2項定理

$$(a+b)^n = a^n + {}_nC_1 a^{n-1} b + {}_nC_2 a^{n-2} b^2 \quad n!$$

$$+ \cdots + {}_nC_k a^{n-k} b^k + \cdots + b^n$$

$$= \cancel{a^n + {}_nC} \, b^n + {}_nC_1 a b^{n-1} + \cdots + {}_nC_k a^k b^{n-k} + \cdots + a^n$$

2項定理. $\cdot \ {}_nC_k = {}_nC_{n-k}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \cdots + x^n$$

$x>0$ のとき 各項は正

$$(1+x)^n > nx$$

$$(1+x)^n > \frac{n(n-1)}{2} x^2$$

$$(1+x)^n > \frac{n(n-1)(n-2)}{6} x^3$$

$r \neq 1$

$$S_n = 1 + r + r^2 + \cdots + r^n$$

$$= \frac{1-r^{n+1}}{1-r} = \frac{1}{1-r} - \frac{r}{1-r} \cdot r^n$$

$\boxed{|r|<1 \text{ のとき } r^n \to 0}$

$$S_n \longrightarrow \frac{1}{1-r} - \frac{r}{1-r} \cdot 0 = \frac{1}{1-r}$$

$|r|<1$

$$\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + \cdots = \frac{1}{1-r}$$

$a_n \longrightarrow \alpha\ ,\quad b_n \longrightarrow \beta \quad (n \to +\infty)$

I) $a_n \pm b_n \longrightarrow \alpha \pm \beta$

II) $a_n \cdot b_n \longrightarrow \alpha\beta$

III) $a_n \neq 0,\ \alpha \neq 0$ atz

$$\frac{1}{a_n} \longrightarrow \frac{1}{\alpha}$$

CT 20 page.

• $x_n \underset{\text{constant}}{=} C \longrightarrow C \quad (n \to +\infty)$

134 : $\frac{1}{n} \longrightarrow 0 \quad (n \to +\infty)$ $\quad \frac{1}{n^2} = \frac{1}{n} \cdot \frac{1}{n} \longrightarrow 0 \cdot 0 = 0$

$$\frac{1}{n^k} \longrightarrow 0 \quad (n \to +\infty)$$

$$(k = 1, 2, 3, \dots)$$

$|r| < 1$ atz $r^n \longrightarrow 0 \quad (n \to +\infty)$

CT 22 page.

$$\frac{n}{1+3n} = \frac{1}{\frac{1}{n}+3} \longrightarrow \frac{1}{0+3} = \frac{1}{3}$$

$$\frac{1}{1+3n} = \frac{1}{n} \cdot \frac{n}{1+3n} = \frac{1}{n} \cdot \frac{1}{\frac{1}{n}+3} \longrightarrow 0 \cdot \frac{1}{0+3} = 0$$

$\frac{x}{y}$ x over y , x sur y

$$\frac{4-n}{5+n} = \frac{\frac{4}{n}-1}{\frac{5}{n}+1} \longrightarrow \frac{0-1}{0+1} = -1.$$

$$\frac{2+3n}{1+n^2} = \frac{1}{n}\cdot\frac{n(2+3n)}{1+n^2} = \frac{1}{n}\cdot\frac{\frac{2}{n}+3}{\frac{1}{n^2}+1}$$

$$\longrightarrow 0\cdot\frac{0+3}{0+1} = 0$$

$|r|<1$

$$T_n = 1+2r+3r^2+\cdots+nr^{n-1}$$

$$-)\ rT_n = \quad r+2r^2+\cdots+(n-1)r^{n-1}+nr^n$$

$$(1-r)T_n = 1+r+r^2+\cdots+r^{n-1}-nr^n$$

$$= \frac{1-r^n}{1-r} - nr^n$$

$r \neq 1$ だから

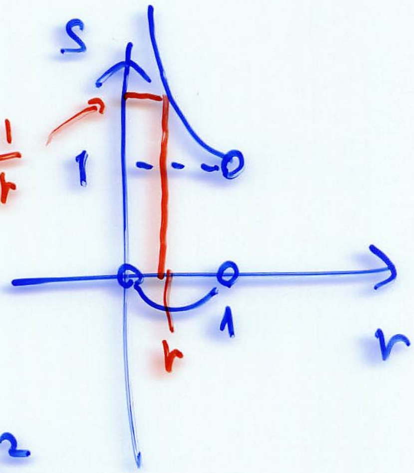
$$T_n = \frac{1-r^n}{(1-r)^2} - \frac{nr^n}{(1-r)}$$

