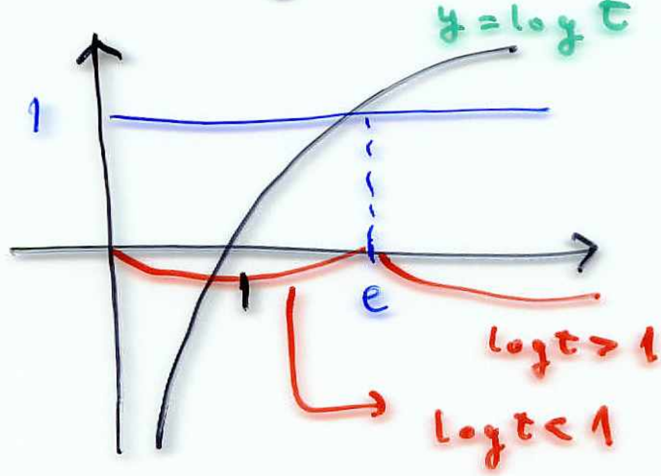
$$y = \frac{\log t}{t}$$

$$y' = \frac{(\log t)' t - \log t \cdot (t)'}{t^2} \quad t^2 > 0$$

$$= \frac{\frac{1}{t}\cdot t - \log t \cdot 1}{t^2} = \frac{1 - \log t}{t^2} \quad t^2 > 0$$

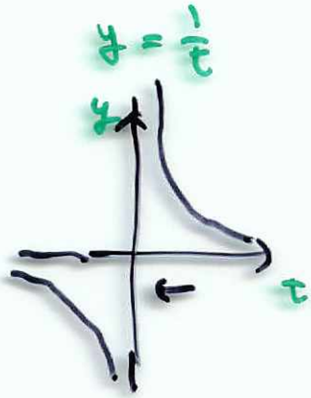


$y' \gtreqless 0$

$\iff 1 - \log t \gtreqless 0$

$\iff t \lesseqgtr e$

y	(0)		e		
y'		$+$	0	$-$	
y''		[illegible]		[illegible]	
y	$(-\infty)$	$\nearrow$	$\frac{1}{e}$	$\searrow$	



$$\left.\begin{array}{l} t \to +0 \quad \frac{1}{t} \to +\infty \\ \log t \to -\infty \end{array}\right\} \Longrightarrow \frac{\log t}{t} \to -\infty$$

$t \to +\infty$ $\quad \log t \to +\infty$ $\quad \frac{+\infty}{+\infty}$?

$t \to +\infty$

$$\frac{\log t}{t} = \frac{s}{e^s} \to 0 \quad (t \to +\infty)$$

$\log t = s$ とおく $s \to +\infty$

$t = e^s$

$$\boxed{\frac{t^n}{e^t} \to 0 \quad (t \to \infty)}$$

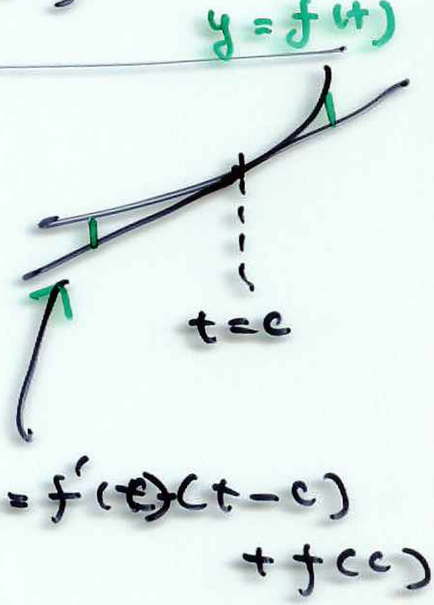
$\cancel{\quad \to 0 \quad (t \to +\infty)}$

CT 118 page

$f : (a, b) \longrightarrow \mathbb{R}$

2階微分可能.

