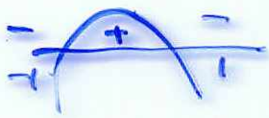
$$y = \frac{x}{1+x^2}$$

$$y' = \frac{(x)'(1+x^2) - x(1+x^2)'}{(1+x^2)^2}$$

$$= \frac{(1+x^2) - x \cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{\boxed{(1+x^2)^2}} > 0$$

符号 sign

la signe



x		-1		1	
y'	$-$	0	$+$	0	$-$
y	$\searrow$	$-\frac{1}{2}$	$\nearrow$	$\frac{1}{2}$	$\searrow$

$$\frac{1}{x^2} \to 0$$

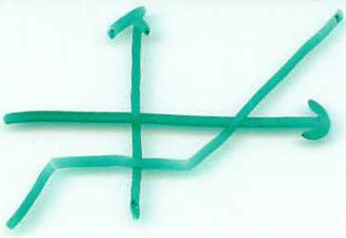
$x \to +\infty$ $\qquad \frac{1}{x} \to 0$.

$$\frac{x}{1+x^2} = \frac{1}{x} \cdot \frac{x^2}{1+x^2} = \frac{1}{x} \cdot \frac{1}{1+\frac{1}{x^2}}$$

$$\to 0 \cdot \frac{1}{1+0} = 0 \cdot 1 = 0$$

$x \to -\infty$ のとき $\qquad \frac{1}{x} \to 0$

$$\frac{x}{1+x^2} = \frac{1}{x} \cdot \frac{1}{1+\frac{1}{x^2}} \to 0 \cdot \frac{1}{1+0^2} =$$

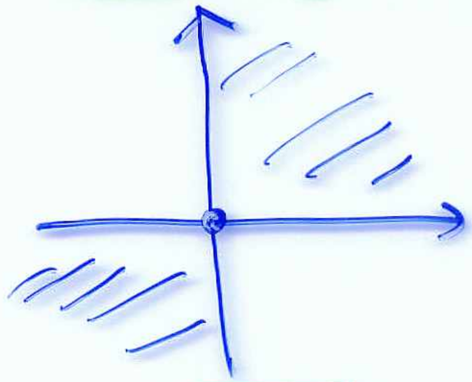


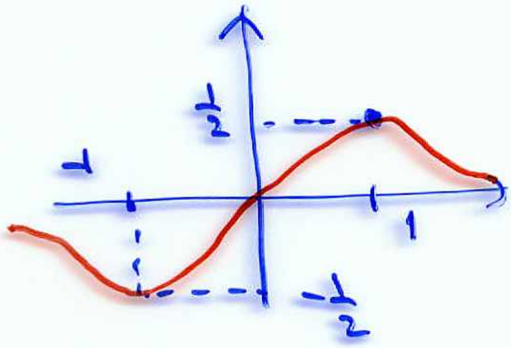
広義の単調増加とは.

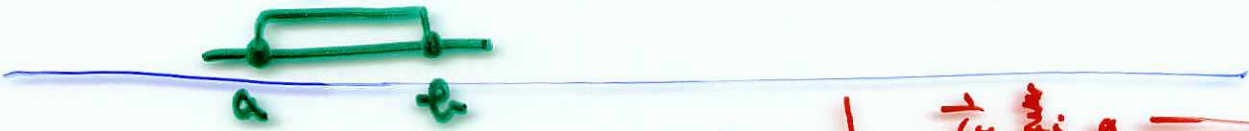
increasing.

$y = \dfrac{x}{1+x^2}$ $1+x^2 > 0$

$y \gtreqless 0 \iff x \gtreqless 0$







$f : [a, b] \longrightarrow \mathbb{R}$

・狭義の単調増加

$x < y \Rightarrow f(x) < f(y)$

広義の ─

$x < y \Rightarrow f(x) \leqq f(y)$

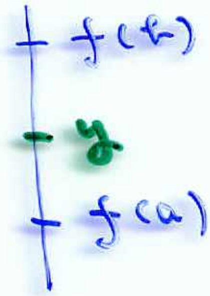
・連続

定理

$f : [a, b] \xrightarrow{1:1} [f(a), f(b)]$

逆関数定理.