$$\boxed{|r|<1 \text{ のとき } nr^n \longrightarrow 0}$$

$$= \frac{1}{(1-r)^2} - \frac{1}{(1-r)^2}\overset{\to 0}{r^n} - \frac{1}{1-r}\cdot\overset{\to 0}{nr^n}$$

$$\longrightarrow \frac{1}{(1-r)^2} - \frac{1}{(1-r)^2}\cdot 0 - \frac{1}{1-r}\cdot 0 = \frac{1}{(1-r)^2}$$

$1 > r > 0$ のとき.　　$s = \frac{1}{r} > 1$　$\frac{1}{r}$



$s = 1 + \theta$ とおくと $\theta > 0$.

$$\frac{1}{r^n} = s^n = (1+\theta)^n > \frac{n(n-1)}{2}\theta^2$$

$$0 < r^n < \frac{2}{n(n-1)} \cdot \theta^2$$

$$\underset{\downarrow \atop 0}{0} < \underset{\downarrow \atop 0}{n r^n} < \frac{n}{n(n-1)} 2\theta^2 = \underbrace{\frac{1}{n-1} \cdot 2\theta^2}_{\downarrow \atop 0}$$

← はさみうち

$$\boxed{\begin{array}{l} a_n \leqq b_n \leqq c_n \quad \text{はさみうち.} \\ a_n \to \alpha,\ c_n \to \alpha \Rightarrow b_n \to \alpha \end{array}}$$

$-1 < r < 1$ のとき　一般の場合.

$r = 0$ のとき　$n 0^n = 0 \longrightarrow 0$　ok.

$r \neq 0$ のとき　$s = |r|$ とする

$0 < s < 1$

$$-s^n \leqq r^n \leqq s^n \qquad -|x| \leqq x \leqq |x|$$

$$\underbrace{-ns^n}_{\downarrow \atop (-1)\cdot 0 = 0} \leqq \underbrace{n r^n}_{\downarrow \atop 0} \leqq \underbrace{n s^n}_{\downarrow \atop 0}$$

上で示した →　　　はさみうち.

