復習

1) $(f \pm g)' = f' \pm g'$

2) $(fg)' = f'g + fg'$

3) $\left(\dfrac{1}{f}\right)' = -\dfrac{f'}{f^2}$

$\Rightarrow \left(\dfrac{g}{f}\right)' = \dfrac{g'f - gf'}{f^2}$

---

(1) $$\left(\frac{1}{x^2+x+1}\right)' = -\frac{(x^2+x+1)'}{(x^2+x+1)^2}$$

$$= -\frac{2x+1}{(x^2+x+1)^2}$$

---

(2) $(\sqrt{x})' = \dfrac{1}{2}\cdot\dfrac{1}{\sqrt{x}}$ $(x > 0)$

$$(x\sqrt{x})' = (x)'\sqrt{x} + x\cdot(\sqrt{x})'$$

$$= \sqrt{x} + \frac{x}{2}\,\frac{1}{\sqrt{x}} = \sqrt{x} + \frac{1}{2}\sqrt{x}$$

$$= \frac{3}{2}\sqrt{x}.$$

$\frac{3}{2} - 1 = \frac{1}{2}$

(3) $$\left(x^{\frac{3}{2}}\right)' = \frac{3}{2}\cdot x^{\frac{3}{2}-1}$$

$$(x^2\sqrt{x})' = (x^2)'\cdot\sqrt{x} + x^2\cdot(\sqrt{x})'$$

$$= 2x\sqrt{x} + x^2\cdot\frac{1}{2}\,\frac{1}{\sqrt{x}}$$

$2 + \frac{1}{2} = \frac{5}{2}$

$$= \frac{5}{2}x\sqrt{x}.$$

$\frac{5}{2} - 1 = \frac{3}{2}$

$$\left(x^{\frac{5}{2}}\right)' = \frac{5}{2}x^{\frac{3}{2}} = \frac{5}{2}x^{\frac{5}{2}-1}$$

$$\boxed{(x^{\alpha})' = \alpha\, x^{\alpha-1}}$$

(4) $$\left(\frac{1}{\sqrt{x}}\right)' = \frac{-(\sqrt{x})'}{(\sqrt{x})^2} = \frac{-\frac{1}{2}\frac{1}{\sqrt{x}}}{x}$$

$$= -\frac{1}{2} \cdot \frac{1}{x\sqrt{x}}$$

$$\boxed{-\frac{1}{2} - 1 = -\frac{3}{2}}$$

$$\left(x^{-\frac{1}{2}}\right)' = -\frac{1}{2}\, x^{-\frac{3}{2}}$$

$$= -\frac{1}{2}\, x^{-\frac{1}{2}-1}$$

---

(5) $$\left(\frac{3x-1}{x+1}\right)' = \frac{(3x-1)'(x+1) - (3x-1)\cdot(x+1)'}{(x+1)^2}$$

$$\boxed{\left(\frac{g}{f}\right)' = \frac{g'f - gf'}{f^2}}$$

$$= \frac{3(x+1) - (3x-1)}{(x+1)^2}$$

$$= \frac{4}{(x+1)^2}$$

$$\frac{f(x) - f(\alpha)}{x - \alpha} \longrightarrow f'(\alpha)$$

$$(x \to \alpha)$$

$\{x_n\}$ は $x_n \neq \alpha$ $x_n \to \alpha$ を満たす。

このとき $$\frac{f(x_n) - f(\alpha)}{x_n - \alpha} \longrightarrow f'(\alpha)$$

I. 合成関数の微分

II. 連続関数の性質

$\{(3x-1)^{10}\}'$ 　　$x=\alpha$ における微分を考える

$x_n \longrightarrow \alpha$ 　$x_n \neq \alpha$ である $\{x_n\}$ をとる.

$$\frac{(3x_n-1)^{10}-(3\alpha-1)^{10}}{x_n-\alpha}$$

$y_n = 3x_n - 1$ とおく.

$y_n \longrightarrow 3\alpha-1 = A$ とおく.

$$= \frac{y_n^{10}-A^{10}}{y_n-A} \cdot \frac{(3x_n-1)-(3\alpha-1)}{x_n-\alpha}$$

$$\longrightarrow (y^{10})'\Big|_{y=A} \cdot 3$$

$$= 10A^9 \cdot 3 = 30(3\alpha-1)^9.$$

$g(x) = (3x-1)^{10} = f(u(x))$

$f(y) = y^{10}$

$y = u(x) = 3x-1$

合成関数 composition

f of y 　y^{10} 　y to 10.

f at y. 　10 the power of y.

$$\frac{f(u(x)) - f(u(\alpha))}{x - \alpha}$$

$$= \frac{f(u(x)) - f(u(\alpha))}{u(x) - u(\alpha)} \cdot \frac{u(x) - u(\alpha)}{x - \alpha}$$