$[f(a), f(b)] \ni y$ に対して
唯1つ x が存在して
$f(x) = y$.



逆関数

$y \longmapsto x$

$f^{-1} : [f(a), f(b)] \longrightarrow [a, b]$

連続

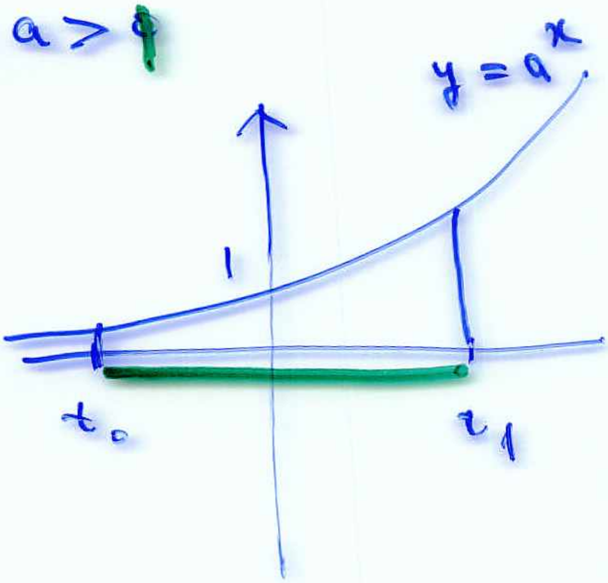
- $a < x < b$

$$\longrightarrow f(a) < f(x) < f(b)$$

$$x \in [a, b] \Longrightarrow f(x) \in [f(a), f(b)]$$

-

(I) $a > 1$



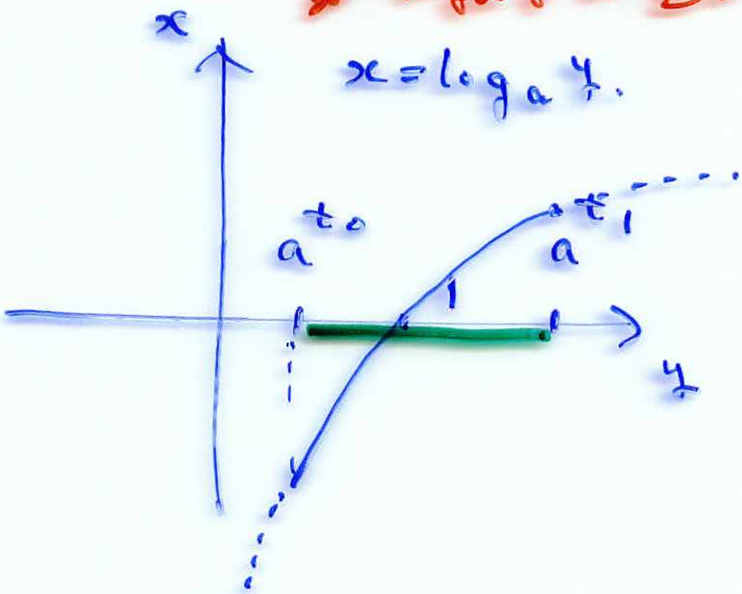
CT 58 page

$y = a^x$ は連続.

$\downarrow$ 定理

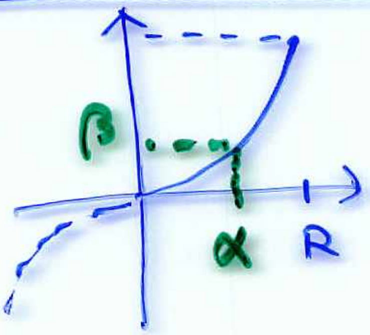
$x = \log_a y$ は連続

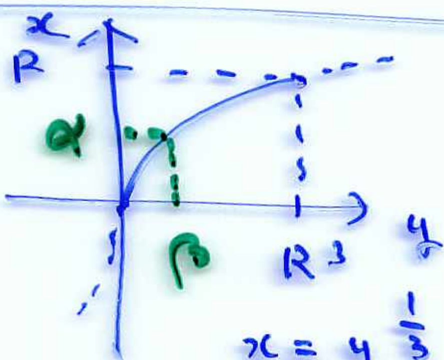
$x = \log_a y.$



$y = x^3$

(II)





