但し $t_0 = 1$, $n = 2$

IV $y = \boxed{t^2 \log t}$ の増減表