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$$a_{n+1} = -2a_n + 1$$

$$\alpha = -2\alpha + 1$$ ← n に依らない定数の解を求めている

$a_n = \alpha$ $(n = 0, 1, 2, \cdots)$

→ $3\alpha = 1$ より $\alpha = \frac{1}{3}$

$$\begin{array}{rrcl} & a_{n+1} &=& -2a_n + 1 \\ -) & \frac{1}{3} &=& -2\cdot\frac{1}{3} + 1 \\ \hline \end{array}$$

$$a_{n+1} - \frac{1}{3} = -2\left(a_n - \frac{1}{3}\right)$$

$b_n = a_n - \frac{1}{3}$ とし $\boxed{b_{n+1} = -2b_n}$

↓

$\{b_n\}$ は公比 (-2) の等比数列

$$b_n = (-2)^n b_0$$

$$a_n - \frac{1}{3} = (-2)^n\left(a_0 - \frac{1}{3}\right)$$

→ $$a_n = \frac{1}{3} + (-2)^n\left(a_0 - \frac{1}{3}\right)$$

2) $\alpha \neq 1$ とする

$$S_n = 1 + \boxed{\alpha + \alpha^2 + \cdots + \alpha^n}$$

$$-)\ \alpha S_n = \boxed{\alpha + \alpha^2 + \cdots + \alpha^n} + \alpha^{n+1}$$

$$S_n \underbrace{(1-\alpha)}_{\neq 0} = 1 \quad \boxed{} \quad - \alpha^{n+1}$$

$$S_n = \frac{1-\alpha^{n+1}}{1-\alpha}$$

乗数効果.

$0 < \alpha < 1$

T : 投資.

$$T + \alpha T + \alpha^2 T + \alpha^3 T + \cdots$$

$$= \frac{T}{1-\alpha}$$

$|\alpha| < 1$ $\quad 1 + \alpha + \alpha^2 + \cdots = \dfrac{1}{1-\alpha}$

$$T_n = 1 + 2\alpha + 3\alpha^2 + \cdots + n\alpha^{n-1}$$

$$-)\ \alpha T_n = \alpha + 2\alpha^2 + \cdots + (n-1)\alpha^{n-1} + n\alpha^n$$

$$(1-\alpha) T_n = 1 + \alpha + \cdots + \alpha^{n-1} - n\alpha^n$$

$$= \frac{1-\alpha^n}{1-\alpha} - n\alpha^n$$

$$T_n = \frac{1-\alpha^n}{(1-\alpha)^2} - \frac{n\alpha^n}{1-\alpha}$$

別の解法.

$$a_{n+2} - 5a_{n+1} + 4a_n = 0$$

$$\underline{a_0,\ a_1} \rightsquigarrow a_2 = 5a_1 - 4a_0$$

$$\vdots$$

特性方程式.

$$\lambda^2 - 5\lambda + 4 = 0$$

$$(\lambda - 4)(\lambda - 1) = 0 \quad \lambda = 4, 1$$

$$\left.\begin{array}{l} 4 \cdot 1 = 4 \\ 4 + 1 = 5 \end{array}\right)$$

$$a_{n+2} - (1+4)a_{n+1} + 4 \cdot 1\, a_n = 0$$

$$\begin{cases} a_{n+2} - a_{n+1} = 4(a_{n+1} - a_n) \\ \underbrace{a_{n+2} - 4a_{n+1}}_{= c_{n+1}} = a_{n+1} - 4a_n \end{cases}$$

$b_n = a_{n+1} - a_n$ とおく

$c_n = a_{n+1} - 4a_n$

$$b_{n+1} = 4b_n \qquad c_{n+1} = c_n$$

$$\longrightarrow b_n = 4^n b_0 \qquad c_n = c_0$$

$$\rightsquigarrow \begin{cases} a_{n+1} - a_n = 4^n(a_1 - a_0) \\ a_{n+1} - 4a_n = a_1 - 4a_0 \end{cases}$$

$$3a_n = 4^n(a_1 - a_0) - (a_1 - 4a_0)$$

$$a_{n+2} - 4a_{n+1} + 4a_n = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$ 特性方程式.

$$(\lambda - 2)^2 = 0 \qquad \lambda = 2$$ (重解)