← 上で示したこと

$|r|<1$ のとき $nr^n \longrightarrow 0 \quad (n \to +\infty)$

まとめ

$$\sum_{n=1}^{+\infty} n r^{n-1} = \frac{1}{(1-r)^2}. \quad (|r|<1)$$

$$= 1+2r+3r^2+\cdots\cdots$$

補足

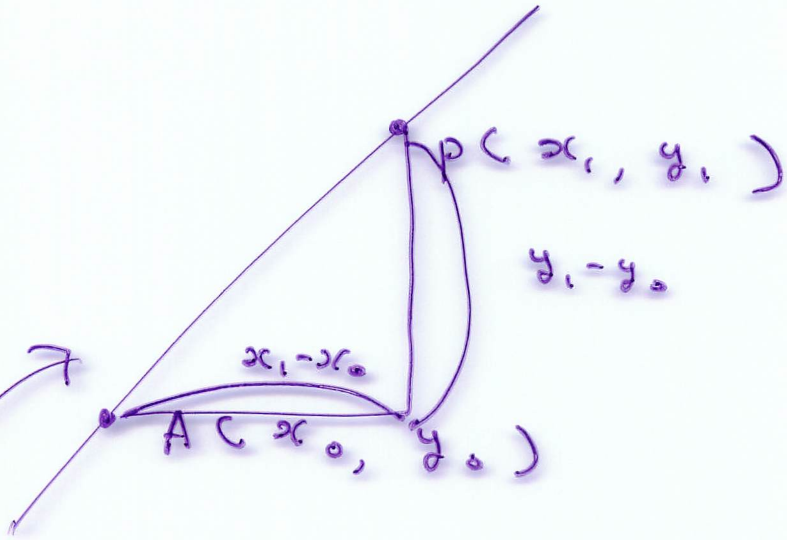
$|r|<1$

$$n^2 r^n \longrightarrow 0 \quad (n \to +\infty)$$

$\theta>0$ のとき

$$(1+\theta)^n > \frac{n(n-1)(n-2)}{6}\theta^3.$$

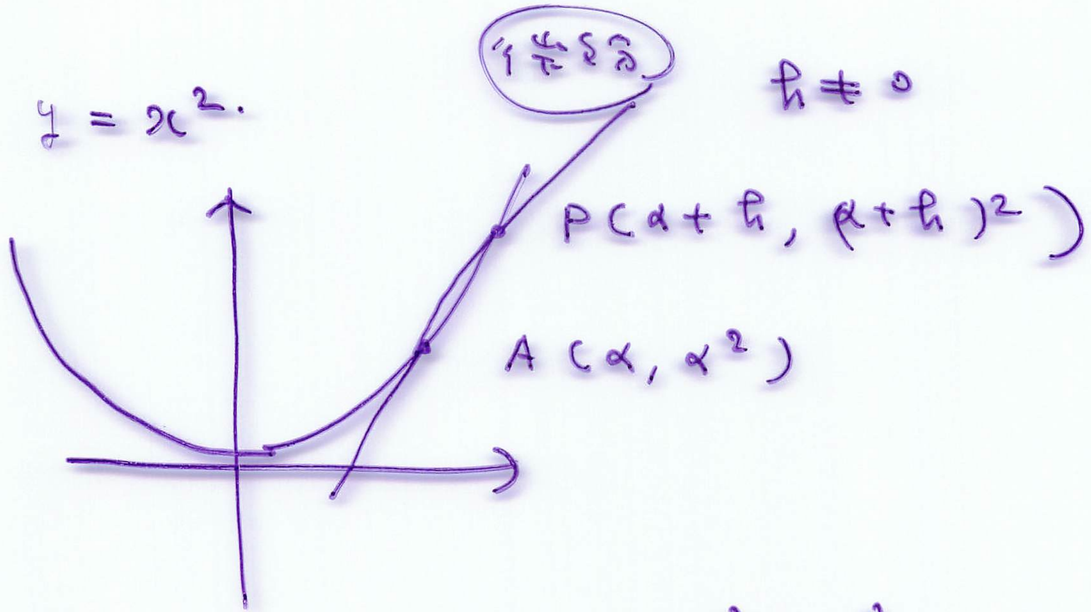
を用いる.　　CT 25 page の演習 1.10.



相似.

$$AP의\ 기울기 = \frac{y_1 - y_0}{x_1 - x_0}$$

기울기.



$h \neq 0$

$$AP\text{の傾き} = \frac{(\alpha + h)^2 - \alpha^2}{(\alpha + h) - \alpha}$$

$$= \frac{\cancel{\alpha^2} + 2\alpha h + h^2 - \cancel{\alpha^2}}{h}$$

$$= \frac{2\alpha h + h^2}{h} = 2\alpha + h.$$

$h_n \to 0$ である任意の数列 $\{h_n\}$ をとる.

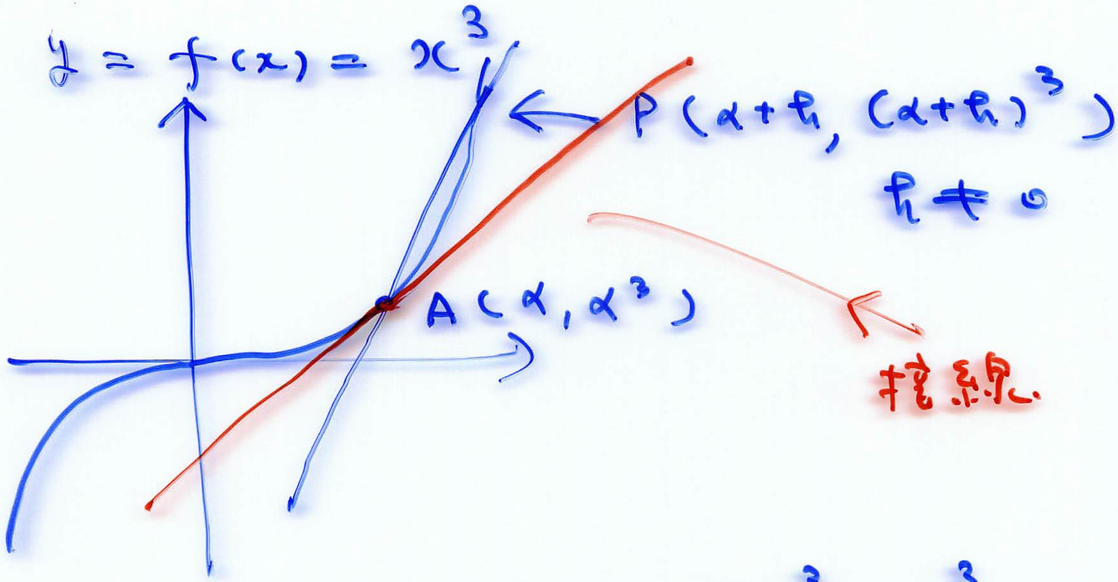
$$AP\text{の傾き} = 2\alpha + h_n \to 2\alpha + 0 = 2\alpha$$

