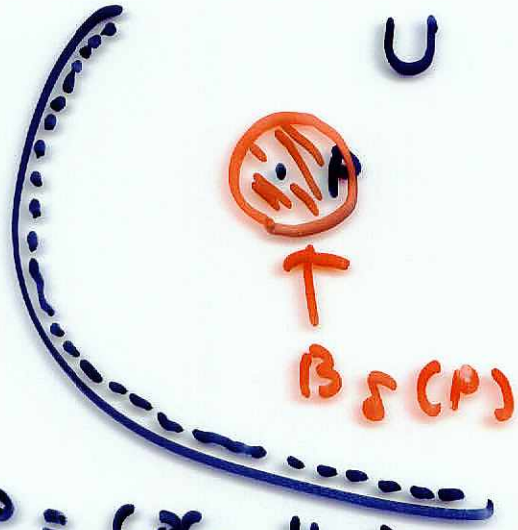
CT 246p.

開集合. $U \subset \mathbb{R}^2$ が開集合

任意の点 $P \in U$

$B_\delta(P) \subset U$

がある $\delta > 0$ に
対して成立.



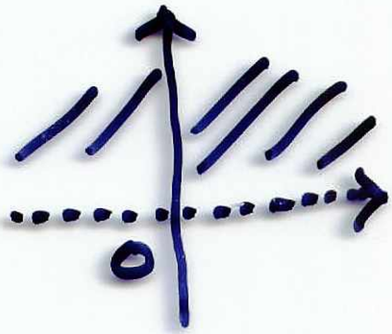
$P = (x_0, y_0)$

$$B_\delta(P) = \left\{ (x, y) ; \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta \right\}$$

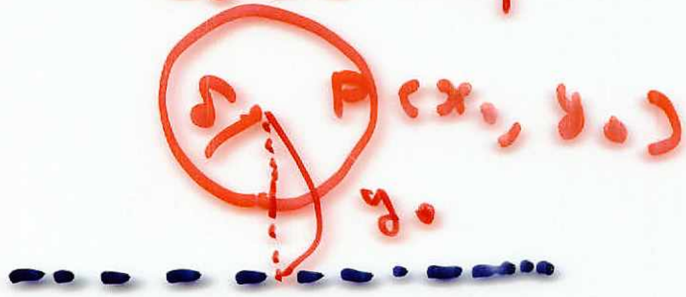
中心 P.
半径 $\delta > 0$ の
開円盤

$P(x_0, y_0)$

$V_1 = \{(x, y) \,;\, y > 0\}$



$B_\delta(P) \subset V_1$



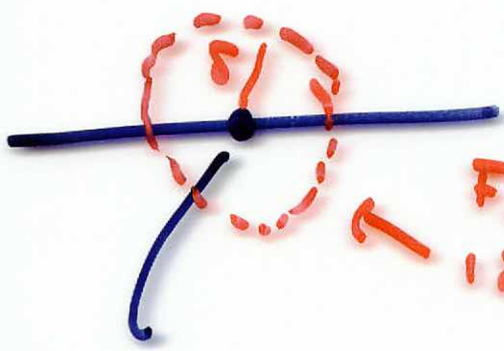
$\longrightarrow$ $P \in V_1$

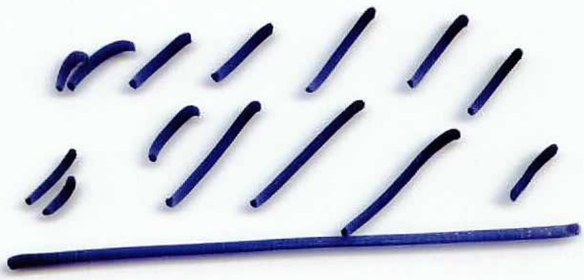
$y_0 > 0$

$\delta = \dfrac{y_0}{2} > 0$

$\rightarrow$ V_1 開集合.

$F_1 = \{(x, y) \,;\, y \geq 0\}$





F_1 からはみでる

$(0, 0) \in F_1$ $\rightsquigarrow$ F_1 は開でない.

偏微分係数.　$U \subset \mathbb{R}^2$ 開

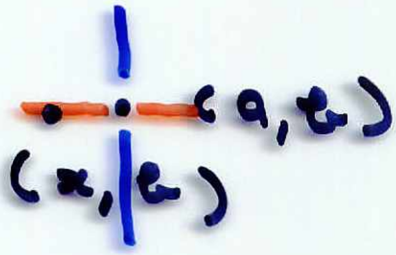
$$f : U \longrightarrow \mathbb{R}$$

定義域　　値域

例　$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

$U = \mathbb{R}^2$

$$F(x) = x^4 + b^4 - 2x^2 + 4xb - 2b^2 = f(x, b)$$



$$F'(x) = 4x^3 + 0 - 4x + 4b + 0 = 4x^3 - 4x + 4b$$

$$F'(a) = 4a^3 - 4a + 4b = f_x(a, b) = \frac{\partial f}{\partial x}(a, b)$$

f の x に関する偏微分係数

$$
\begin{aligned}
G(y) &= f(a, y) \\
&= a^4 + y^4 - 2a^2 \\
&\quad + 4ay - 2y^2
\end{aligned}
$$