仮定 $f''(x) > 0 \quad (x \in (a, b))$



$$F(t) := f(t) - f'(c)(t - c) - f(c)$$

$\bigcirc := \square$

$\bigcirc$ を $\square$ で定義する

$F'(t) = f'(t) - f'(c) \longrightarrow$ (a, b) 上単調増加.

$F''(t) = f''(t) > 0$

$t < c \Longrightarrow F'(t) < F'(c) = 0$

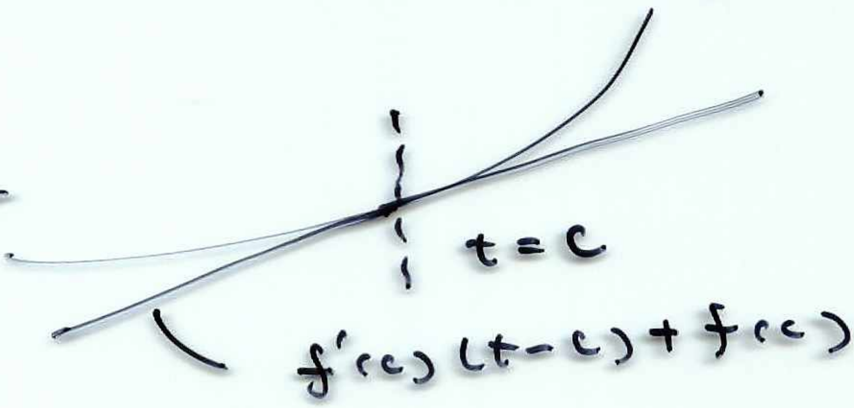
$c < t \Longrightarrow 0 = F'(c) < F'(t)$

t		c	
F'	$-$	0	$+$
F	$\searrow$	0	$\nearrow$

$t \neq c \Longrightarrow \underset{\overset{\|}{f(t) - f'(c)(t-c) - f(c)}}{F(t)} > 0$

