復習　$a_n \to \alpha,\ b_n \to \beta$ とする.　①

(1) $a_n \pm b_n \longrightarrow \alpha \pm \beta.$

(2) $a_n \cdot b_n \longrightarrow \alpha\beta$

(3) $a_n \neq 0,\ \alpha \neq 0$ のとき $\dfrac{1}{a_n} \to \dfrac{1}{\alpha}.$

I (1) $\dfrac{3\cdot 2^n - 5}{2^n + 3} = \dfrac{3 - 5\cdot\left(\frac{1}{2}\right)^n}{1 + 3\cdot\left(\frac{1}{2}\right)^n} \to \dfrac{3-5\cdot 0}{1+3\cdot 0}$

$= 3.$

$|r| < 1$ のとき $r^n \longrightarrow 0$

(2) $\dfrac{3n^2+4n}{2n^2-3} = \dfrac{3 + 4\cdot\frac{1}{n}}{2 - 3\cdot\frac{1}{n^2}} \longrightarrow \dfrac{3+4\cdot 0}{2-3\cdot 0}$

$= \dfrac{3}{2}$

$$\boxed{\frac{1}{n^k} \longrightarrow 0 \quad (n \to +\infty)}$$

(3) $\dfrac{3n-4}{2n^2+1} = \dfrac{3\cdot\frac{1}{n} - 4\cdot\frac{1}{n^2}}{2+\frac{1}{n^2}} \longrightarrow \dfrac{3\cdot 0 - 4\cdot 0}{2+0} = 0$

Ⅱ(1) $(n+1)n(n-1) - n(n-1)(n-2)$

$= n(n-1)\{(n+1)-(n-2)\}$

$= 3n(n-1)$

よって $n(n-1) = \frac{1}{3}\{(n+1)n(n-1) - n(n-1)(n-2)\}$

(2) $\sum_{k=1}^{n} k(k-1) = \frac{1}{3}\sum_{k=1}^{n}(f(k) - f(k-1))$

$f(k) = (k+1)k(k-1)$

$= \frac{1}{3}(f(n) - \boxed{f(0)})$ $\leftarrow 0$

後に説明 $= \frac{1}{3}(n+1)n(n-1)$

$$
\begin{array}{rll}
 & \boxed{f(n)} - \cancel{f(n-1)} & k=n \\
 & \cancel{f(n-1)} - \cancel{f(n-2)} & k=n-1 \\
 & \cancel{f(n-2)} - \cancel{f(n-3)} & k=n-2 \\
 & \vdots \qquad \vdots & \\
+) & \cancel{f(1)} \quad \boxed{-f(0)} & \\
\hline
 & f(n) - f(0) &
\end{array}
$$

公式

$$\sum_{k=1}^{n}(f(k) - f(k-1)) = f(n) - f(0)$$

$$\sum_{k=1}^{n} k^2 = \sum_{k=1}^{n} k(k-1) + \sum_{k=1}^{n} k$$

$$= \frac{1}{3}(n^3 - n) + \frac{n(n+1)}{2}$$

$$\begin{array}{rl} & 1 + 2 + 3 + \cdots + n = S \\ +\big) & n + (n-1) + (n-2) + \cdots + 1 = S \\ \hline \end{array}$$

$$2S = (n+1) + (n+1) + (n+1) + \cdots + (n+1)$$

$$= n(n+1)$$

D_n: 国債の残高.

P_n: プライマリー・バランス.

r: 利率 一定とする

$$D_{n+1} = (1+r)D_n - P_{n+1}$$

$$D_n = (1+r)^n D_0 - \left\{ (1+r)^{n-1}P_1 + (1+r)^{n-2}P_2 + \cdots \cdots + (1+r)P_{n-1} + P_n \right\}$$