$$\boxed{\lim_{h \to 0} (AP\text{の傾き}) = 2\alpha}$$

$h_n \to 0$ である $\{h_n\}$ の取り方に依らない.

$$= f'(\alpha) \qquad f(x) = x^2$$

$y = f(x)$ の $x = \alpha$ における微分係数.



$$(\text{APの傾き}) = \frac{(\alpha+h)^3 - \alpha^3}{(\alpha+h) - \alpha}$$

$$= \frac{\cancel{\alpha^3} + 3\alpha^2 h + 3\alpha h^2 + h^3 - \cancel{\alpha^3}}{h}$$

$$= 3\alpha^2 + 3\alpha h + h^2$$

$\{h_n\}$ は $h_n \to 0 \quad (n \to +\infty)$

をみたすとする

$\{h_n\}$ に依らない

$$= 3\alpha^2 + 3\alpha h_n + h_n^2$$

$$\xrightarrow[n \to +\infty]{} 3\alpha^2 + 3\alpha \cdot 0 + 0 \cdot 0$$

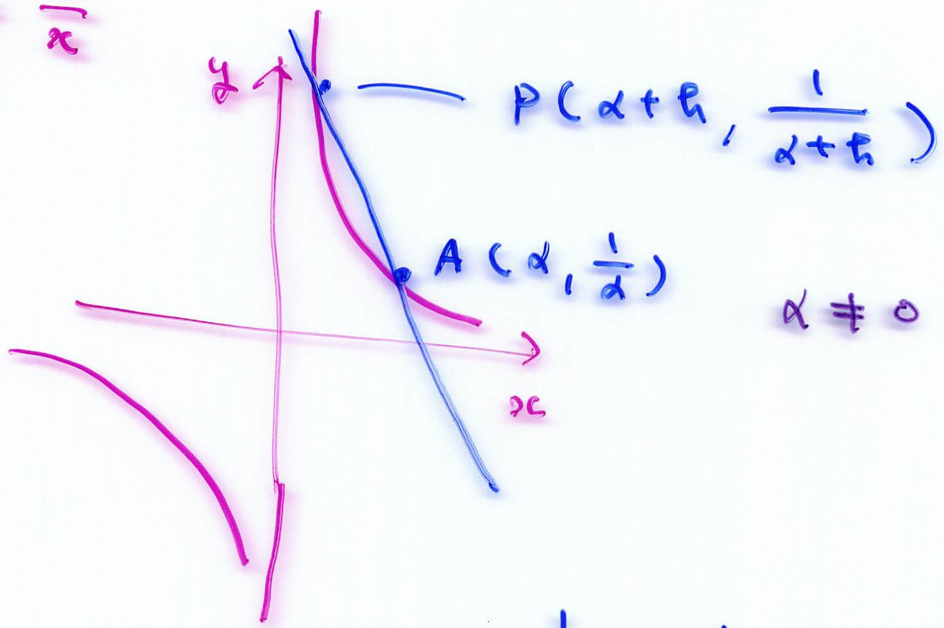
$$= 3\alpha^2.$$

$$\lim_{h \to 0} \text{APの傾き} = 3\alpha^2$$

$= f'(\alpha) =$ ($y = f(x)$ の $x = \alpha$ における 接線の傾き

$f(x)$ の $x = \alpha$ における 微分係数

$y = \frac{1}{x}$



$h \neq 0.$

$\alpha \neq 0$

$$(\text{AP の傾き}) = \frac{\frac{1}{\alpha + h} - \frac{1}{\alpha}}{(\alpha + h) - \alpha}$$