(a, b)

(a, y)

$$G'(y) = 0 + 4y^3 - 0 + 4a - 4y$$

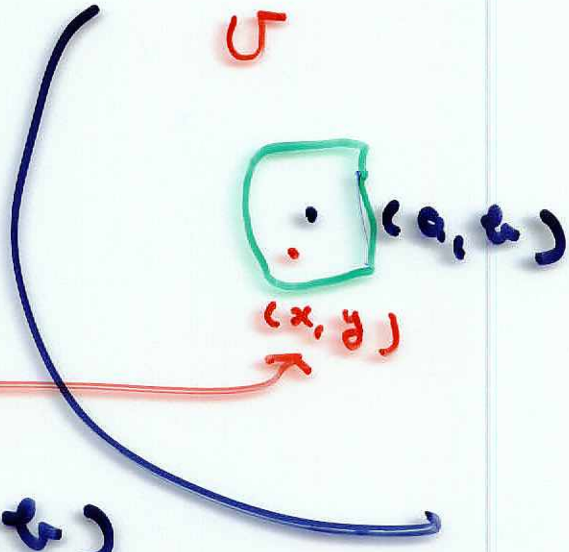
$$
\begin{aligned}
G'(b) &= 4b^3 + 4a - 4b. \\
&= f_y(a, b) = \frac{\partial f}{\partial y}(a, b)
\end{aligned}
$$

f の <u>y に関する</u>偏微分係数

$$
\begin{cases}
\dfrac{\partial f}{\partial x} = 4x^3 - 4x + 4y \\[2ex]
\dfrac{\partial f}{\partial y} = 4y^3 + 4x - 4y
\end{cases}
$$

2変数の極大・極小

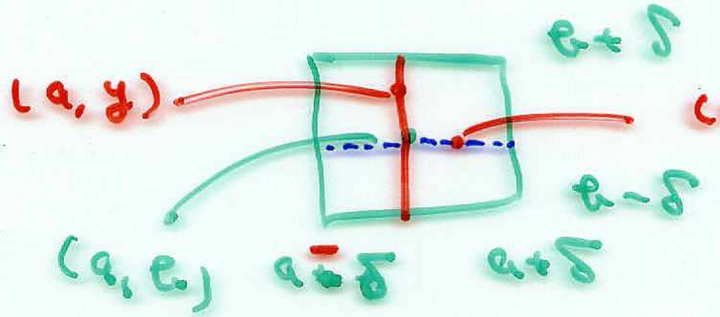
$f : U \longrightarrow \mathbb{R}$



f が (a, b) で
極小（大）であるとは

$$f(x, y) \geq f(a, b)$$
$$(\leq)$$

$$\left((x, y) \in [a-\delta, a+\delta] \times [b-\delta, b+\delta] \right)$$



$\longrightarrow \begin{cases} \bullet\ F(x) = f(x, b) \text{ は } x = a \text{ で極小（大）} \\ \bullet\ G(y) = f(a, y) \text{ は } y = b \text{ で極小（大）} \end{cases}$

$\rightsquigarrow \begin{cases} F'(a) = \dfrac{\partial f}{\partial x}(a, b) = 0 \\ G'(b) = \dfrac{\partial f}{\partial y}(a, b) = 0 \end{cases}$

$$f(x,y) = x^2 + xy + y^2 - 4x - 2y$$

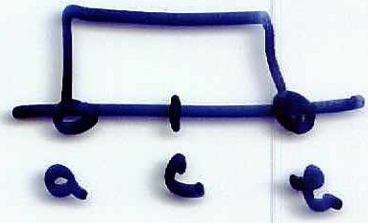
$$\frac{\partial f}{\partial x} = 2x + y \cdot 1 + 0 - 4 - 0$$

y：定数. $\quad = 2x + y - 4.$

$$\frac{\partial f}{\partial y} = 0 + x \cdot 1 + 2y - 0 - 2$$

x：定数. $\quad = x + 2y - 2.$

定理 $\quad (a, b) \xrightarrow{f} \mathbb{R}$



$f(c)$ が 極大値 (小) $\Longrightarrow f'(c) = 0$

定理 $\quad f : U \longrightarrow \mathbb{R}$

$P(a, b)$ が f の 極小 (大)

