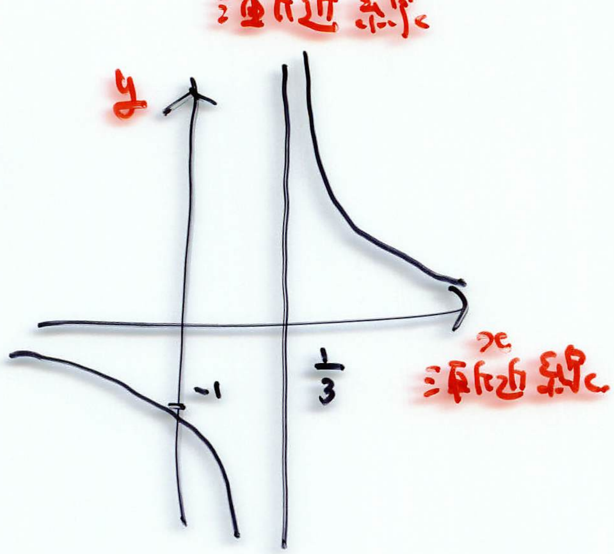
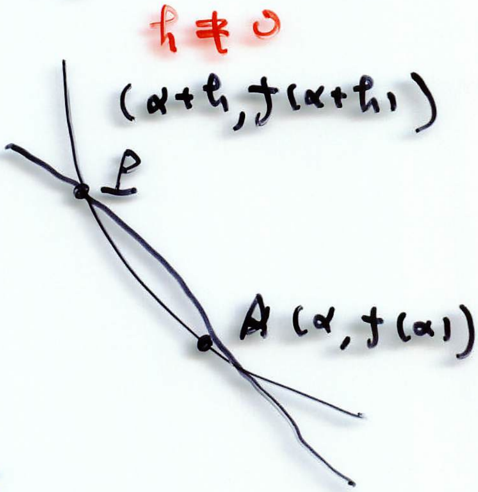
$$y = \frac{1}{3x-1} = f(x) \qquad x \neq \frac{1}{3}$$

$$\frac{1}{3x-1} = \frac{1}{3} \cdot \frac{1}{x - \frac{1}{3}}$$



$\alpha \neq \frac{1}{3}$ に対して

$$f'(\alpha) = \lim_{h \to 0} \boxed{\frac{f(\alpha+h) - f(\alpha)}{h}}$$



$$\text{AP の傾き} = \frac{f(\alpha+h) - f(\alpha)}{h}$$

$$= \frac{\dfrac{1}{3(\alpha+h)-1} - \dfrac{1}{3\alpha-1}}{h}$$

$$= \frac{1}{h} \cdot \frac{(3\alpha-1) - (3(\alpha+h)-1)}{(3\alpha-1)(3(\alpha+h)-1)}$$

$(-1)^n$

$\left(\frac{1}{2}\right)^n$

$\cdot(-1)$

$$= \frac{1}{h} \cdot \frac{-3h}{(3\alpha-1)\{3(\alpha+h)-1\}}$$

$$= -\frac{3}{(3\alpha-1)} \cdot \frac{1}{3(\alpha+h)-1}$$

$\frac{1}{3\alpha-1}$

$h \to 0$ とは $\{h_n\}$ $h_n \neq 0$, $h_n \to 0$ $(n \to +\infty)$
をみたす数列とり $h = h_n$ とおく

$$\text{AP の傾き} = -\frac{3}{(3\alpha-1)} \cdot \boxed{\frac{1}{3(\alpha+h_n)-1}}$$

$\{h_n\}$ のとり方
に依らず

$$\to -\frac{3}{(3\alpha-1)} \cdot \frac{1}{3\alpha-1} = -\frac{3}{(3\alpha-1)^2}$$

$a_n \longrightarrow \alpha$, $h_n \longrightarrow \beta$ $(n \longrightarrow +\infty)$ のとき.

1) $a_n \pm h_n \longrightarrow \alpha \pm \beta$.

2) $a_n \cdot h_n \longrightarrow \alpha \cdot \beta$

3) $a_n \neq 0$ (全ての n), $\alpha \neq 0$ であるとき

$$\frac{1}{a_n} \longrightarrow \frac{1}{\alpha}$$

$\{h_n\}$ $h_n \neq 0$, $h_n \to 0$ とする.

$$\text{AP の傾き} = -\frac{3}{3\alpha - 1} \cdot \frac{1}{3(\alpha + h_n) - 1}$$

$$\longrightarrow -\frac{3}{(3\alpha - 1)^2}$$ ← $\{h_n\}$ によらず..

