5月2日 演習 小テスト

I

(1) $\dfrac{3\cdot 2^n-5}{2^n+3}=\dfrac{3-5\cdot\frac{1}{2^n}}{1+3\cdot\frac{1}{2^n}}\longrightarrow\dfrac{3-0}{1+0}=3$

(2) $\dfrac{3n^2+4n}{2n^2-3}=\dfrac{3+4\cdot\frac{1}{n}}{2-3\cdot\frac{1}{n^2}}\longrightarrow\dfrac{3+4\cdot 0}{2-3\cdot 0}=\dfrac{3}{2}$

(3) $\dfrac{3n-4}{n^2+1}=\dfrac{3\cdot\frac{1}{n}-4\cdot\frac{1}{n^2}}{1+\frac{1}{n^2}}\longrightarrow\dfrac{3\cdot 0-4\cdot 0}{1+0}=\dfrac{0}{1}=0$

II (1) $n(n-1)=\dfrac{1}{3}\{(n+1)n(n-1)-n(n-1)(n-2)\}$

$(n+1)n(n-1)-n(n-1)(n-2)$

$=n(n-1)\{(n+1)-(n-2)\}$

$=3n(n-1)$

から分かる.

(2) $\displaystyle\sum_{k=1}^{n}k(k-1)=?$

$f(k)=(k+1)k(k-1)$ とすると

$$\sum_{k=1}^{n}k(k-1)=\frac{1}{3}\sum_{k=1}^{n}\{f(k)-f(k-1)\}$$

$$=\frac{1}{3}(f(n)-f(0))$$

$$=\frac{1}{3}(n+1)n(n-1)$$

$$=\frac{1}{3}(n^3-n)$$

を得る.

注) $\displaystyle\sum_{k=1}^{n} k^2 = \sum_{k=1}^{n}(k^2-k) + \sum_{k=1}^{n} k$

$$= \frac{1}{3}(n^3-n) + \frac{n(n+1)}{2}$$

$$= \frac{1}{6}\{(2n^3-2n)+(3n^2+3n)\}$$

$$= \frac{1}{6}(2n^3+3n^2+n)$$

$$= \frac{1}{6}n(n+1)(2n+1).$$

III $f(x)=\dfrac{1}{x^2}$ $(x \neq 0)$ $\alpha \neq 0$ とするとき $f'(\alpha)$ を求めよ.

$$\frac{1}{h}\cdot\frac{\dfrac{1}{(\alpha+h)^2}-\dfrac{1}{\alpha^2}}{h} = \frac{1}{h}\cdot\frac{\alpha^2-(\alpha+h)^2}{\alpha^2(\alpha+h)^2}$$

$$= \frac{-2\alpha-h}{\alpha^2(\alpha+h)^2}$$

から $h_n \to 0$ $(h_n \neq 0)$ である任意の数列 $\{h_n\}$ をとると.

$$\frac{f(\alpha+h_n)-f(\alpha)}{h} = \frac{-2\alpha-h_n}{\alpha^2(\alpha+h_n)^2}$$

$$\longrightarrow \frac{-2\alpha-0}{\alpha^2(\alpha+0)^2} = -\frac{2\alpha}{\alpha^4} = -\frac{2}{\alpha^3}$$

を得るので

$$f'(\alpha) = \lim_{h\to 0}\frac{f(\alpha+h)-f(\alpha)}{h} = -\frac{2}{\alpha^3}$$

IV $f(x)=\dfrac{1}{1+x^2}$ に対して $f'(\alpha)=?$

$h\neq 0$ のとき

$$\frac{f(\alpha+h)-f(\alpha)}{h}=\frac{1}{h}\cdot\left(\frac{1}{1+(\alpha+h)^2}-\frac{1}{1+\alpha^2}\right)$$

$$=\frac{1}{h}\cdot\frac{(1+\alpha^2)-(1+(\alpha+h)^2)}{(1+\alpha^2)(1+(\alpha+h)^2)}$$

$$=\frac{1}{h}\cdot\frac{-2\alpha h-h^2}{(1+\alpha^2)(1+(\alpha+h)^2)}=\frac{-2\alpha-h}{(1+\alpha^2)(1+(\alpha+h)^2)}$$

と計算される。ここで $h_n\to 0$, $h_n\neq 0$ をみたす数列 $\{h_n\}$ を任意にとると

$$\frac{f(\alpha+h_n)-f(\alpha)}{h_n}=\frac{-2\alpha-h_n}{(1+\alpha^2)(1+(\alpha+h_n)^2)}$$

$$\longrightarrow\frac{-2\alpha-0}{(1+\alpha^2)(1+(\alpha+0)^2)}=\frac{-2\alpha}{(1+\alpha^2)^2}$$

$$(n\to+\infty)$$

が成立するので

$$f'(\alpha)=\lim_{h\to 0}\frac{f(\alpha+h)-f(\alpha)}{h}=\frac{-2\alpha}{(1+\alpha^2)^2}$$

を得る。