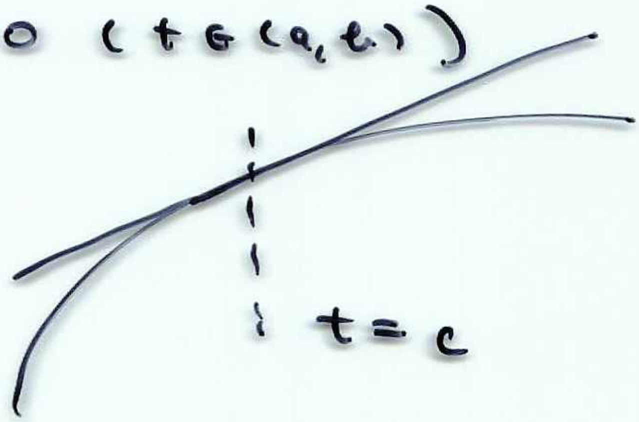
$f(t) > f'(c)(t - c) + f(c)$

$f''(t) > 0 \quad (t \in (a, b))$

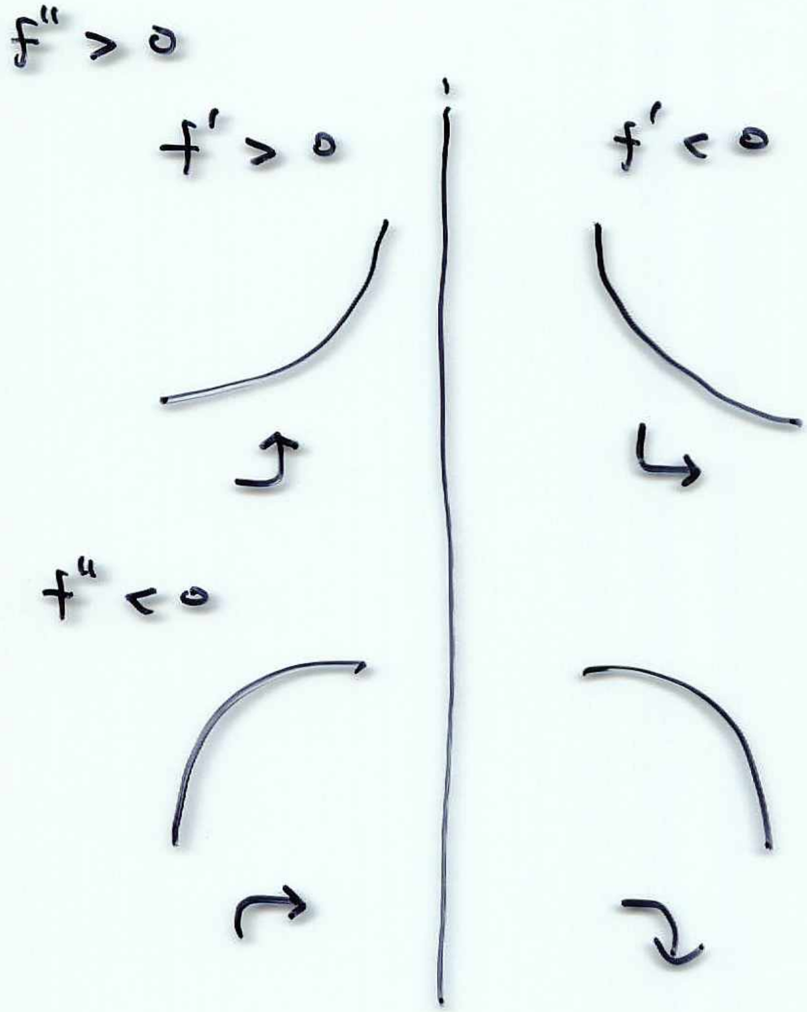


$f(t)$ は (a, b) 上 下に凸

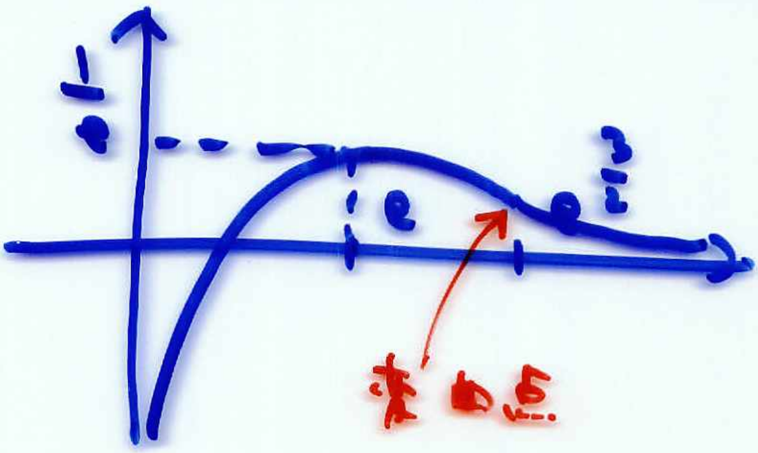
$f''(t) < 0 \quad (t \in (a, b))$



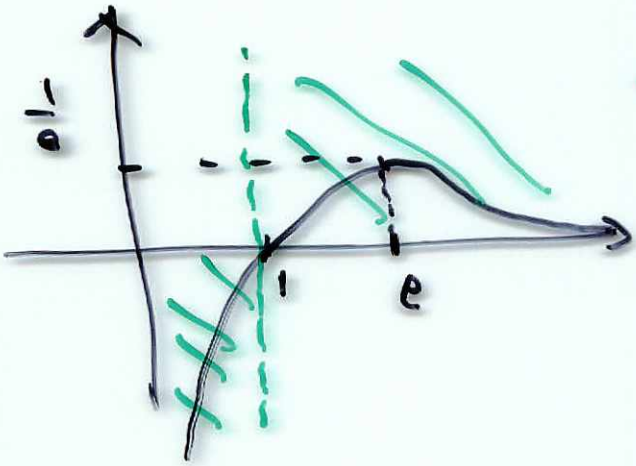
$f(t)$ は (a, b) 上で上に凸.