$$\rightsquigarrow \lim_{h \to 0} (\text{AP の傾き}) = -\frac{3}{(3\alpha - 1)^2}.$$

$$\text{AP の傾き} = -\frac{3}{3\alpha - 1} \cdot \frac{1}{3(\alpha + h) - 1}$$

$h \to 0$

$$3(\alpha + h) \longrightarrow 3 \cdot (\alpha + 0) = 3\alpha$$

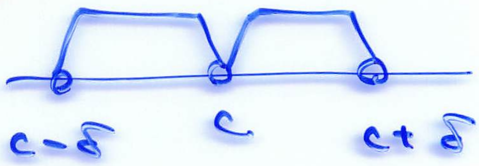
$$3(\alpha + h) - 1 \longrightarrow 3(\alpha + 0) - 1 = 3\alpha - 1 \neq 0$$

$$\frac{1}{3(\alpha + h) - 1} \longrightarrow \frac{1}{3\alpha - 1}$$

まとめ

$\delta > 0$

$$f : \underbrace{\boxed{(c-\delta,\ c) \cup (c,\ c+\delta)}}_{= I \text{ とおく}} \to \mathbb{R}$$



$$\lim_{t \to c} f(t) = A$$

$\Longleftrightarrow$ 定義

$t_n \in I$, $t_n \to c$ をみたす $\{t_n\}$ に対し

$$f(t_n) \to A \quad (n \to +\infty)$$

$I = (c-\delta,\ c) \cup (c,\ c+\delta)$

$\cup$ cup.

定理

$g : I \to \mathbb{R}$

CT 46 page 定理2.1

$f(t) \to A$, $g(t) \to B$ $(t \to c)$

$A \cup B$ the union of A and B.

(i) $f(t) \pm g(t) \to A \pm B$.

(ii) $f(t) \cdot g(t) \to A \cdot B$. ← CT 47 p.

(iii) $f(t) \neq 0$ $(t \in I)$, $A \neq 0$

$$\frac{1}{f(t)} \to \frac{1}{A}$$

$\frac{1}{f(t)} \to \frac{1}{A}$ $(t \to c)$

(iii) $t_n \to c$, $t_n \in I$ をみたす $\{t_n\}$ をとる

$\boxed{f(t) \to A \ (t \to A)}$ 仮定

$$\underset{\neq 0}{f(t_n)} \to \underset{\neq 0}{A} \rightsquigarrow \frac{1}{f(t_n)} \to \frac{1}{A}.$$

微分の公式.

1) $(f \pm g)' = f' \pm g'$

2) $(fg)' = f'g + fg'$

3) $\left(\dfrac{1}{f}\right)' = \dfrac{-f'}{f^2}$

5月16日.

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2) の証明は演習のスライドをみる.

3) の証明. CT 85ページ.

$$\frac{1}{h}\left(\frac{1}{f(\alpha+h)} - \frac{1}{f(\alpha)}\right) = \frac{1}{h}\cdot\frac{f(\alpha) - f(\alpha+h)}{f(\alpha)\cdot f(\alpha+h)}$$

$$= \boxed{-\frac{1}{f(\alpha)}}\cdot\boxed{\frac{1}{f(\alpha+h)}}\cdot\boxed{\frac{f(\alpha+h) - f(\alpha)}{h}}$$

$$\downarrow \qquad\qquad \downarrow\,? \qquad\qquad \downarrow$$

$$-\frac{1}{f(\alpha)} \qquad \frac{1}{f(\alpha)} \qquad f'(\alpha)$$

$h \to 0$ この極限

CT 定理3.1

定理 $f(t)$ が $t = \alpha$ で微分できるとする.

$f(t) \to f(\alpha)$ $(t \to \alpha)$

$$f(\alpha+h) \longrightarrow f(\alpha) \quad (h \to 0)$$

$$\frac{1}{f(\alpha+h)} \longrightarrow \frac{1}{f(\alpha)}$$ が分かる

$\Rightarrow$ montrer

It remains to show the theorem.

prove $\longleftrightarrow$ 名 proof.

$\updownarrow$

démontrer $\longleftrightarrow$ la démonstration

$h \to 0$ のとき.

$$\left.\frac{f(\alpha+h)-f(\alpha)}{h} \longrightarrow f'(\alpha)\right] \text{仮定}$$

が成立する.