---

u: 微分可能 $\rightsquigarrow$ 連続

$x \to \alpha$ のとき $u(x) \to u(\alpha)$

$y = u(x) \longrightarrow A = u(\alpha)$

$$= \frac{f(y) - f(A)}{y - A} \cdot \frac{u(x) - u(\alpha)}{x - \alpha}$$

$$\longrightarrow f'(A) \cdot u'(\alpha)$$

$$= f'(u(\alpha)) \cdot u'(\alpha)$$

公式

$$\frac{d f(u(x))}{dx} = \frac{d f(y)}{dy} \cdot \frac{d u(x)}{dx}.$$

$$\frac{d(3x-1)^{10}}{dx} = \left.\frac{d\, y^{10}}{dy}\right|_{y = 3x-1} \cdot \frac{d(3x-1)}{dx}$$

$$= 10 y^{9} \cdot 3$$

$$= 30 (3x-1)^{9}.$$

5月23日 EX0 解説

Traduction Dirigente
Travaux Dirigents
TD.

(3) $\left\{(1+t^2)^4\right\}'$ $\qquad y=(1+t^2)^4 \quad u=1+t^2$

$$= u^4.$$

$$\frac{dy}{dt} = \frac{du^4}{du}\cdot\frac{du}{dt}$$

$$= 4u^3\cdot 2t = 8t(1+t^2)^3$$

(4) $$y = \frac{1}{(2t+1)^4} = \frac{1}{u^4} \qquad u = 2t+1.$$

$$\frac{dy}{dt} = \frac{dy}{du}\cdot\frac{du}{dt} = -\frac{4}{u^5}\cdot 2$$

$$\boxed{\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}}} \qquad = -8\frac{1}{(2t+1)^5}$$

(5) $$y = \frac{1}{\sqrt{1+t^2}} = \frac{1}{\sqrt{u}} \qquad (u = 1+t^2)$$

$$\frac{dy}{dt} = \frac{dy}{du}\cdot\frac{du}{dt}$$

$$= -\frac{1}{2}\frac{1}{u\sqrt{u}}\cdot 2t = -\frac{t}{(1+t^2)\sqrt{1+t^2}}$$

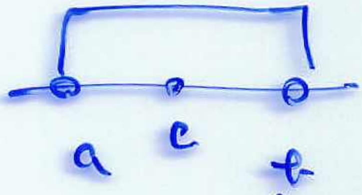
(6) $$y = \frac{1}{(1+t^2)^2} = \frac{1}{u^2} \qquad u = 1+t^2$$

$$\frac{dy}{dt} = \frac{dy}{du}\cdot\frac{du}{dt} = -\frac{2}{u^3}\cdot 2t \qquad \text{“}u'\text{”}$$

$$= -\frac{4t}{(1+t^2)^3}$$

CT51 page.

$f : (a, b) \longrightarrow \mathbb{R}$



$(a,b) = \{ x \in \mathbb{R} ; a < x < b \}$ the open interval
開区間

$f(x) \longrightarrow f(c) \quad (x \to c)$

をみたすと

f は $x = c$ で連続

$x_n > 0$

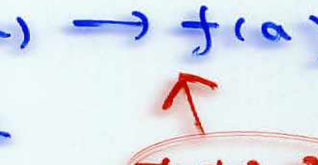
$x_n = \frac{1}{n}$ とする. $f(x_n) = 1 \longrightarrow 1 \neq f(0) = 0$

$\longrightarrow 0$

f は $x = 0$ で不連続.
discontinuous.

閉区間.

$f : [a, b] \longrightarrow \mathbb{R}$



f が $x = a$ で連続

x は右から a に近付く

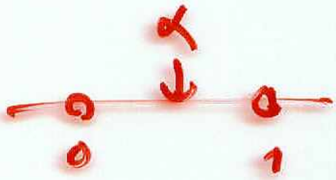
$x \to a+0$ のとき $f(x) \longrightarrow f(a)$

右側極限

$a < x_n \leqq b$, $x_n \longrightarrow a$ のとき

$f(x_n) \longrightarrow f(a)$

$$(0,1) \xrightarrow{f} \mathbb{R}$$



$\alpha \in (0,1)$

$$x \longmapsto \frac{1}{x}$$

連続 $x_n \longrightarrow \alpha$

$\alpha \neq x_n$

$$f(x_n) = \frac{1}{x_n} \longrightarrow \frac{1}{\alpha} = f(\alpha)$$

$\rightsquigarrow$ f は $x = \alpha$ で連続

CT 60 page, 61 page

① 最大値の定理.