$f'' > 0$
$f' > 0$
$f' < 0$
$f'' < 0$



$\frac{1}{e}$
e
$e^{\frac{3}{2}}$
変曲点



$t>0$ 真数条件

$$\frac{\log t}{t} \gtreqless 0 \quad (0<t)$$

$$\Longleftrightarrow \log t \gtreqless 0$$

$$\Longleftrightarrow t \gtreqless 1$$

$$y = \frac{\log t}{t}$$

$$y' = \frac{1-\log t}{t^2}$$

$$y'' = \frac{(1-\log t)'t^2 - (1-\log t)\cdot 2t}{t^4}$$

$$= \frac{-\frac{1}{t}\cdot t + (\log t - 1)2}{t^3} \qquad t^3>0$$

$$= \frac{2\log t - 3}{t^3}$$

$$y'' \gtreqless 0 \Longleftrightarrow \log t \gtreqless \frac{3}{2}$$

$$\Longleftrightarrow t \gtreqless e^{\frac{3}{2}} \qquad e^{\frac{3}{2}}$$

	(0)		e		$e^{\frac{3}{2}}$	
y'		+	0	−	−	−
y''		−	−	−	0	+
	$-\infty$	↗	$\frac{1}{e}$	↘		↘

$$\frac{3}{2}\cdot\frac{1}{e\sqrt{e}}$$

$$y = \frac{\log t}{t}, \quad y' = \frac{1-\log t}{t^2}$$

$$y'' = \frac{(1-(\log t))' \cdot t^2 - (1-\log t)\cdot (t^2)'}{t^4}$$

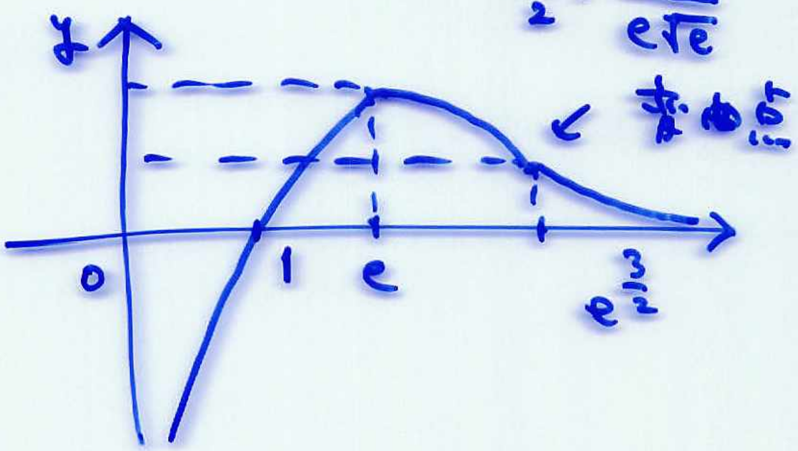
$$= \frac{-\frac{1}{t}\cdot t^2 - (1-\log t)\cdot 2t}{t^4}$$

$$= \frac{-t + (\log t - 1)\cdot 2t}{t^4}$$

$$= \frac{2\log t - 3}{t^3}$$

$$y'' \gtreqless 0 \iff \log t \gtreqless \frac{3}{2} \iff t \gtreqless e^{\frac{3}{2}}$$

	(0)		e		$e^{\frac{3}{2}}$	
y'	/	$+$	0	$-$	$-$	$-$
y''	/	$-$	$-$	$-$	0	$+$
y	$-\infty$	↗	$\frac{1}{e}$	↘	$\frac{3}{2}\cdot\frac{1}{e\sqrt{e}}$	↘



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$$f' = f^{(1)},\quad f'' = f^{(2)},\quad \cdots$$

$$f^{(n+1)} = \left(f^{(n)}\right)'$$

continuous

C^1級　　$f : (a, b) \to \mathbb{R}$

微分可能. f' も連続.

→ f : 連続

C^2級　　$f \in C^1$級, f' : C^1級

f'' が存在し連続

. . . .