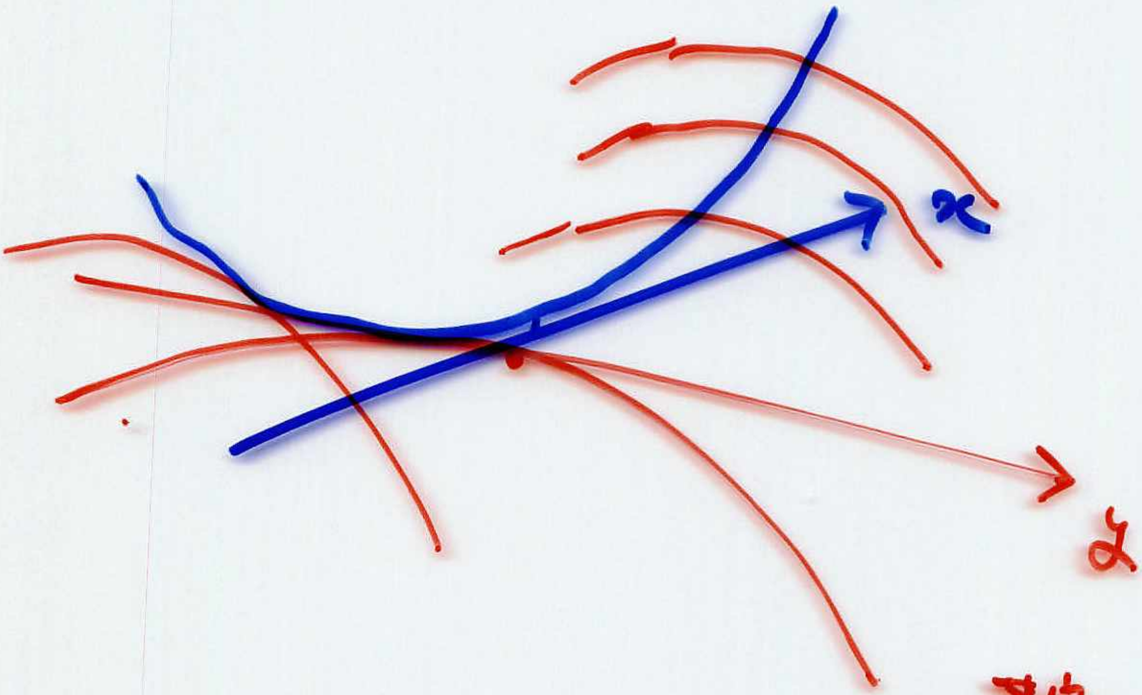
$\in U \quad \Longrightarrow \boxed{\frac{\partial f}{\partial x}(a, b) = \frac{\partial f}{\partial y}(a, b) = 0}$

逆 $\Leftarrow$ は成立 (せず..)

停留点.

stationary point

$z = f(x, y) = x^2 - y^2$



鞍点

$z = f(x, 0) = x^2$

$z = f(0, y) = -y^2$

saddle point

le point de col.

$z_x = 2x$

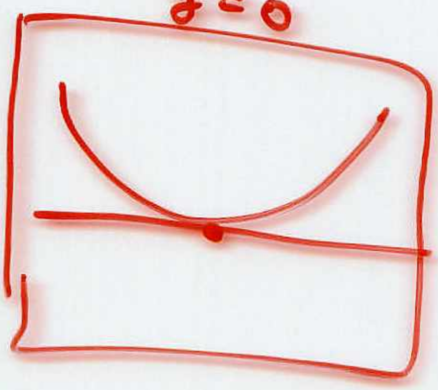
$z_y = 2y$

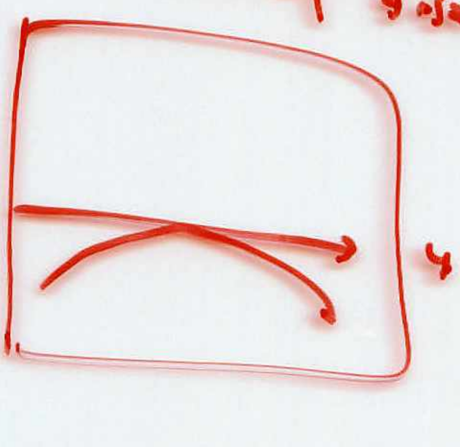
$z_x(0,0) = z_y(0,0) = 0$

だけど $(0,0)$ では極大でも

$x=0$ すなわち、でもない

$y=0$





$F'(a) = 0$

$F''(a) > 0$

$\quad\quad < 0$

$\Longrightarrow$ $F(a)$ は 極小値.