$$f : [a, b] \longrightarrow \mathbb{R}.$$

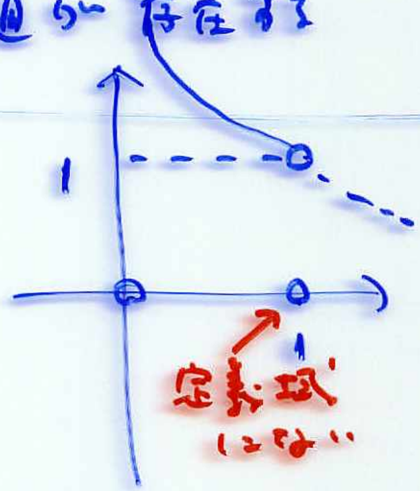
$[a, b]$の各点で連続とする.

fには最大値と最小値が存在する

定義域

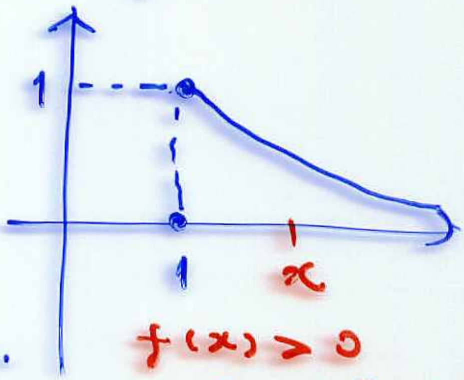
$$f : (0, 1) \longrightarrow \mathbb{R}$$
$$x \longmapsto \frac{1}{x}$$

最大値・最小値なし.



$$f : [1, +\infty) \longrightarrow \mathbb{R}$$
$$x \longmapsto \frac{1}{x}$$

$f(x) = 0$ をみたす
x は $[1, +\infty)$ にない.



$f(x) > 0$
$(x \in [1, +\infty))$

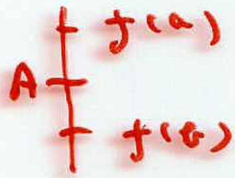
⇝ 最小値なし.

・最大値 1.

($1 = f(1)$, $x \in [1, +\infty)$
$\Rightarrow f(x) \leq f(1)$)

② 中間値の定理.

$f : [a, b] \longrightarrow \mathbb{R}$ 各点で連続

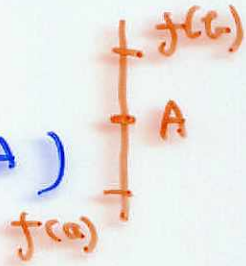


$f(a) \neq f(b)$

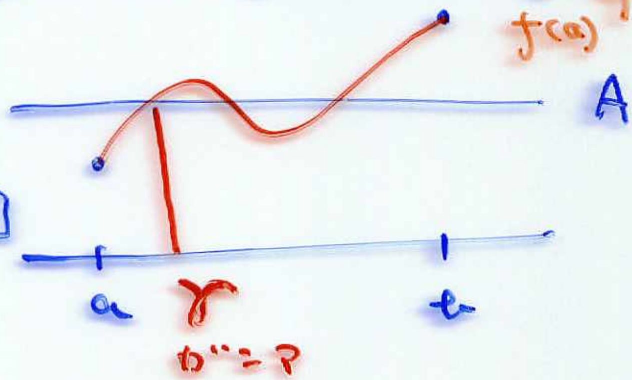
$f(a) > f(b)$ $\quad (f(a) > A$ かつ $f(b) < A)$

OR

$f(a) < f(b)$ $\quad (f(a) < A$ かつ $f(b) > A)$



$\Longrightarrow$ $f(\gamma) = A$ である $\gamma \in [a, b]$ が在る.



CT 65 page.

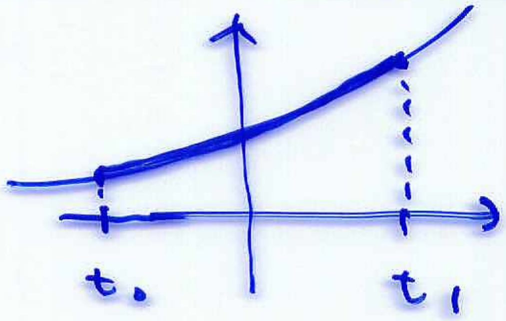
逆関数の定理.

$f: [a, b] \longrightarrow \mathbb{R}$ 連続

狭義の単調増加

$s < t \Rightarrow f(s) < f(t)$

例 $a > 1$

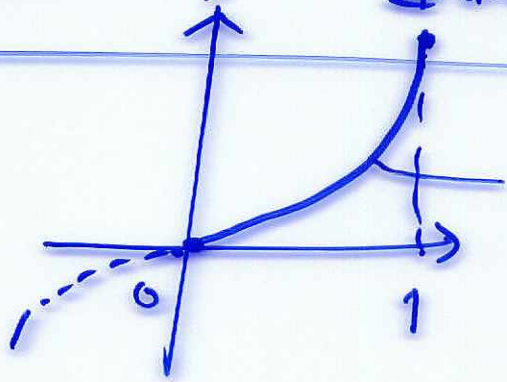


$y = f(t) = a^t$

$(t_0 \leqq t \leqq 1)$

連続関数, 狭義の単調増加.