$$a_{n+2} - (2+2)a_{n+1} + 2\cdot 2\, a_n = 0$$

$$\boxed{a_{n+2} - 2a_{n+1} = 2(a_{n+1} - 2a_n)}$$

$$\frac{1}{2^{n+1}}$$

$$\frac{a_{n+2} - 2a_{n+1}}{2^{n+1}} = \frac{a_{n+1} - 2a_n}{2^n}$$

$$a_{n+1} - 2a_n = 2^n(a_1 - 2a_0)$$

$$\frac{1}{2^{n+1}}$$

$$\boxed{\frac{a_{n+1}}{2^{n+1}} - \frac{a_n}{2^n} = \frac{1}{2}(a_1 - 2a_0)}$$

$c_n = \dfrac{a_n}{2^n}$ は 等差 数列

$$c_{n+1} - c_n = \underbrace{\frac{1}{2}(a_1 - 2a_0)}_{\text{公差}}$$

$$b_{n+1} - b_n = \alpha$$

$$b_n = n\alpha + b_0$$

$$c_n = \frac{n}{2}(a_1 - 2a_0) + c_0$$

$$\frac{a_n}{2^n} = \frac{n}{2}(a_1 - 2a_0) + \frac{a_0}{1}$$

$$a_n = n\,2^{n-1}(a_1 - 2a_0) + 2^n a_0$$

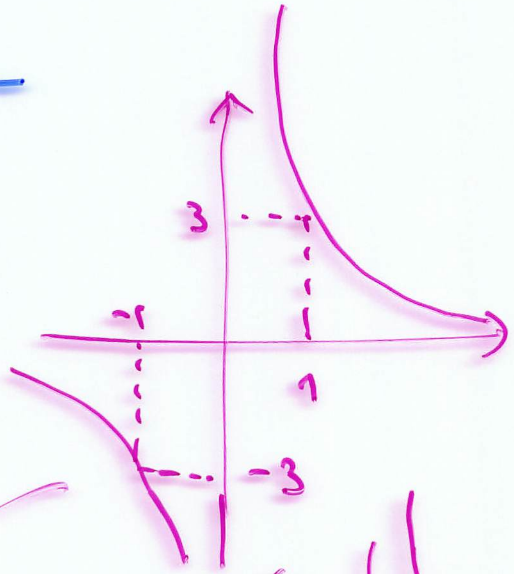
[illegible]

$$y = \frac{2x+1}{x-1} \quad (x \neq 1)$$

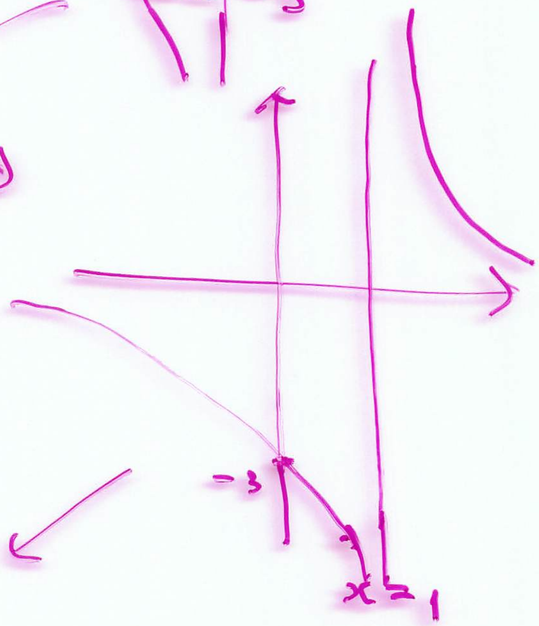
$$= \frac{2(x-1)+3}{x-1}$$

$$= 2 + \frac{3}{x-1}$$

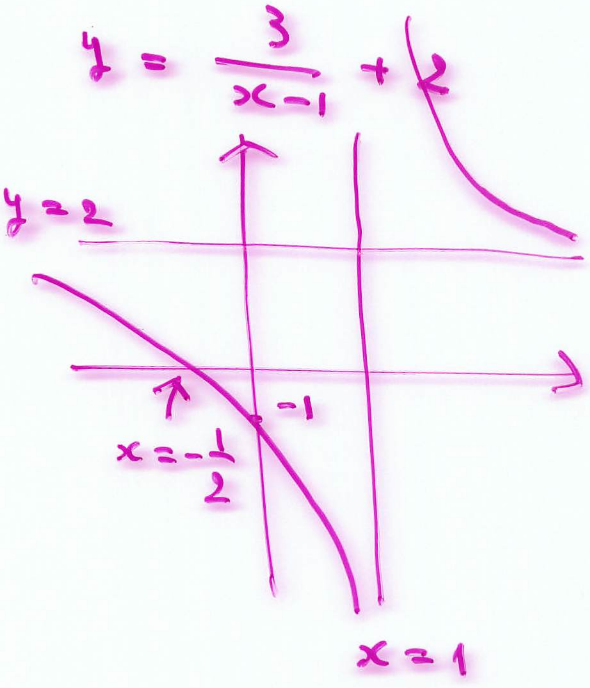
$$y = \frac{3}{x}$$



$$y = \frac{3}{x-1}$$



$$y = \frac{3}{x-1} + 2$$



$$y = \frac{2x+1}{x-1}$$

$$y = 0 \iff x = -\frac{1}{2}$$

予習

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 \\
 & & & & & \vdots & & & & &
\end{array}$$

2項定理

$$(a+b)^n = a^n + {}_nC_1 a^{n-1}b + {}_nC_2 a^{n-2}b^2 + \cdots$$

$$\cdots + {}_nC_k a^{n-k}b^k + \cdots + b^n$$

$$\overbrace{(a+b)(a+b)\cdots\cdots(a+b)}^{n\text{因子}}$$ factor

$$\underline{{}_nC_k}\, a^{n-k} b^k$$

I $a_{n+1} = 4a_n - 2$ $(n = 0, 1, 2, \cdots)$
の解を n と a_0 で表す.

II $S_N = \sum_{n=1}^{N} n\left(\frac{1}{3}\right)^n = 1 \cdot \frac{1}{3} + 2 \cdot \left(\frac{1}{3}\right)^2 + 3 \cdot \left(\frac{1}{3}\right)^3 + \cdots + N\left(\frac{1}{3}\right)^N$

III $a_{n+2} - a_{n+1} - 6a_n = 0$
を解く. a_0, a_1 で表す.

IV $a_{n+2} - 6a_{n+1} + 9a_n = 0$
を解く.

V. $y = \dfrac{3x-1}{x+1}$ のグラフを求める.

VI $(a+b)^{10}$ の $a^4 b^6$ の係数.