大

これに対応する2変数の定理は

参考書 I_A.

$$f(x,y) = x^2 + xy + y^2 - 4x - 2y$$

$$\begin{cases} \dfrac{\partial f}{\partial x} = 2x + y - 4 = 0 \\[2ex] \dfrac{\partial f}{\partial y} = x + 2y - 2 = 0 \end{cases}$$

$$\Longleftrightarrow \quad (x,y) = (2,0)$$

停留点

II. $z = f(x,y) = x^3 + y^3 + 6xy$

$$f_x = 3x^2 + 0 + 6y \cdot 1$$

$$= 3x^2 + 6y = 0 \quad \cdots ①$$

$$f_y = 0 + 3y^2 + 6x$$

$$= 3(y^2 + 2x) = 0 \quad \cdots ②$$

$$\begin{cases} ① \rightsquigarrow \ y = -\dfrac{1}{2}x^2 \cdot 2 \\ ② \rightsquigarrow \ y^2 + 2x = 0 \text{ に代入.} \end{cases}$$

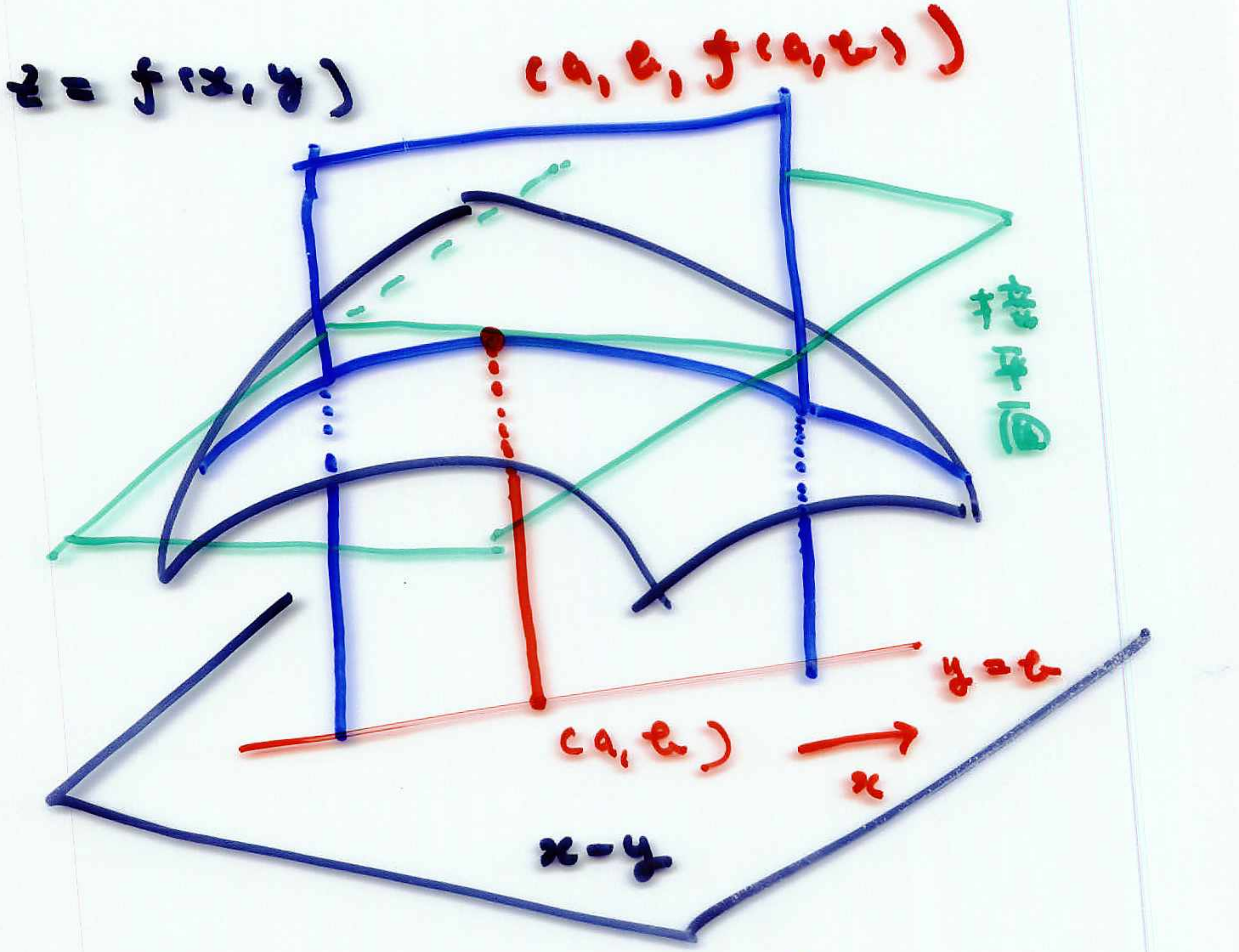
$$\frac{1}{4}x^4 + 2x = 0$$

$$\Longleftrightarrow \quad x(x^3 + 8) = 0$$

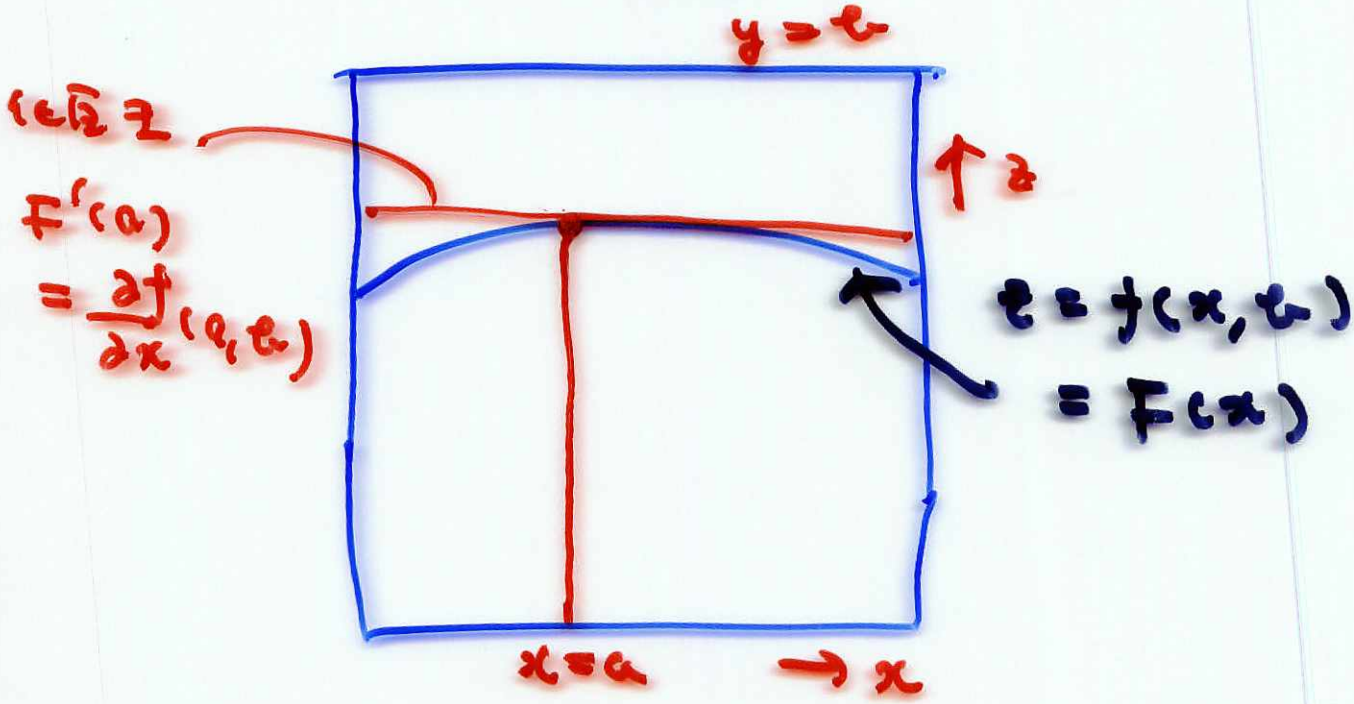
$$\Longleftrightarrow \quad x = 0, -2$$

$x = 0$ のとき $y = 0$

$x = -2$ のとき $y = -2$



z = f(x, y)
(a, b, f(a, b))
接平面
y = b
(a, b)
x
x − y



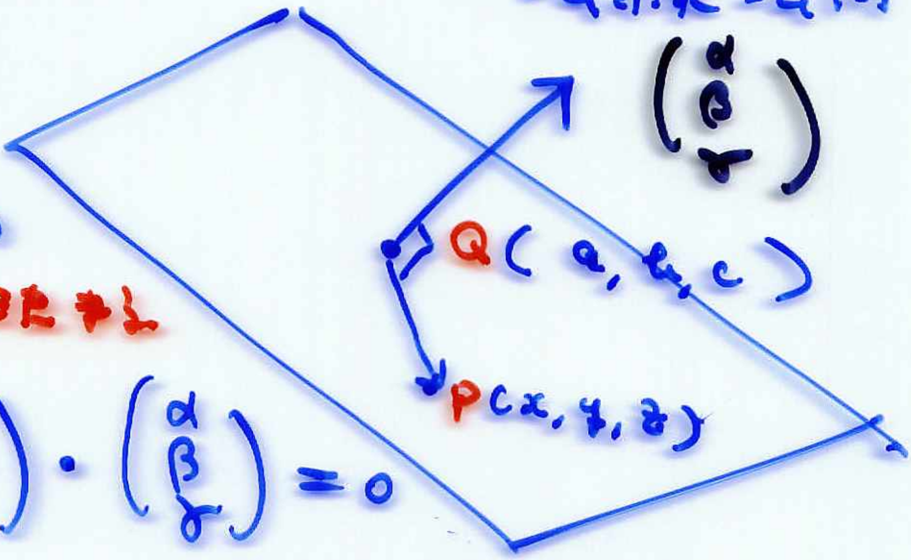
y = b
傾き
F'(a)
= ∂f/∂x (a, b)
z
z = f(x, b)
= F(x)
x = a
x

平面の方程式.

