Est-ce que je pourrais commencer?

復習

(I) 数列の極限.

基本定理. $\alpha, \beta \in \mathbb{R}$.

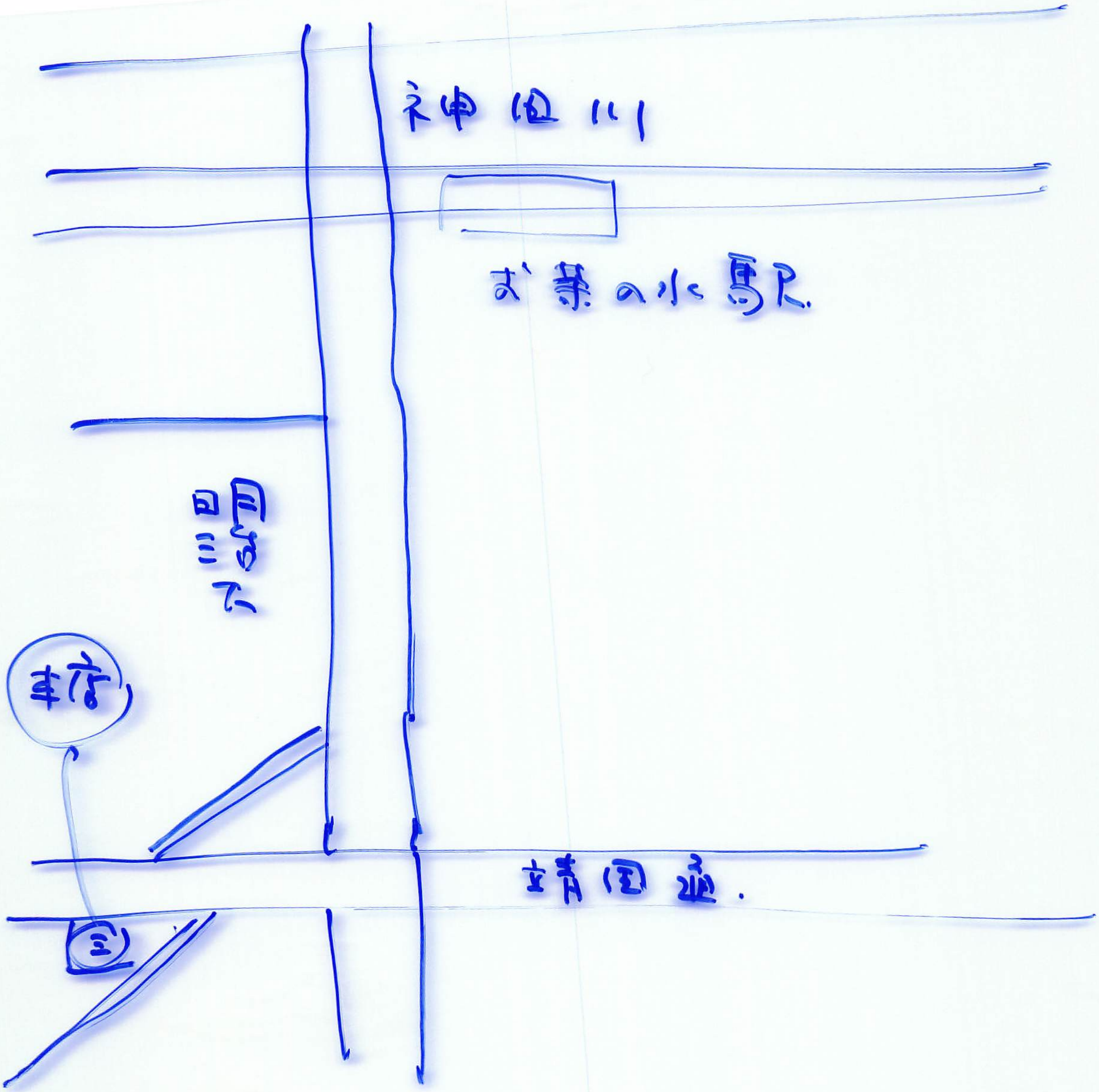
$$a_n \longrightarrow \alpha \ , \ b_n \longrightarrow \beta \ (n \to +\infty)$$