C^{k+1}級　$f \in C^k$. $f' \in C^k$級

---

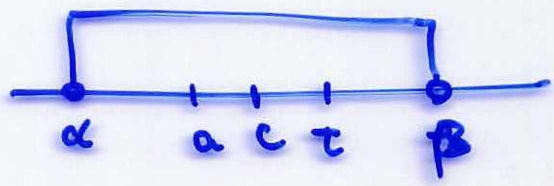
f : C^∞級　全ての k で C^k級に入っている

無限階微分可能.

f : C^k級.　$f, f', f'', \cdots, f^{(k)}$ が存在して連続.

Taylorの定理.

$f : (\alpha, \beta)$ 上 C^∞ 級



$$f(t) = f(a) + f'(a)(t-a) + \frac{1}{2!} f^{(2)}(a)(t-a)^2$$

$$+ \frac{1}{3!} f^{(3)}(a)(t-a)^3 + \cdots\cdots$$

$$\cdots + \frac{1}{n!} f^{(n)}(a)(t-a)^n$$

$$+ \underbrace{\boxed{\frac{1}{(n+1)!} f^{(n+1)}(c)(t-a)^{n+1}}}_{= R_{n+1}}$$

注 c

をみたす c が
t と a の間に有り

剰余項
Remainder Term

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$f(t) = e^t, \quad f'(t) = e^t$

$\cdots \quad f^{(n)}(t) = e^t$

$f^{(n)}(0) = 1$ 　　 $a = 0$ の場合.

$$e^t = 1 + t + \frac{1}{2!}t^2 + \cdots + \frac{1}{n!}t^n + \boxed{\frac{1}{(n+1)!}e^c t^{n+1}}$$

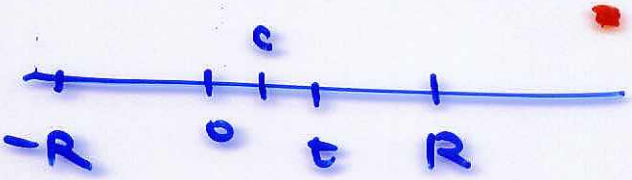
" R_{n+1}

Iみたす c が 0 と t の間にある.

c は n による!!

$R_{n+1} \longrightarrow 0 \quad (n \longrightarrow +\infty)$

$n \to +\infty$ のとき $R_n \to 0$



$|t| \leqq R$

$$R_{n+1} = \frac{1}{(n+1)!} e^{c} t^{n+1}$$

$$0 \leqq |R_{n+1}| \leqq \frac{R^{n+1}}{(n+1)!} e^{R} \longrightarrow 0$$

$$\boxed{\lim_{n \to +\infty} \frac{R^n}{n!} = 0 \qquad (R > 0)}$$

$R > 0$ 　 $N \geqq R$. 　 $n \geqq N$ で考える

$$0 \leqq \frac{R^n}{n!} = \boxed{\frac{R}{1} \cdot \frac{R}{2} \cdot \frac{R}{3} \cdot \cdots \cdot \frac{R}{N}} \cdot \underbrace{\frac{R}{N+1} \cdot \frac{R}{N+2} \cdots \frac{R}{n}}$$

$N+2 \to N+1$, $n \to N+1$

$$\leqq \frac{R^N}{N!} \cdot \frac{R^{n-N}}{(N+1)^{n-N}}$$

$$= \underbrace{\frac{R^N}{N!} \cdot \left(\frac{R}{N+1}\right)^{-N}}_{定数} \cdot \left(\frac{R}{N+1}\right)^{n} \longrightarrow 0$$

$\frac{R}{N+1} < 1$

$$\boxed{e^t = 1 + t + \frac{1}{2!} t^2 + \frac{1}{3!} t^3 + \cdots\cdots = \sum_{n=0}^{+\infty} \frac{1}{n!} t^n}$$

$0! = 1$ と約束する.