$$= \frac{1}{h} \cdot \frac{\alpha - (\alpha + h)}{\alpha \cdot (\alpha + h)}$$

$$= \frac{1}{h} \cdot \frac{-h}{\alpha(\alpha + h)}$$

$$= -\frac{1}{\alpha} \cdot \frac{1}{\alpha + h}$$

$h_n \to 0$ である $\{h_n\}$ をとる.

$$= -\frac{1}{\alpha} \cdot \frac{1}{\alpha + h_n}$$

$$\longrightarrow -\frac{1}{\alpha} \cdot \frac{1}{\alpha} = -\frac{1}{\alpha^2}$$

$$\boxed{\lim_{h \to 0} (\text{AP の傾き}) = -\frac{1}{\alpha^2}}$$

$y = f(x)$ "$\frac{1}{x}$" のとき

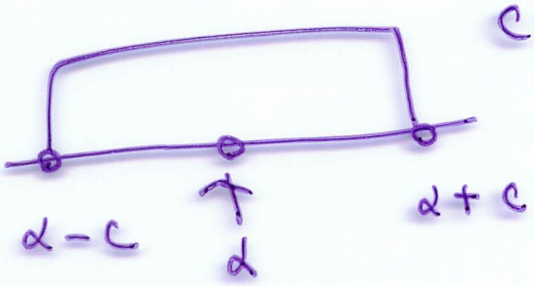
$\alpha \neq 0$

$f'(\alpha) = -\frac{1}{\alpha^2}$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

極限

$c > 0$



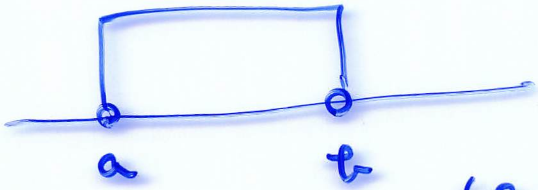
$x \to \alpha$ のとき

~~$f(x) : (\alpha - c, \alpha + c)$~~

$f : (\alpha - c, \alpha) \cup (\alpha, \alpha + c) \to \mathbb{R}$

または
和集合 union

実数全体の集合.



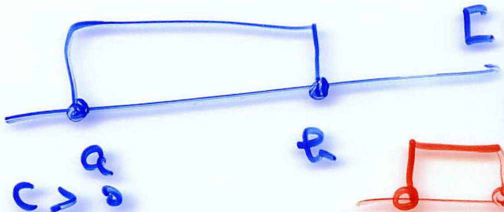
(a, b) , $]a, b[$

開区間
an open interval

$= \{x \mid a < x < b\}$

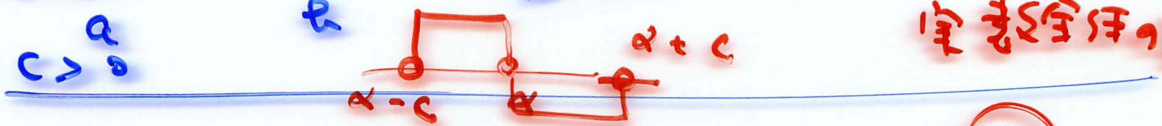
$[a, b] = \{x \mid a \leqq x \leqq b\}$

閉区間 a closed interval



実数全体の集合.

$c > 0$



$f : (\alpha - c, \alpha) \cup (\alpha, \alpha + c) \to \mathbb{R}.$

和集合 union

$x \to \alpha$ のとき $f(x) \to A$ とは

$$\left(\lim_{x \to \alpha} f(x) = A \right)$$

任意の数列 $\{x_n\}$ が

$x_n \to \alpha$

$\boxed{x_n \in (\alpha - c, \alpha) \cup (\alpha, \alpha + c)}$

をみたすならば $f(x_n) \to A \ (n \to +\infty)$

$f(x_n)$ が
定義されてい