① $a_n \pm b_n \longrightarrow \alpha \pm \beta$

② $a_n \cdot b_n \longrightarrow \alpha\beta$

③ $a_n \neq 0 , \alpha \neq 0$

$$\frac{1}{a_n} \longrightarrow \frac{1}{\alpha}.$$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① $\frac{2n-5}{n} = 2 - \frac{5}{n} \longrightarrow 2 - 5 \times 0 = 2$

$$\boxed{\frac{1}{n} \longrightarrow 0 \quad (n \rightarrow +\infty)}$$

② $\frac{4n^2-3n-5}{n^2} = 4 - 3 \cdot \frac{1}{n} - 5 \cdot \frac{1}{n^2}$

$\longrightarrow 4 - 3 \cdot 0 - 5 \cdot 0 = 4.$

$\frac{1}{n^2} = \frac{1}{n} \times \frac{1}{n} \longrightarrow 0 \times 0 = 0$

③ $\frac{3n-1}{2n^2+3} = \frac{1}{n} \cdot \frac{n(3n-1)}{2n^2+3}.$

$= \frac{1}{n} \cdot \frac{3 - \frac{1}{n}}{2 + 3 \cdot \frac{1}{n^2}}$

$\longrightarrow 0 \times \frac{3-0}{2+3 \cdot 0} = 0 \times \frac{3}{2} = 0$

Ⅱ) $|r| < 1.$ $\quad r^n \longrightarrow 0$

$S_n = 1 + r + r^2 + \cdots + r^n$

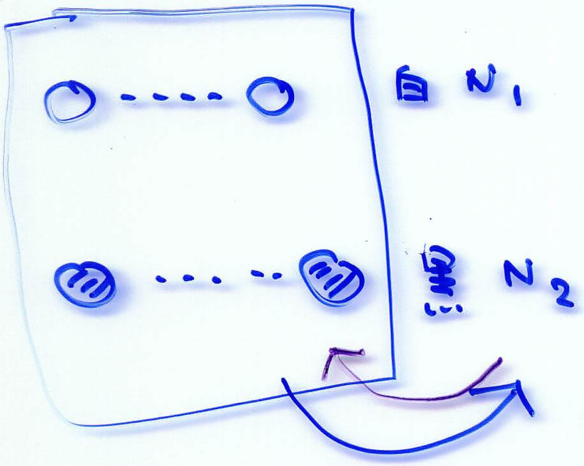
$= \frac{1-r^{n+1}}{1-r} = \frac{1}{1-r} - \frac{r}{1-r} \cdot r^n$

$\longrightarrow \frac{1}{1-r} - \frac{r}{1-r} \cdot 0 = \frac{1}{1-r}$

$|r|<1$ のとき

まとめ $\quad 1 + r + r^2 + \cdots\cdots = \frac{1}{1-r}$

幾何分布.



$p = \dfrac{N_1}{N_1+N_2}$ 白が出る確率

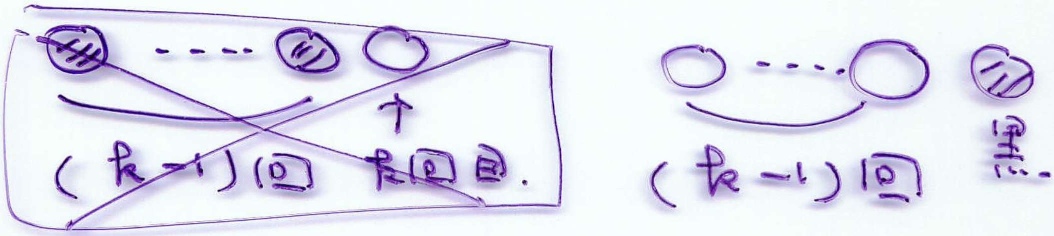
$q = \dfrac{N_2}{N_1+N_2}$ 黒が出る確率.