$$f(\alpha+h)-f(\alpha) = h \times \frac{f(\alpha+h)-f(\alpha)}{h}$$

$$\longrightarrow 0 \times f'(\alpha) = 0$$

$$f(\alpha+h) = \big(f(\alpha+h)-f(\alpha)\big) + f(\alpha)$$

$$\longrightarrow 0 + f(\alpha) = f(\alpha)$$

計算 1) $f(x) = x^n$. $\quad f'(x) = n x^{n-1}$

$$\frac{(\alpha+h)^n - \alpha^n}{h}$$

$$= \frac{1}{h}\left(\cancel{\alpha^n} + {}_nC_1\,\alpha^{n-1}h + {}_nC_2\,\alpha^{n-2}h^2 + \cdots + h^n - \cancel{\alpha^n}\right)$$

$$= {}_nC_1\,\alpha^{n-1} + {}_nC_2\,\alpha^{n-2}h + {}_nC_3\,\alpha^{n-3}h^2 + \cdots$$

$$\cdots + h^{n-1}$$

$$\longrightarrow {}_nC_1\,\alpha^{n-1} = n\,\alpha^{n-1}$$

1) $f(x) = x^n \quad (n = 1, 2, \cdots)$

$$\frac{(\alpha+h)^n - \alpha^n}{h} = \frac{1}{h}\left\{ (\cancel{\alpha^n} + {}_nC_1 \alpha^{n-1} h + {}_nC_2 \alpha^{n-2} h^2 + \cdots \right.$$

$$\left. \cdots + {}_nC_{n-1} \alpha h^{n-1} + h^n) - \cancel{\alpha^n} \right\}$$

$$= {}_nC_1 \alpha^{n-1} + {}_nC_2 \alpha^{n-2} \cdot h + {}_nC_3 \alpha^{n-3} h^2 + \cdots$$

$$\cdots + h^{n-1}.$$

$$\longrightarrow {}_nC_1 \alpha^{n-1} = n \alpha^{n-1}$$

公式 $(x^n)' = n x^{n-1}$

帰納法 $(x)' = 1 = 1 \cdot x^0 = 1 \cdot x^{1-1}$

$n=1$ OK

nの時OK $(x^n)' = n x^{n-1}$ ← 仮定

$$(x^{n+1})' = (x \cdot x^n)'$$

$$= (x)' \cdot x^n + x \cdot (x^n)'$$

$$= x^n + x \cdot n x^{n-1}$$

$$= (n+1) x^n$$

$(fg)' = f'g + fg'$

$\rightsquigarrow (n+1)$ の時 OK.

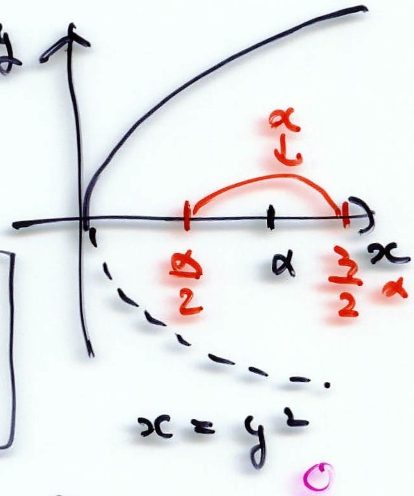
1) $f(x) = \dfrac{1}{x^n}$ $\qquad\qquad \left(\dfrac{1}{f}\right)' = \dfrac{-f'}{f^2}$

$$\left(\frac{1}{x^n}\right)' = \frac{-(x^n)'}{(x^n)^2} = \frac{-n x^{n-1}}{x^{2n}}$$

$$= -\frac{n}{x^{n+1}}$$

$$\boxed{(x^{-n})' = -n\, x^{-n-1}}$$ となる.

2) $\sqrt{x}$ $(x > 0)$

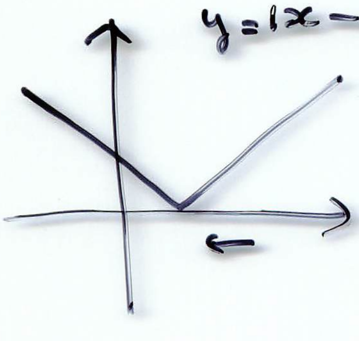


$$\boxed{\begin{array}{l} \alpha > 0 \\ \sqrt{x} \to \sqrt{\alpha} \quad (x \to \alpha) \end{array}}$$

50 page. $\qquad x \neq \alpha$, $\dfrac{\alpha}{2} < x < \dfrac{3}{2}\alpha$

はさみうち $\to 0$

$$0 \leqq \left|\sqrt{x} - \sqrt{\alpha}\right| = \frac{|x-\alpha|}{\sqrt{x}+\sqrt{\alpha}} \leqq \frac{|x-\alpha|}{\sqrt{\frac{\alpha}{2}}+\sqrt{\alpha}} \to 0$$



