先々週

5月21日 EXO 2005

$$S_N = \sum_{n=1}^{N} n\left(\frac{1}{3}\right)^n = 1\cdot\left(\frac{1}{3}\right) + 2\cdot\left(\frac{1}{3}\right)^2 + \cdots + N\cdot\left(\frac{1}{3}\right)^N$$

$$\frac{1}{3}S_N = 1\cdot\left(\frac{1}{3}\right)^2 + \cdots + (N-1)\left(\frac{1}{3}\right)^N + \underbrace{N\left(\frac{1}{3}\right)^{N+1}}$$

$$\frac{2}{3}S_N = 1\cdot\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \cdots + \left(\frac{1}{3}\right)^N - N\left(\frac{1}{3}\right)^{N+1}$$

$$N\cdot\left(\frac{1}{3}\right)^{N+1}$$

$$= \frac{\frac{1}{3}\left(1-\left(\frac{1}{3}\right)^N\right)}{1-\frac{1}{3}} - N\left(\frac{1}{3}\right)^{N+1}$$

$$\boxed{r \neq 1 \qquad 1 + r + r^2 + \cdots + r^N = \frac{1-r^{N+1}}{1-r}}$$

$$a + a\cdot r + a\cdot r^2 + \cdots + a r^N = \frac{a(1-r^{N+1})}{1-r}$$

先週 II.

$$S_N = 1 + 2\cdot\left(\frac{1}{2}\right) + 3\cdot\left(\frac{1}{2}\right)^2 + \cdots + N\left(\frac{1}{2}\right)^{N-1}$$

$$-\big)\quad \frac{1}{2}S_N = 1\cdot\frac{1}{2} + 2\cdot\left(\frac{1}{2}\right)^2 + \cdots + (N-1)\left(\frac{1}{2}\right)^{N-1} + N\left(\frac{1}{2}\right)^N$$

$$\frac{1}{2}S_N = 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \cdots + \left(\frac{1}{2}\right)^{N-1} - N\left(\frac{1}{2}\right)^N$$

$$= \frac{1-\left(\frac{1}{2}\right)^N}{1-\frac{1}{2}} - N\left(\frac{1}{2}\right)^N$$

$$S_N = 2\cdot 2\left(1-\left(\frac{1}{2}\right)^N\right) - 2N\left(\frac{1}{2}\right)^N$$

$$\longrightarrow 4(1-0) - 2\cdot 0 = 4$$

$$|r|<1 \qquad r^N \longrightarrow 0 \quad (N\to+\infty) \qquad (N\to+\infty)$$

$$N r^N \longrightarrow 0 \quad (N\to+\infty)$$

$$|r|<1 \qquad \sum_{n=1}^{\infty} n r^{n-1} = \frac{1}{(1-r)^2}.$$

III. $\boxed{\dfrac{r^n}{1+r^n} \longrightarrow \ ? \qquad r \neq -1}$

・$|r|<1$ のとき $r^n \to 0 \quad (n \to +\infty)$

$$\frac{r^n}{1+r^n} \longrightarrow \frac{0}{1+0} = 0 .$$

・$r=1$ $\quad \dfrac{r^n}{1+r^n} = \dfrac{1}{2} \longrightarrow \dfrac{1}{2}$

・$\underline{|r|>1}$ $\quad \dfrac{r^n}{1+r^n} = \dfrac{1}{\left(\frac{1}{r}\right)^n + 1} \longrightarrow \dfrac{1}{0+1} = 1$

$\left|\dfrac{1}{r}\right| < 1$ だから $\left(\dfrac{1}{r}\right)^n \longrightarrow 0$

IV. $a_0 = 0, \quad a_{n+1} = \dfrac{1}{2} a_n + 1$

$\boxed{\begin{array}{l} \lambda = \frac{1}{2}\lambda + 1 \\ \lambda = 2 \end{array}}$ $\rightarrow$ $2 = \dfrac{1}{2} \cdot 2 + 1$

$$a_{n+1} - 2 = \frac{1}{2}(a_n - 2)$$

$\{a_n - 2\}_{n=0}^{+\infty}$ は公比 $\frac{1}{2}$ の等比数列

$$a_n - 2 = \left(\frac{1}{2}\right)^n (a_0 - 2)$$

だから

$$a_n = 2 + \left(\frac{1}{2}\right)^n (a_0 - 2)$$

収束することを
示している.

