I (1) $\left(\dfrac{1}{2x+1}\right)' = \dfrac{-(2x+1)'}{(2x+1)^2}$

$= \dfrac{-2}{(2x+1)^2}$

(2) $\left(\dfrac{x}{x-1}\right)' = \dfrac{(x)'(x-1) - x(x-1)'}{(x-1)^2}$

$= \dfrac{(x-1) - x}{(x-1)^2}$

$= -\dfrac{1}{(x-1)^2}$

---

(3) $\{(x^2+1)(x-1)\}'$

$= (x^2+1)'(x-1) + (x^2+1)(x-1)'$

$= 2x(x-1) + (x^2+1)\cdot 1$

$= 3x^2 - 2x + 1$

(4) $\left(\dfrac{1}{1+x+x^2}\right)' = \dfrac{-(1+x+x^2)'}{(1+x+x^2)^2}$

$= \dfrac{-1-2x}{(1+x+x^2)^2}$

$$\{(2x-1)^{10}\}' = ?$$

$$\lim_{x \to \alpha} \frac{f(x) - f(\alpha)}{x - \alpha} = f'(\alpha)$$

$x_n \to \alpha$, $x_n \neq \alpha$ である $\{x_n\}$ をとる.

$$\frac{f(x_n) - f(\alpha)}{x_n - \alpha} = \frac{(2x_n - 1)^{10} - (2\alpha - 1)^{10}}{x_n - \alpha}$$

$y_n = 2x_n - 1$
とおく

$y_n \to 2\alpha - 1 = A$ とおく

$$= \underbrace{\frac{y_n^{10} - A^{10}}{y_n - A}}_{\to (y^{10})'|_{y=A}} \cdot \underbrace{\frac{(2x_n - 1) - (2\alpha - 1)}{x_n - \alpha}}_{= 2}$$

$$(y^{10})'\big|_{y=A} = 10y^9\big|_{y=A} = 10A^9$$

$$\to 2 \cdot 10A^9$$

$$= 2 \cdot 10(2\alpha - 1)^9$$

$$\underset{\displaystyle\overset{\|}{f(x)}}{(2x-1)^{10}} = g(u(x))$$

$$g(y) = y^{10}, \quad \underset{\displaystyle\overset{\|}{y}}{u(x)} = 2x-1.$$

合成関数.

$$A = u(\alpha),$$

$$\frac{f(x)-f(\alpha)}{x-\alpha} = \frac{g(u(x)) - g(u(\alpha))}{x-\alpha}$$

$$= \boxed{\frac{g(y)-g(A)}{y-A}} \cdot \boxed{\frac{u(x)-u(\alpha)}{x-\alpha}}$$

$x \to \alpha$ のとき. $y = u(x) \longrightarrow u(\alpha) = 2\alpha - 1 = A$

$\to g'(A)$ $\quad \to u'(\alpha)$

$$\longrightarrow g'(A) \cdot u'(\alpha)$$

一般的に. $g(u(x))$. $\left.\begin{array}{l} z = g(y) \\ y = u(x) \end{array}\right\}$ 合成

$$\frac{d\, g(u(x))}{dx} = \left.\frac{d\, g(y)}{dy}\right|_{y=u(x)} \cdot \frac{du(x)}{dx}.$$

---

① $\left(\sqrt{x^2+1}\right)' = ?$

$$f(x) = \sqrt{x^2+1} = g(u(x))$$

$$g(y) = \sqrt{y}, \quad \underset{\overset{\shortparallel}{y}}{u(x)} = x^2+1.$$

$$\left(\sqrt{x^2+1}\right)' = \left.\frac{d\sqrt{y}}{dy}\right|_{y=u(x)} \cdot \frac{du}{dx}$$

$$= \frac{1}{2}\frac{1}{\sqrt{y}} \cdot 2x = \frac{x}{\sqrt{x^2+1}}.$$

---

② $(1+t)^n = g(u(t))$

$$g(x) = x^n, \quad \underset{\overset{\shortparallel}{x}}{u(t)} = 1+t.$$

$$\left\{(1+t)^n\right\}' = \frac{d(x^n)}{dx} \cdot \frac{du}{dt}$$

$$= n\,x^{n-1} \cdot 1$$

$$= n(1+t)^{n-1}.$$

$$(1+2t)^n = g(u(t))$$

$$g(x) = x^n, \quad x = u(t) = 1+2t.$$

$$\{(1+2t)^n\}' = \frac{d(x^n)}{dx} \cdot \frac{du(t)}{dt}$$

$$= n x^{n-1} \cdot 2 = 2n(1+2t)^{n-1}$$

$$\sum_{k=0}^{n} {}_nC_k\, a^{n-k} t^k = (a+t)^n.$$

2項定理.

$$\sum_{k=0}^{n} k\, {}_nC_k\, a^{n-k} t^k = ?$$

$$(a+t)^n = \sum_{k=0}^{n} {}_nC_k\, a^{n-k} t^k$$

両辺をtで微分

$$\{(a+t)^n\}' = \frac{d(x^n)}{dx} \cdot \frac{d(a+t)}{dt}$$

$x = a+t$

$$= n x^{n-1} \cdot 1$$

$$= n(a+t)^{n-1}$$

$$n(a+t)^{n-1} = \left(\sum_{k=0}^{n} {}_nC_k\, a^{n-k} t^k\right)'$$

$$= \sum_{k=0}^{n} \left({}_nC_k\, a^{n-k} t^k\right)' = \sum_{k=0}^{n} k\, {}_nC_k\, a^{n-k} t^{k-1}.$$

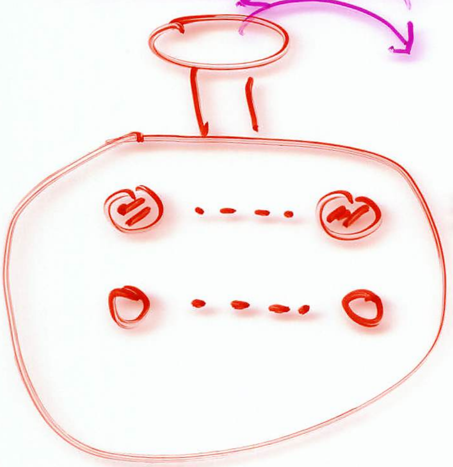
まとめ $n(a+t)^{n-1} = \sum_{k=0}^{n} k\,{}_nC_k\, a^{n-k}\, t^{k-1}$

N.B.

両辺をt倍する

$$n(a+t)^{n-1}\,t = \sum_{k=0}^{n} k\,{}_nC_k\, a^{n-k}\, t^{k}$$

2項分布の期待値.



赤玉 N_1

白玉 N_2

$$p = \frac{N_1}{N_1+N_2}$$

$$q = \frac{N_2}{N_1+N_2}.$$

ℓ回出し入れする　赤玉の個数 X

$$P(X=k) = {}_\ell C_k\, q^{\ell-k}\, p^k$$

X=kである確率.

期待値 ~~P(X)~~ $E(X) = \sum_{k=0}^{\ell} k\, P(X=k)$

$$= \sum_{k=0}^{\ell} k\,{}_\ell C_k\, q^{\ell-k}\, p^k$$

小行 ゆえ. $\rightarrow = n(p+q)^{n-1}\cdot p$

$$= np.$$

$a=q,\ t=p$ にあたる

レポート #3

$$k\,{}_{l}C_{k} = l\,{}_{l-1}C_{k-1}$$

を示して

$$\sum_{k=0}^{l} k\,{}_{l}C_{k}\, q^{l-k} p^{k}$$

を計算する.

cf 98 page 参照