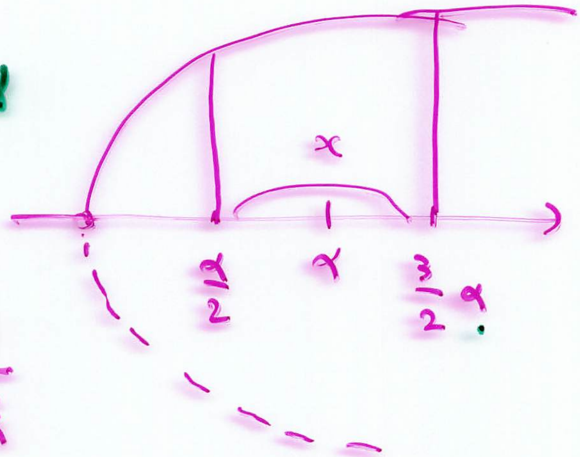
$$\boxed{\lim_{x \to \alpha} |x - \alpha| = 0}$$

まとめ $\quad |\sqrt{x} - \sqrt{\alpha}| \longrightarrow 0$

$$-|\sqrt{x}-\sqrt{\alpha}| \leqq \sqrt{x} - \sqrt{\alpha} \leqq |\sqrt{x}-\sqrt{\alpha}|$$

$0 \leftarrow$ $\qquad \downarrow 0 \qquad \nearrow 0$

$\alpha > 0$ $\quad \dfrac{\alpha}{2} < x < \dfrac{3}{2}\alpha$



$$0 \leqq \left| \sqrt{x} - \sqrt{\alpha} \right| = \frac{|x-\alpha|}{\sqrt{x}+\sqrt{\alpha}} \leqq \frac{|x-\alpha|}{\sqrt{\frac{\alpha}{2}}+\sqrt{\alpha}}$$

$x_n \to \alpha$, $x_n \neq \alpha$ となる列 $\{x_n\}$ をとる.

- $\boxed{|x_n - \alpha| \longrightarrow 0 \quad (n \to +\infty)}$

言い換え

$\cancel{|x_n - \alpha| \leqq x_n}$

$$0 \leqq \left| \sqrt{x_n} - \sqrt{\alpha} \right| \leqq \frac{|x_n - \alpha|}{\sqrt{\frac{\alpha}{2}}+\sqrt{\alpha}} \longrightarrow 0$$

$0 \to 0$, $\left| \sqrt{x_n} - \sqrt{\alpha} \right| \to 0$

はさみうち.

$$-\left|\sqrt{x_n}-\sqrt{\alpha}\right| \leqq \sqrt{x_n}-\sqrt{\alpha} \leqq \left|\sqrt{x_n}-\sqrt{\alpha}\right|$$

$\downarrow \qquad \downarrow \qquad \downarrow$

$0 \qquad 0 \qquad 0$

$$\sqrt{x_n} = \left(\sqrt{x_n}-\sqrt{\alpha}\right) + \sqrt{\alpha} \longrightarrow 0 + \sqrt{\alpha} = \sqrt{\alpha}.$$

$\leadsto$ $\{x_n\}$ は任意

$$\boxed{\sqrt{x} \longrightarrow \sqrt{\alpha} \quad (x \to \alpha)}$$

$$\sqrt{x} = \left(\sqrt{x} - \sqrt{a}\right) + \sqrt{a} \longrightarrow 0 + \sqrt{a} = \sqrt{a}$$

$a > 0$

$$\frac{\sqrt{a+h} - \sqrt{a}}{h} = \frac{(\cancel{a} + h) - \cancel{a}}{h\left(\sqrt{a+h} + \sqrt{a}\right)}$$

$$= \frac{1}{\sqrt{a+h} + \sqrt{a}} \longrightarrow \frac{1}{\sqrt{a} + \sqrt{a}} = \frac{1}{2}\frac{1}{\sqrt{a}}$$

$$\sqrt{a+h} \longrightarrow \sqrt{a}$$

公式

$$\left(\sqrt{x}\right)' = \frac{1}{2}\frac{1}{\sqrt{x}}$$

$$\left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\left(\frac{1}{3x-1}\right)' = \frac{-(3x-1)'}{(3x-1)^2} = \frac{-3}{(3x-1)^2}$$

$$(FG)' = F'G + FG'$$

$$\left(\frac{g}{f}\right)' = \left(g \cdot \frac{1}{f}\right)' = g'\left(\frac{1}{f}\right) + g\left(\frac{1}{f}\right)'$$

$$= \frac{g'}{f} + \frac{g \cdot (-f')}{f^2} = \frac{g'f - gf'}{f^2}$$

公式 $\left(\frac{g}{f}\right)' = \frac{g'f - gf'}{f^2}$

I) 1) $\dfrac{1}{(x^2+x+1)}$

$$\boxed{(\sqrt{x})' = \frac{1}{2}\,\frac{1}{\sqrt{x}}}$$

2) $x\sqrt{x}$

3) $x^2\sqrt{x}$ 4) $\dfrac{1}{\sqrt{x}}$

5) $\dfrac{3x-1}{x+1}$

$\cancel{\left(\dfrac{1}{\sqrt{x}}\right)'}$