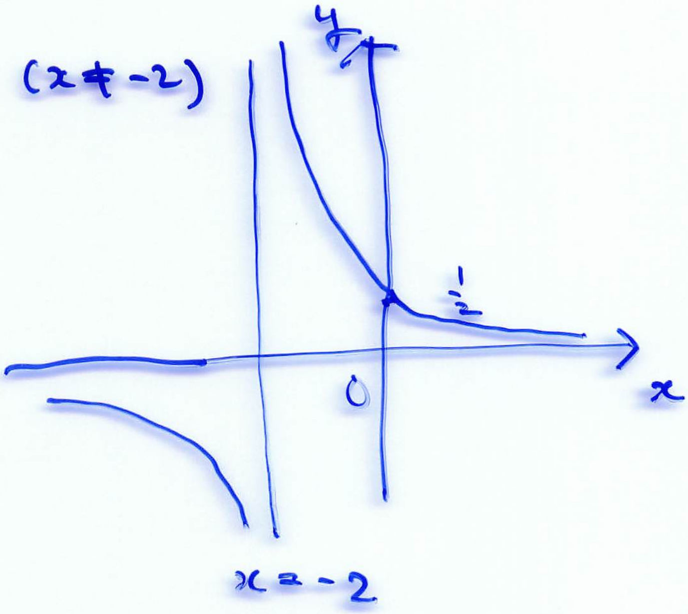
5月16日演習.

II $\quad f(x) = \dfrac{1}{2+x} \quad (x \neq -2)$



$\alpha \neq -2$)

$$\frac{f(\alpha+h) - f(\alpha)}{h}$$

$$= \frac{\dfrac{1}{2+\alpha+h} - \dfrac{1}{2+\alpha}}{h}$$

$$= \frac{1}{h} \cdot \frac{(2+\alpha) - (2+\alpha+h)}{(2+\alpha)(2+\alpha+h)} \qquad h \neq 0$$

$$= \frac{1}{h} \cdot \frac{(-h)}{(2+\alpha)(2+\alpha+h)}$$

$$= -\frac{1}{2+\alpha} \cdot \frac{1}{2+\alpha+h} = \#$$

$\{h_n\}$ $h_n \neq 0$, $h_n \to 0 \ (n \to +\infty)$ とする.

$$\# = -\frac{1}{2+\alpha} \cdot \frac{1}{2+\alpha+h_n} \longrightarrow -\frac{1}{2+\alpha} \cdot \frac{1}{2+\alpha+0}$$

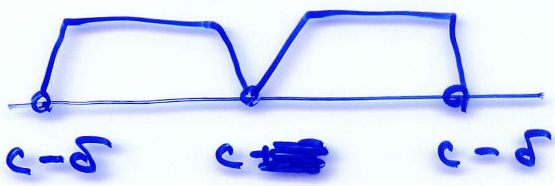
$$= -\frac{1}{(2+\alpha)^2}$$

$$\left(\lim_{h \to 0} \frac{f(\alpha+h) - f(\alpha)}{h} = -\frac{1}{(2+\alpha)^2} \right.$$

The limit does not depend on the choice of $\{h_n\}$.

復習

$I = (c-\delta, c) \cup (c, c+\delta)$



$I \xrightarrow[f]{\quad} \mathbb{R}$.

f(tₙ) が定義されるように。

$$\lim_{t \to c} f(t) = A \iff \begin{cases} \{t_n\} : t_n \in I. \\ \lim_{n\to+\infty} t_n = c \\ \text{[みたす]}\ \{t_n\}\ \text{に対して} \\ \lim_{n\to+\infty} f(t_n) = A \end{cases}$$

def.

definition

la définition

$a_n \to \alpha$, $b_n \to \beta$.

(1) $a_n \pm b_n \longrightarrow \alpha \pm \beta$

(2) $a_n \cdot b_n \longrightarrow \alpha\beta$

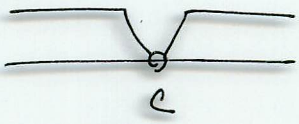
(3) $a_n \neq 0$ (全ての n), $\alpha \neq 0$ のとき

$$\frac{1}{a_n} \longrightarrow \frac{1}{\alpha}.$$

$\delta > 0$ とする.

$I = (c-\delta, c) \cup (c, c+\delta)$ とする.