法線方向 $\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}$

通る1点
+
法線方向

で平面が決まる



$$\begin{pmatrix}x-a\\y-b\\z-c\end{pmatrix}\cdot\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}=0$$

$$\alpha(x-a)+\beta(y-b)+\gamma(z-c)=0 \quad ※$$

$$\boxed{Ax+By+Cz+D=0}$$

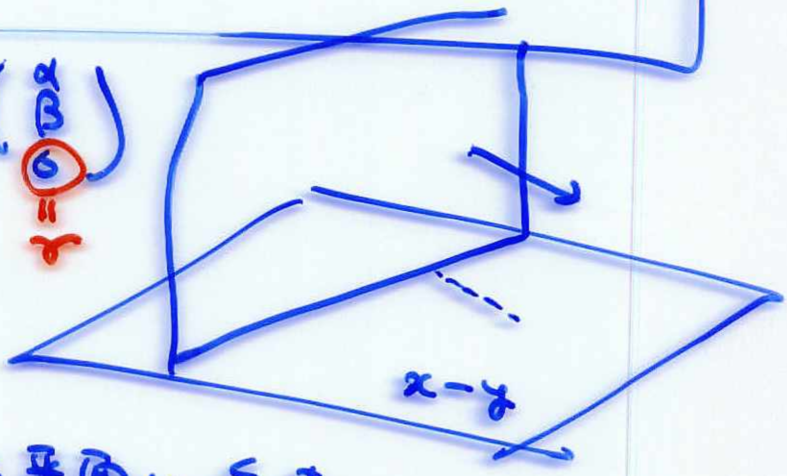
平面の方程式.