例



$y = x^3$

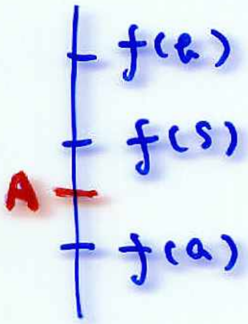
$(0 \leqq x \leqq 1)$

$$
\begin{aligned}
f : [a,b] &\longrightarrow \mathbb{R}. \\
s &\longmapsto f(s)
\end{aligned}
$$

$$a < s < b \implies f(a) < f(s) < f(b)$$

↑
単調増加

$$s \in [a,b] \implies \cancel{f}\ f(s) \in [f(a), f(b)]$$



$$
\begin{aligned}
f : [a,b] &\longrightarrow [f(a), f(b)] \\
s &\longmapsto f(s)
\end{aligned}
$$

- $f(a) < A < f(b) \longrightarrow$ 中間値の定理

  ある $r \in (a,b)$ に対し

  $f(r) = A$.

- $f(a)$, $f(b)$ も値をとる.

$$
\begin{aligned}
f : [a,b] &\longrightarrow [f(a), f(b)] \\
& \qquad\quad \rotatebox{90}{$\in$} \\
r &\longmapsto A = f(r)
\end{aligned}
$$

↑
任意.

全射

$s \neq t \quad f(s) \neq f(t)$

$s < t \longrightarrow f(s) < f(t)$

$s > t \longrightarrow f(s) > f(t)$

$\rightsquigarrow f : [a, b] \longrightarrow [f(a), f(b)]$

逆関数. f^{-1} は単射.

$$\longleftarrow$$

$$f : [a, b] \longrightarrow [f(a), f(b)]$$

$$\underset{1:1}{\longleftrightarrow}$$

定理 $f : [a, b] \longrightarrow \mathbb{R}$.

連続, 狭義の単調増加

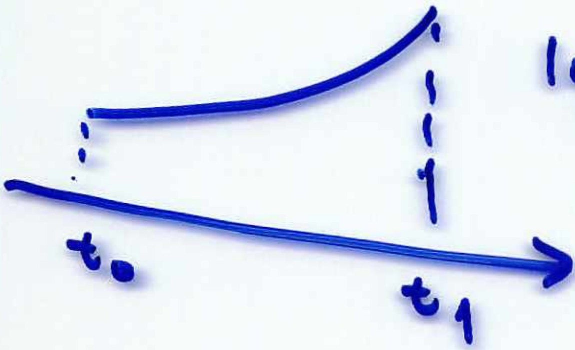
1) $f : [a, b] \underset{1:1}{\longrightarrow} [f(a), f(b)]$

2) $f^{-1} : [f(a), f(b)] \longrightarrow [a, b]$ は連続

中間値の定理

$a > 1$

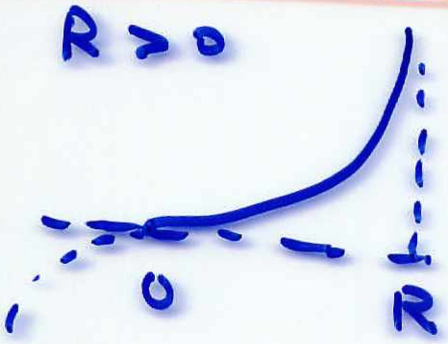
$$f : [t_0, t_1] \longrightarrow [a^{t_0}, a^{t_1}]$$
$$t \longmapsto a^t$$
$$\log_a s \longleftarrow s$$



a^t : 連続性

$\xrightarrow[\text{定理}]{}$ $\log_a s$ の 連続性

$R > 0$



$$[0, R] \longrightarrow [0, R^3]$$
$$t \longmapsto t^3$$
$$s^{\frac{1}{3}} \longleftarrow s$$
$$\parallel$$
$$\sqrt[3]{s} \leftarrow \text{連続.}$$

La semaine prochaine :

$$(f^{-1})' = ?$$

I (1) $\left\{(1-2x)^5\right\}'$

(2) $\left\{\dfrac{1}{(3x-2)^5}\right\}'$

(3) $\left\{\left(\dfrac{x-1}{x}\right)^4\right\}'$

(4) $\left(\dfrac{1}{\sqrt{1+t^{\cancel{2}}+t^2}}\right)'$

~~(5)~~

II $y = \dfrac{x}{1+x^2}$ の増減表,

$$\left(\frac{1}{\sqrt{1+t+t^2}}\right)'$$