両辺を

$(1+r)^{-n-1}$

$$\frac{D_{n+1}}{(1+r)^{n+1}} - \frac{D_n}{(1+r)^n} = -\frac{P_{n+1}}{(1+r)^{n+1}}$$

$$\sum_{k=1}^{n} \left(\frac{D_k}{(1+r)^k} - \frac{D_{k-1}}{(1+r)^{k-1}} \right) = -\sum_{k=1}^{n} \frac{P_k}{(1+r)^k}$$

$$\frac{D_n}{(1+r)^n} - D_0 = \frac{-P_n}{(1+r)^n} + \frac{-P_{n-1}}{(1+r)^{n-1}} + \cdots + \frac{-P_1}{(1+r)}$$

III. $f(x) = \dfrac{1}{x^2}$

$$\boxed{f'(x) = -\frac{2}{x^3}}$$

$h \neq 0$ とする.

$$\frac{1}{h}\cdot\left(f(\alpha + h) - f(\alpha) \right)$$

$$= \frac{\dfrac{1}{(\alpha+h)^2} - \dfrac{1}{\alpha^2}}{h} = \frac{1}{h}\cdot\frac{\alpha^2 - (\alpha+h)^2}{\alpha^2(\alpha+h)^2}$$

$$= \frac{(2\alpha + h)(-h)}{h\ \alpha^2\ (\alpha+h)^2}$$

$A^2 - B^2$
$= (A+B)(A-B)$

$$= -\frac{2\alpha + h}{\alpha^2(\alpha+h)^2}$$

$h_n \to 0$, $h_n \neq 0$ をみたす $\{h_n\}$ に対して

$$\frac{f(\alpha + h_n) - f(\alpha)}{h_n} = -\frac{2\alpha + h_n}{\alpha^2(\alpha + h_n)^2}$$

$$\longrightarrow -\frac{2\alpha}{\alpha^2\cdot\alpha^2} = \underbrace{-\frac{2}{\alpha^3}}$$

$\Downarrow$

$$\lim_{h\to 0}\frac{f(\alpha+h) - f(\alpha)}{h} = -\frac{2}{\alpha^3}$$

$\{h_n\}$ のとり方
に依らない.

$$\boxed{f'(\alpha) = -\frac{2}{\alpha^3}}$$

IV $\quad \boxed{f(x) = \dfrac{1}{1+x^2}}$

$h_n \to 0,\ h_n \neq 0$ をみたす $\{h_n\}$

$$\frac{f(\alpha+h_n) - f(\alpha)}{h_n} = \frac{1}{h}\left(\frac{1}{1+(\alpha+h)^2} - \frac{1}{1+\alpha^2}\right)$$

$$= \frac{1}{h}\left(\frac{(1+\alpha^2) - (1+(\alpha+h)^2)}{\{1+(\alpha+h)^2\}(1+\alpha^2)}\right)$$

$$= \frac{1}{h}\,\frac{(2\alpha+h)\cdot(-h)}{\{1+(\alpha+h)^2\}(1+\alpha^2)}$$

$$= -\frac{2\alpha+h_n}{(1+(\alpha+h_n)^2)(1+\alpha^2)}$$

$$\longrightarrow -\frac{2\alpha+0}{\{1+(\alpha+0)^2\}(1+\alpha^2)}$$

$$= -\frac{2\alpha}{(1+\alpha^2)^2}$$

$$f'(\alpha) = \lim_{h\to 0}\frac{1}{h}\left(f(\alpha+h) - f(\alpha)\right)$$

$$= -\frac{2\alpha}{(1+\alpha^2)^2}$$

矢上キャンパス $\longrightarrow$ 東大駒場

外国人.

$$f(k) - f(k-1) = (k+1)k(k-1)$$

$$(k+1)k(k-1) = k^3 - k$$

I. (1) $\displaystyle\sum_{k=1}^{n}(k^3 - k)$ を求めよ.

(2) $\displaystyle\sum_{k=1}^{n} k^3$ を求めよ.

II. $f(x) = \dfrac{1}{2+x}$ $(x \neq -2)$

$f'(\alpha) = ?$ $(\alpha \neq -2)$

(定義に従って計算する)

III. $g(x) = \dfrac{1}{x^3}$ $x \neq 0$

$g'(\alpha) = ?$ $(\alpha \neq 0)$