$\gamma=0$ の場合

$\begin{pmatrix}\alpha\\\beta\\0\end{pmatrix}$ // x-y 平面

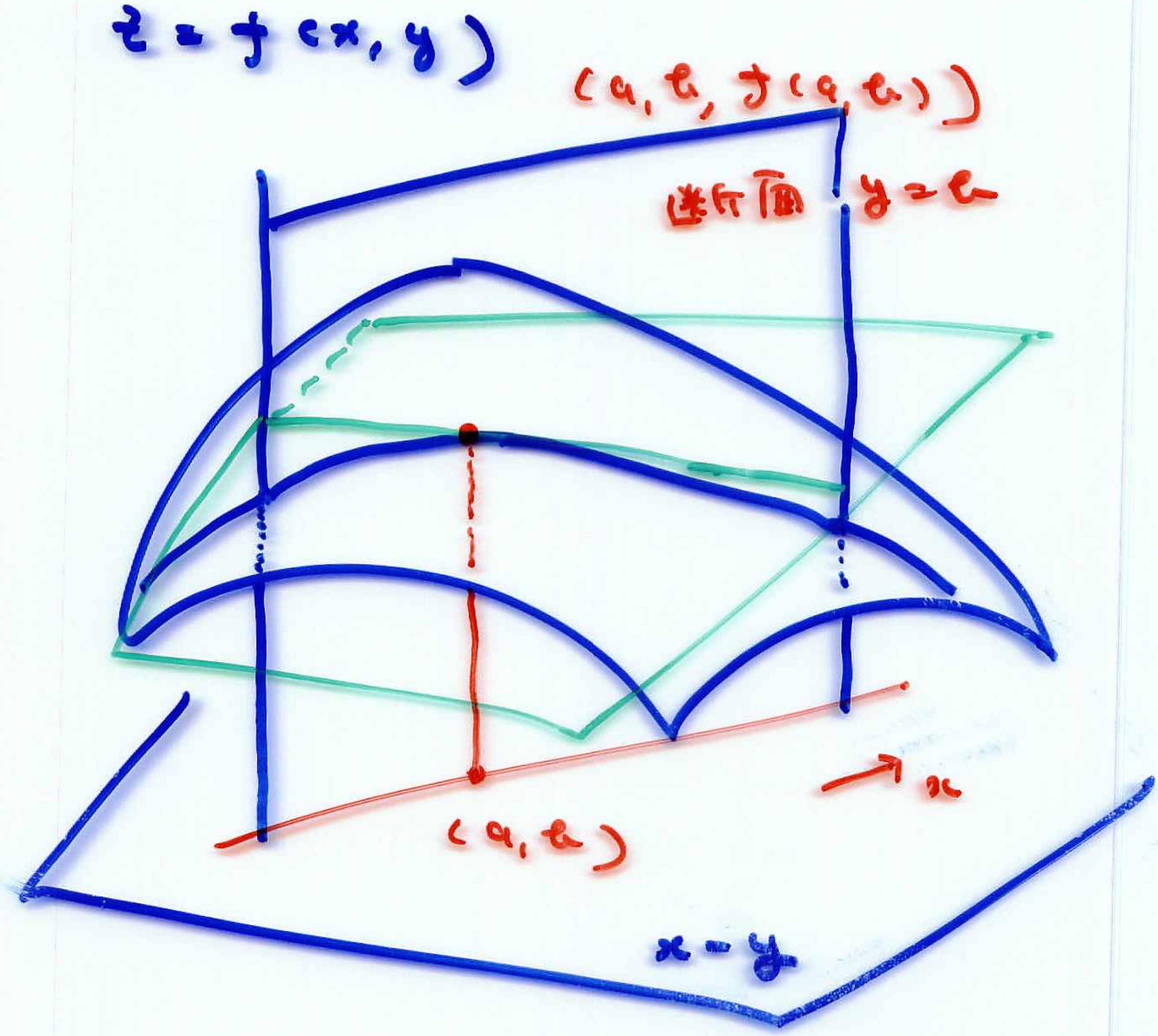
γ

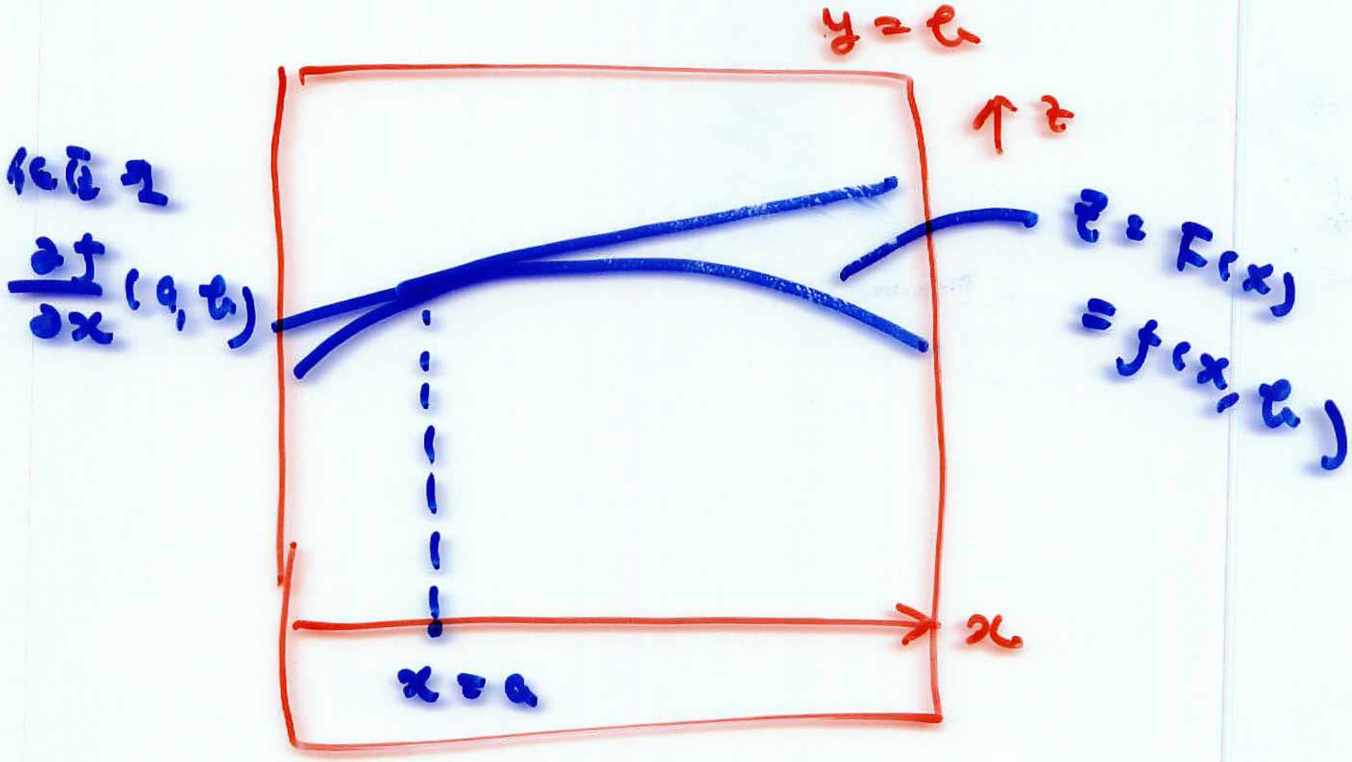
$\begin{pmatrix}\alpha\\\beta\\0\end{pmatrix}$ $\underset{\gamma}{\shortparallel}$



$\rightsquigarrow$ ※は x-y 平面に垂直

接平面を考える場合、このケースは不要.