$$\longrightarrow 2 + 0 \cdot (a_0 - 2) = 2 .$$

注 この $\{a_n\}$ が収束していると仮定

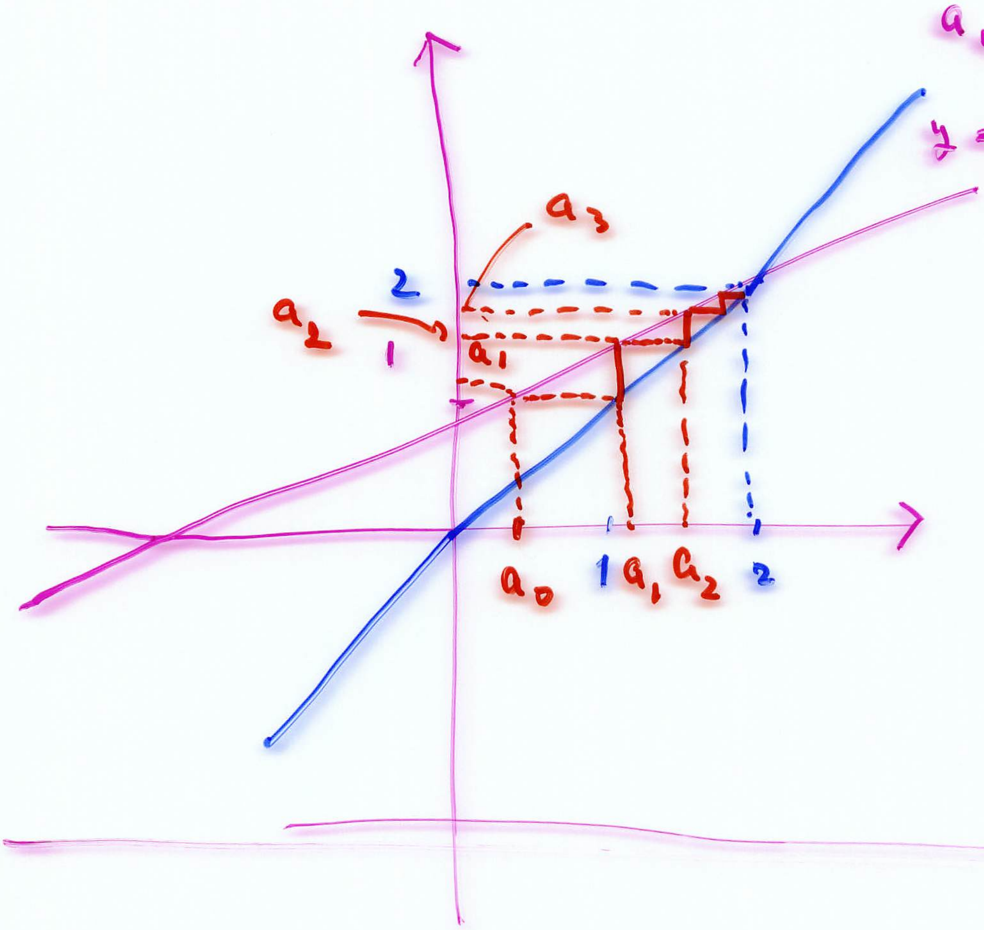
$a_n \longrightarrow \alpha$ とする

$$a_{n+1} = \frac{1}{2} a_n + 1 \xrightarrow{n \to +\infty} \alpha = \frac{1}{2} \cdot \alpha + 1$$