$x = y^{\frac{1}{3}}$

$x = y^{\frac{1}{3}}$ は連続

$x_n \longrightarrow \alpha$

$x_n^3 \longrightarrow \alpha^3$

$\rightsquigarrow f(x) = x^3$ は $\underbrace{\ }_{x=\alpha\text{ で}}$ 連続

$y = f(x) = x^3 \qquad x \in [0, R]$

$0 < \alpha < R \qquad \beta = \alpha^3 = f(\alpha)$

$x = g(y) = f^{-1}(y) = y^{\frac{1}{3}}$.

f:連続
↓ 逆関数定理
g:連続

$\boxed{g'(\beta) = ?}$

$y \to \beta \rightsquigarrow x = g(y) \to g(\beta) = \alpha$

$$\frac{g(y) - g(\beta)}{y - \beta} = \frac{x - \alpha}{f(x) - f(\alpha)}$$

$$= \frac{1}{\frac{f(x) - f(\alpha)}{x - \alpha}} \to \frac{1}{f'(\alpha)}$$

$f'(x) = 3x^2 \qquad \alpha = \beta^{\frac{1}{3}} \qquad \frac{1}{3\alpha^2} = \frac{1}{3} \cdot \frac{1}{\beta^{\frac{2}{3}}}$

$$= \frac{1}{3} \cdot \beta^{-\frac{2}{3}} = \frac{1}{3} \beta^{\frac{1}{3} - 1}$$

まとめ

$$\left(y^{\frac{1}{3}}\right)' = \frac{1}{3} \cdot y^{\frac{1}{3} - 1} = \frac{1}{3} y^{-\frac{2}{3}}$$

$y = f(x) = x^n \qquad x \in [0, R]$

$x = g(y) = f^{-1}(y) = y^{\frac{1}{n}}$

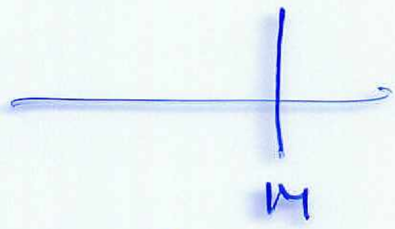
$\rightsquigarrow \qquad \left(y^{\frac{1}{n}}\right)' = \frac{1}{n} y^{\frac{1}{n} - 1}$

を示すことができる。

CT 91 page

定理. $a_0 \leqq a_1 \leqq a_2 \leqq a_3 \leqq \cdots$

単調増加.



$a_n \leqq M \quad (n=0,1,2,\cdots)$

上に有界.

$\Longrightarrow$ $\{a_n\}$ は収束する.

ℝの完備性.

年利

$100a$ %	a 単利.	元利合計 $1+a$
	半年複利	$\left(1+\frac{a}{2}\right)^2$
	$\frac{1}{3}$年 〃	$\left(1+\frac{a}{3}\right)^3$
		$\vdots$
$\frac{1}{n}$年複利		$\left(1+\frac{a}{n}\right)^n$.

$\downarrow$

瞬間複利 $\quad e^a$

$a=1$

$a_n = \left(1+\frac{1}{n}\right)^n$

単調増加, $a_n \leqq 3$. $\Leftarrow$ CT でみる.

$$e = \lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n$$

$$\left(x^{\frac{m}{n}}\right)' = \frac{du^m}{du}\cdot\frac{du}{dx} = m u^{m-1}\cdot\frac{1}{n}x^{\frac{1}{n}-1}$$

$u = x^{\frac{1}{n}}$

$$= \frac{m}{n}\ x^{\frac{m-1}{n}}\cdot x^{\frac{1}{n}-1}$$

$m = 0, \pm 1, \pm 2, \cdots$

$$\begin{cases} (x^m)' = m x^{m-1}. \\ \quad m = 0, 1, 2, \cdots \\ (x^{-m})' = -m x^{-m-1}. \end{cases}$$

$$= \frac{m}{n}\ x^{\frac{m}{n}-1}$$

$\alpha = \frac{m}{n}$ とおく.