$f : I \longrightarrow \mathbb{R}, \quad g : I \longrightarrow \mathbb{R}$



に対して

$$\lim_{t \to c} f(t) = A, \quad \lim_{t \to c} g(t) = B$$

が成立するとする. このとき次の定理が成立する.

定理.

1) $f(t) \pm g(t) \longrightarrow A \pm B \quad (t \to c)$

2) $f(t) \cdot g(t) \longrightarrow AB \quad (t \to c)$

3) $f(t) \neq 0 \ (t \in I), \ A \neq 0$ とすると

$$\frac{1}{f(t)} \longrightarrow \frac{1}{A} \quad (t \to c)$$

$f(t_n)$, $g(t_n)$ 定義される.

2) の証明 $\{t_n\}$ を $\boxed{t_n \in I \ (n = 1, 2, \cdots)}$, $\lim_{n \to +\infty} t_n = c$ を満たす数列とする. このとき

$$f(t_n) \longrightarrow A, \quad g(t_n) \longrightarrow B \quad (n \to +\infty)$$

から

$$f(t_n)\, g(t_n) \longrightarrow AB.$$

$\lim_{t \to c}$ の定義

を得る. これは

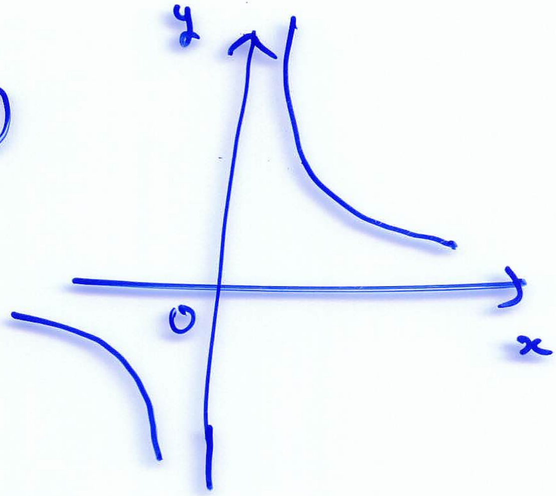
$$\lim_{t \to c} f(t)\, g(t) = AB.$$

を意味する.

数列の極限.

III. $g(x) = \dfrac{1}{x^3} \quad (x \neq 0)$

$\alpha \neq 0$



$$\frac{g(\alpha + h) - g(\alpha)}{h}$$

$$= \frac{1}{h} \cdot \left(\frac{1}{(\alpha + h)^3} - \frac{1}{\alpha^3} \right)$$

$$= \frac{1}{h} \cdot \frac{\alpha^3 - (\alpha + h)^3}{\alpha^3 (\alpha + h)^3}$$

$$= \frac{1}{h} \cdot \frac{\alpha^3 - (\alpha^3 + 3\alpha^2 h + 3\alpha h^2 + h^3)}{\alpha^3 (\alpha + h)^3}$$

$$= \underbrace{\frac{1}{\alpha^3}}_{\downarrow \, \frac{1}{\alpha^3}} \cdot \underbrace{\frac{1}{(\alpha + h)^3}}_{\frac{1}{\alpha^3}} \cdot \underbrace{(-3\alpha^2 - 3\alpha h - h^2)}_{\downarrow \, -3\alpha^2 - 3\alpha^2 \cdot 0 - 0^2}$$

$$\longrightarrow \frac{1}{\alpha^3} \cdot \frac{1}{\alpha^3} \cdot (-3\alpha^2) = -\frac{3}{\alpha^4}$$

$$f'(x) = -\frac{3}{x^4}$$

公式

(1) $(f \pm g)' = f' \pm g'$

(2) $(fg)' = f'g + fg'$

(3) $\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$

例 $(1)' = 0$, $x' = 1$

(I) $(x^2)' = 2x$, $(x^3)' = 3x^2$.

$(x^n)' = nx^{n-1}$ $(n = 0, 1, 2, \cdots)$

数学的归纳法

$$(x^{n+1})' = (x^n \cdot x)'$$

$$= (x^n)' \cdot x + x^n \cdot (x)'$$

$$= nx^{n-1} \cdot x + x^n \cdot 1$$

$$= (n+1)x^n.$$

$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$, $\left(\frac{1}{x^2}\right)' = -\frac{2}{x^2}$, $\left(\frac{1}{x^3}\right)' = -\frac{3}{x^4}$.