$t > 0$ $\boxed{\lim_{t\to\infty} \frac{t^n}{e^t} = 0}$

1°

$$e^t = \boxed{1 + t + \frac{1}{2!}t^2 + \cdots + \frac{1}{n!}t^n} + \frac{t^{n+1}}{(n+1)!} + \underbrace{\cdots}_{\geq 0}$$

$$> \frac{t^{n+1}}{(n+1)!}$$

$t > 0$

$$0 < \frac{t^n}{e^t} < \frac{t^n}{\frac{t^{n+1}}{(n+1)!}} = \underbrace{\frac{1}{t}\cdot (n+1)!}_{\downarrow\, 0}$$

$0 \downarrow 0$, $\frac{t^n}{e^t} \downarrow 0$

はさみうち

に式

$$\lim_{t\to+\infty} \frac{t^n}{e^t} = 0$$

下の例3

$$f(t) = \log(1+t)$$

$$f'(t) = \frac{1}{1+t}, \quad f'' = -\frac{1}{(1+t)^2}$$

$$f^{(3)} = \frac{2}{(1+t)^3}, \quad f^{(4)} = -\frac{2\cdot 3}{(1+t)^4}$$

$$f^{(n)} = \frac{(-1)^{n-1}(n-1)!}{(1+t)^n}$$

$$f^{(n)}(0) = (-1)^{n-1}(n-1)!$$

$$\log(1+t) = t - \frac{1}{2}\cdot t^2 + \frac{1}{3}t^3 - \frac{1}{4}t^4$$

$$+ - \cdots + \frac{1}{k}(-1)^k t^k + \cdots + \frac{(-1)^{n+1}}{n+1}\frac{t^{n+1}}{(1+c)^{n+1}}$$

$f(t) = \log(1+t)$ 131 p.

$f' = \frac{1}{1+t}$, $f'' = -\frac{1}{(1+t)^2}$, $f^{(3)} = \frac{2}{(1+t)^3}$

$f^{(4)}(t) = \frac{-2\cdot 3}{(1+t)^4}$ $f^{(5)} = \frac{+2\cdot 3\cdot 4}{(1+t)^5}$

$$f^{(n)} = (-)^{n+1}\frac{(n-1)!}{(1+t)^n} \quad (n=1,2,\cdots)$$

$$\underline{f^{(k)}(0) = (-1)^{k+1}(k-1)!}$$

$$\log(1+t) = t - \frac{1}{2}t^2 + \frac{1}{3}t^3 - \frac{1}{4}t^4$$

$a=0$

$$+ - \cdots + (-)^{k+1}\frac{1}{k}t^k + \cdots$$

$\frac{(k-1)!}{k!} = \frac{1}{k}$

$$+ (-)^{n+1}\frac{1}{n}t^n + \boxed{(-1)^{n+2}\frac{1}{n+1}\frac{1}{(1+c)^{n+1}}t^{n+1}}$$

$$\parallel$$

$$R_{n+1}$$

$|t|<1$ のとき $R_{n+1} \longrightarrow 0$ が言える.

(定理 4.11 cf 127)

$|t| < 1$ のとき $\quad R_n \longrightarrow 0$

$$\log(1+t) = t - \frac{1}{2}t^2 + \frac{1}{3}t^3 - + \cdots$$
$$+ \frac{(-1)^{n-1}}{n} t^n + \cdots$$

$$f(t) = \begin{cases} 0 & (t \leqq 0) \\ e^{-\frac{1}{t}} & (t > 0) \end{cases}$$

$\mathbb{R}$ 上 C^∞ 級.

$$f^{(n)}(0) = 0 \quad (n = 0, 1, 2, \cdots)$$

$$\sum_{n=0}^{+\infty} \frac{1}{n!} f^{(n)}(0)\, t^n \equiv 0$$

0

I　$f(t)=\sqrt{1+t}$ に対して Taylor の定理
を用いよ.
$a=0,\ n=2$

II.　$g(t)=\dfrac{1}{1+t}$ に対して Taylor の定理
を用いよ　$a=0,\ n=3.$

III　$f(t)=te^{t}$ に対して $f^{(n)}(t)$ を
求めよ.

IV.　$f(t)=t^{2}e^{t}$ の増減表を求めよ.

V　$f(t)=\dfrac{\log t}{t^{2}}$ の増減表を求めよ.