$0<p<1,\ 0<q<1$

$p+q=1$

k回目で初めて~~白~~黒が出る確率

$= p^{k-1}\cdot q$



$$\sum_{k=1}^{+\infty} p^{k-1} q = q(1+p+p^2+\cdots)$$

$$= q\,\frac{1}{1-p} = q\cdot\frac{1}{q} = 1$$

全確率 $=1$.

$$\text{期待値} = \sum_{k=1}^{+\infty} k\cdot p^{k-1} q = \cdots$$

$$= q\cdot\frac{1}{(1-p)^2} = \frac{1}{q}\ ?$$

予備 $|r|<1$

$n r^n \longrightarrow 0 \quad (n\to+\infty)$

$n r^{n-1} \longrightarrow 0 \quad (n\to+\infty)$

$$T_N = \sum_{n=1}^{N} n r^{n-1} = 1 + 2\cdot r^1 + 3r^2 + \cdots + N r^{N-1}$$

$$rT_N = r + 2r^2 + \cdots + (N-1) r^{N-1} + N r^N$$

$$(1-r)T_N = 1 + r + r^2 + \cdots + r^{N-1} - N r^N$$

$$= \frac{1-r^N}{1-r} - N r^N$$

$$T_N = \frac{1-r^N}{(1-r)^2} - \frac{1}{1-r} N r^N$$

$$= \frac{1}{(1-r)^2} - \frac{1}{(1-r)^2} \underbrace{r^N}_{\to 0} - \frac{1}{1-r} \underbrace{N r^N}_{\to 0}$$

$$\longrightarrow \frac{1}{(1-r)^2} - \frac{1}{(1-r)^2}\times 0 - \frac{1}{1-r}\times 0$$

$$= \frac{1}{(1-r)^2}$$

よって $\displaystyle\sum_{n=1}^{+\infty} n r^{n-1} = \frac{1}{(1-r)^2} \quad (|r|<1)$

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$$\frac{4}{x-3} < 2x+1$$

$(x \neq 3.)$

$(x-3)^2 > 0$

$$2x+1-\frac{4}{x-3} > 0$$

$$= \frac{(2x+1)(x-3)-4}{x-3}$$

$4 < (x-3)^2(2x+1)$

$(x-3)$

$$= \frac{2x^2-5x-7}{x-3}$$

1 1

2 −7

$$= \frac{(x+1)(2x-7)}{x-3}$$

$-1, 3, \frac{7}{2}$

		-1		3		$\frac{7}{2}$	
$x+1$	−	0	+		+	+	+
$x-3$	−	−	−		+	+	+
$2x-7$	−	−	−		−	0	+
$\frac{(x+1)(2x-7)}{x-3}$	−	0	+		−		+

$$\frac{(x+1)(2x-7)}{x-3}$$

$-1 < x < 3$

$\frac{7}{2} < x$

I $\dfrac{2}{x-1} - \dfrac{2}{x} \geqq 1$ を解け.

II. $\displaystyle\sum_{n=1}^{+\infty} n\left(\frac{1}{2}\right)^{n-1} = ?$

III. $r \neq -1$ とする. r について場合分けをして

$\dfrac{r^n}{1+r^n}$ の $n \to +\infty$ の極限を求めよ.

$\left(|r| > 1 \text{ のとき } R = \frac{1}{r} \text{ は } |R| < 1 \text{ をみたす} \right)$

IV $a_0 = 0$, $a_{n+1} = \frac{1}{2} a_n + 1$ で定まる数列の極限を求めよ.

Find the limit of the sequence $\{a_n\}$ satisfying $a_0 = 0$ and $a_{n+1} = \frac{1}{2} a_n + 1$