$\underset{\sim\sim\sim}{\alpha = 2}$

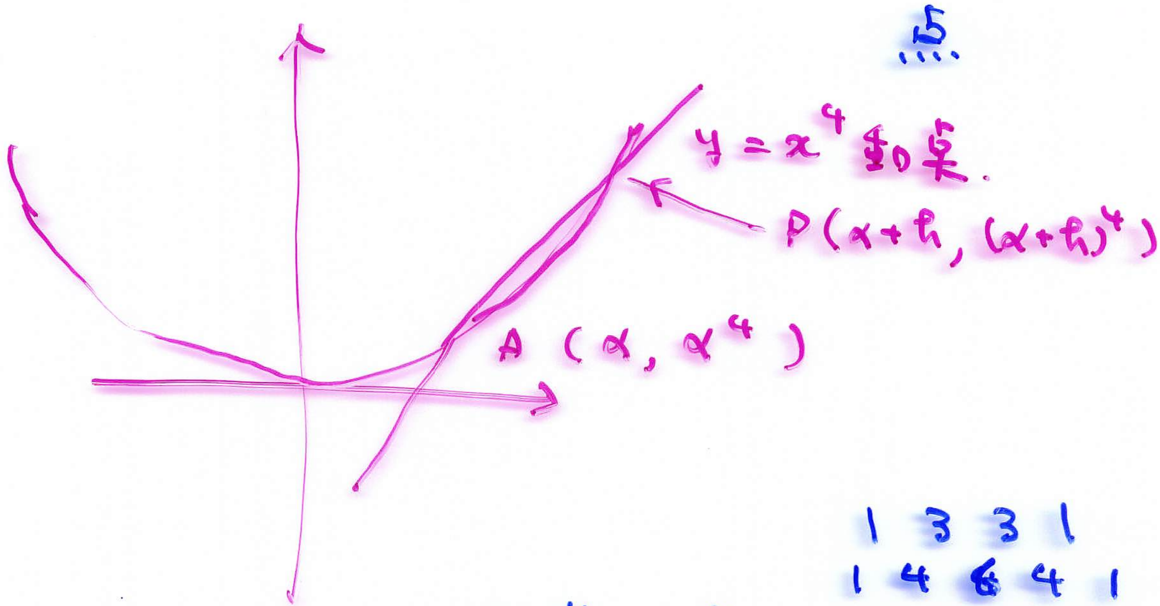
$$a_{n+1} = \frac{1}{2}a_n + 1$$

$$y = \frac{1}{2}x + 1$$



$y = x^4$

点



1 3 3 1
1 4 6 4 1

$$AP の傾き = \frac{(\alpha + h)^4 - \alpha^4}{(\alpha + h) - \alpha}$$

$$= \frac{1}{h}(\cancel{\alpha^4} + 4\alpha^3 h + 6\alpha^2 h^2 + 4\alpha h^3 + h^4 \cancel{- \alpha^4})$$

$$= 4\alpha^3 + 6\alpha^2 h + 4\alpha h^2 + h^3$$

任意に $h_n \to 0$ である $\{h_n\}$ をとる。
$h_n \neq 0$

$$傾き = 4\alpha^3 + 6\alpha^2 h_n + 4\alpha h_n^2 + h_n^3$$

$$\longrightarrow 4\alpha^3 + 6\alpha^2 \cdot 0 + 4\alpha \cdot 0^2 + 0^3$$

$$= \boxed{4\alpha^3}$$ ← $\{h_n\}$ に依らない

$f(x) = x^4$ のとき $f'(x) = 4x^3$.

$$\lim_{h \to 0} AP の傾き = 4\alpha^3.$$

I 次の $\{a_n\}$ の極限を求めよ.

(1) $\dfrac{3\cdot 2^n - 5}{2^n + 3}$ (2) $\dfrac{3n^2 + 4n}{2n^2 - 3}$

(3) $\dfrac{3n-4}{n^2+1}$

II (1) $n(n-1) = \dfrac{1}{3}\left\{(n+1)n(n-1) - n(n-1)(n-2)\right\}$

を示せ

(2) $S_n = \displaystyle\sum_{k=1}^{n} k(k-1)$ を求めよ.

III $f(x) = \dfrac{1}{x^2}$ とする. $(x \neq 0)$

$f'(\alpha) = ?$

IV $f(x) = \dfrac{1}{1+x^2}$ とする.

$f'(\alpha) = \underline{?}$

Ⅱ $a_k = k$

$$n = \frac{1}{2}\{(n+1)n - n(n-1)\}$$

$$\frac{1}{2}\left((n+1)n - \cancel{n(n-1)}\right)$$

$$S_n = \sum_{k=1}^{n} k = ?$$

$$\frac{1}{2}(\cancel{4\cdot 3} - \cancel{3\cdot 2})$$

$$\frac{1}{2}(\cancel{3\cdot 2} - \cancel{2\cdot 1})$$

$$+)\ \frac{1}{2}(\cancel{2\cdot 1} - 1\cdot 0)$$

$$S_n = \frac{1}{2}(n+1)n - 1\cdot 0$$

$$= \frac{1}{2}(n+1)n.$$