$z = f(x, y)$
$(a, b, f(a, b))$
断面 $y = b$
x
(a, b)
$x - y$


$y = b$
z
傾き
$\frac{\partial f}{\partial x}(a, b)$
$z = F(x)$
$= f(x, b)$
x
$x = a$

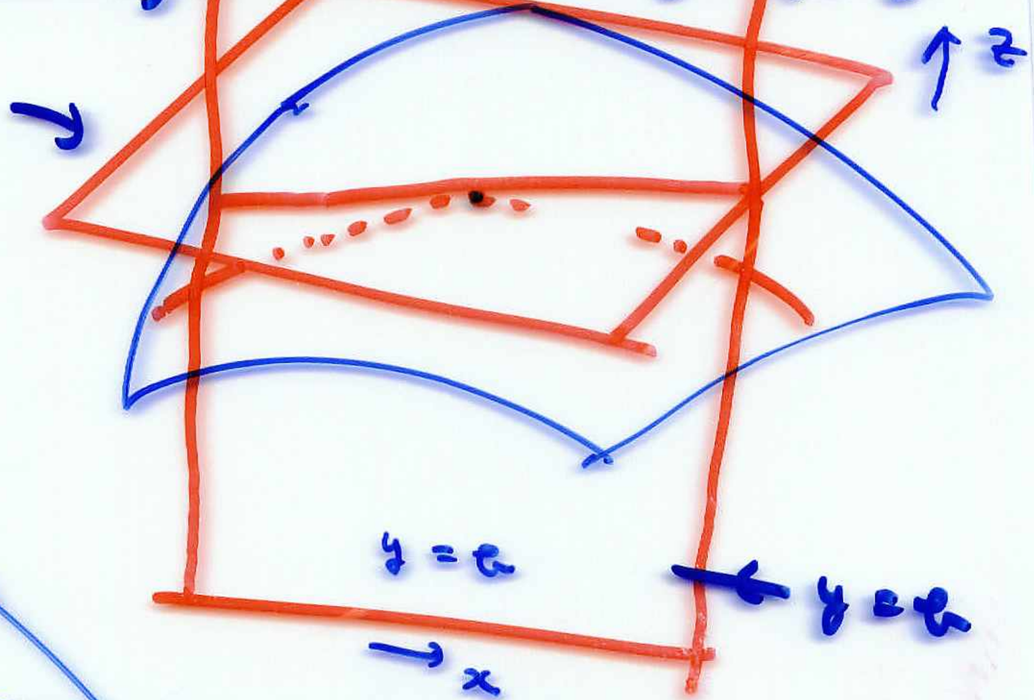
$\gamma \neq 0$ ならば

$$z = \underbrace{-\frac{\alpha}{\gamma}}_{A}(x-a)\underbrace{-\frac{\beta}{\gamma}}_{B}(y-b)+c$$

$$z = A(x-a)+B(y-b)+c$$

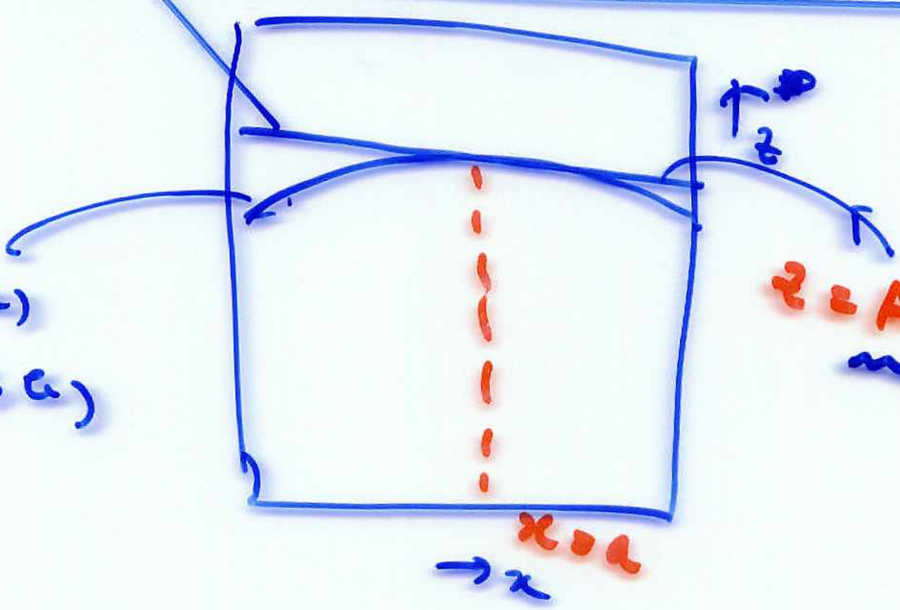
と書ける

$$z = A(x-a)+B(y-b)+f(a,b)$$



に近い

$$\frac{\partial f}{\partial x}(a,b) = A$$



$z = f(x,y)$ の $(a, b, c = f(a,b))$ における接平面：

$$z = A(x-a) + B(y-b) + \overset{f(a,b)}{\overset{\shortparallel}{C}}$$

$$\rightarrow A = \frac{\partial f}{\partial x}(a,b)$$

同様 $B = \dfrac{\partial f}{\partial y}(a,b)$

接平面の方程式.

$$z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$

I　$z = f(x,y) = x^4 + y^4 - 4xy + 3$ の停留点

II　$z = x^2 + xy - y^2 - 4x - 2y$ の停留点.