α: 有理数 $\quad (x^\alpha)' = \alpha x^{\alpha-1}.$

$\alpha = \frac{m}{n}$ $\quad n \in \mathbb{N} = \{1, 2, 3, \cdots\}$

有理数. $\quad m \in \mathbb{Z} = \{0, \pm 1, \pm 2, \cdots\}$

無理数

正.

$$\rightsquigarrow \lim_{h \to 0} (1+h)^{\frac{1}{h}} = e.$$

前page.

対数関数の連続性 ── $\log x$ の $x = e$ での連続性.

$$\log (1+h)^{\frac{1}{h}} = \frac{\log(1+h)}{h}$$

$$\longrightarrow \log e = 1.$$

まとめ ①

$$\frac{\log(1+h)}{h} \longrightarrow 1 \quad (h \to 0)$$

$$x = \log(1+h)$$

$$\rightsquigarrow 1+h = e^{x} \rightsquigarrow h = e^{x} - 1$$

指数関数の連続性. $x \to 0$, $e^{x} \to e^{0} = 1$

$$\frac{e^{x}-1}{x} = \frac{h}{\log(1+h)} \quad \rightsquigarrow h \to 0$$

$$= \frac{1}{\frac{\log(1+h)}{h}} \longrightarrow \frac{1}{1} = 1$$

まとめ ②

$$\frac{e^{x}-1}{x} \to 1 \quad (x \to 0)$$

$y = e^x$ の微分.

$$e^{\alpha+h} = e^{\alpha} \cdot e^{h} \qquad h \to 0$$

$$\frac{e^{\alpha+h} - e^{\alpha}}{(\alpha+h) - \alpha} = \frac{e^{\alpha}(e^{h} - 1)}{h}$$

$$= e^{\alpha} \cdot \boxed{\frac{e^{h} - 1}{h}} \to e^{\alpha} \cdot 1 \qquad \frac{e^{h} - 1}{h} \to 1$$

$$(e^x)' = e^x$$

$$\log_e x = \log x$$
$$= \ln x$$

$e = 2.712 \cdots$

Neper

$x - \alpha = h$

$$\frac{\log x - \log \alpha}{x - \alpha} = \frac{\log(\alpha + h) - \log \alpha}{h}$$

$$= \frac{\log\left(1 + \frac{h}{\alpha}\right)}{h}$$

$g = \frac{h}{\alpha}$ とおく

$h \to 0$ のとき

$g \to 0$

$$= \frac{\log(1+g)}{g} \cdot \frac{1}{\alpha} \longrightarrow \frac{1}{\alpha} \qquad \frac{\log(1+g)}{g} \to 1$$

$$\boxed{(\log x)' = \frac{1}{x}}$$

96 page　$x > 0,\ \alpha \in \mathbb{R}$.

$$(x^\alpha)' = \{(e^{\log x})^\alpha\}'$$

$$= (e^{\alpha \log x})'$$

$u = \alpha \log x$

$$\frac{dx^\alpha}{dx} = \frac{de^u}{du} \cdot \frac{du}{dx}$$

$$= e^u \cdot \alpha \cdot \frac{1}{x} = x^\alpha \cdot \alpha \cdot \frac{1}{x}$$

$$= \alpha\, x^{\alpha-1}.$$

まとめ

$$(x^\alpha)' = \alpha\, x^{\alpha-1}.$$

$a \neq 1,\ a > 0$

$$(\log_a x)' = \left(\frac{\log_e x}{\log_e a}\right)'$$

$$= \frac{1}{\log a} \cdot \frac{1}{x}$$

まとめ

$$(\log_a x)' = \frac{1}{\log a} \cdot \frac{1}{x}.$$

I　(1) e^{-x}　(2) e^{3x}

(3) e^{-x^2}　(4) xe^{3x}.

$(4\frac{1}{2})$ $\log(1+x)$　(5) $\log(1+x^2)$

(6) $x\log x$.　(7) $x^{\frac{2}{3}}$　$(x^{\frac{1}{3}})' = \cdots$ を使う.

II　$y = xe^{-x}$ の増減表を求めよ.

$e^{-x} > 0$.