$\left(\frac{1}{x^4}\right)' =$

$$\left(\frac{1}{x^n}\right)' = \frac{-(x^n)'}{x^{2n}} = \frac{-n x^{n-1}}{x^{2n}}$$

$$= -\frac{n}{x^{n+1}}$$

$$\left(\frac{1}{2+x}\right)' = -\frac{(2+x)'}{(2+x)^2} = -\frac{1}{(2+x)^2}.$$

$(fg)' = f'g + fg'$. の証明.

$$\frac{f(\alpha+h)g(\alpha+h) - f(\alpha)g(\alpha)}{h}.$$

$$= \frac{f(\alpha+h)g(\alpha+h) - f(\alpha)\cdot g(\alpha+h)}{h}$$

$$+ \frac{f(\alpha)g(\alpha+h) - f(\alpha)g(\alpha)}{h}$$

$$= g(\alpha+h)\cdot \boxed{\frac{f(\alpha+h) - f(\alpha)}{h}} \rightarrow f'(\alpha)$$

$$+ f(\alpha) \boxed{\frac{g(\alpha+h) - g(\alpha)}{h}}.$$

$g(\alpha)$

$\downarrow$ $f(\alpha)$ $\qquad$ $\downarrow$ $g'(\alpha)$

$\longrightarrow g(\alpha)f'(\alpha) + f(\alpha)g'(\alpha)$

Il nous reste à montrer

$$g(\alpha+h) \longrightarrow g(\alpha) \quad (h \to 0)$$

微分可能 ⟹ 連続..

この極限があるのは前提.

$$\frac{g(\alpha+h)-g(\alpha)}{h} \longrightarrow g'(\alpha)$$

$$g(\alpha+h)-g(\alpha) = \frac{g(\alpha+h)-g(\alpha)}{h} \times h$$

$$\longrightarrow g'(\alpha) \cdot 0 = 0$$

$$g(\alpha+h) = \big(g(\alpha+h)-g(\alpha)\big) + g(\alpha)$$

$$\longrightarrow 0 + g(\alpha) = g(\alpha)$$

$$\boxed{\left(\frac{1}{1+x^2}\right)' = -\frac{2x}{(1+x^2)^2}} \qquad (1+x^2)'$$

$$\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$$

$$\left(\frac{1}{1+x^2}\right)' = -\frac{(1+x^2)'}{(1+x^2)^2} = -\frac{2x}{(1+x^2)^2}$$

$$\left\{(1+x^2)^4\right\}' = \ ?$$

$$x \longrightarrow \alpha$$

$$\left(\begin{array}{l} x_n \longrightarrow \alpha. \\ x_n \neq \alpha \end{array}\right.$$

$$\frac{(1+x^2)^4 - (1+\alpha^2)^4}{x - \alpha}$$

$$y_n = 1 + x_n^2$$

$$\frac{(1+x_n^2)^4 - (1+\alpha^2)^4}{x_n - \alpha}$$

$$\longrightarrow 1+\alpha^2 = A \quad \text{とおく.}$$

$$(n \to +\infty)$$

$$= \frac{y_n^4 - A^4}{x_n - \alpha} = \frac{y_n^4 - A^4}{y_n - A} \cdot \frac{(1+x_n^2) - (1+\alpha^2)}{x_n - \alpha}$$

$$\longrightarrow (y^4)'\Big|_{y=A} \cdot (1+x^2)'\Big|_{x=\alpha}$$

$$= 4A^3 \cdot 2\alpha$$

$$= 4(1+\alpha^2)^3 \cdot 2\alpha$$

$$\boxed{\left\{(1+x^2)^4\right\}' = 4(1+x^2) \cdot 2x}$$

I (1) $\left(\dfrac{1}{2x+1}\right)' = ?$

(2) $\left(\dfrac{x}{x-1}\right)' = ?$

$$\left(\frac{g}{f}\right)' = \left(g \cdot \frac{1}{f}\right)' = g' \cdot \frac{1}{f} + g \cdot \left(\frac{1}{f}\right)'$$

$$= \frac{g'}{f} + g \cdot \left(-\frac{f'}{f^2}\right)$$

$$= \frac{f g' - f' g}{f^2}$$

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(3) $\{(x^2+1)(x-1)\}' = ?$

(4) $\left(\dfrac{1}{1+x+x^2}\right)' = ?$

II. $\{(2x-1)^{10}\}' = ?$

$(y^{10})' = 10y^9.$

$(2x+1)' = 2.$

$x_n \to \alpha.$ とおくと

$y_n = 2x_n - 1$

$\to 2\alpha - 1 = A$

とおく.