Exo

(1) $y = \dfrac{x}{x^2+1}$ $\qquad \left(\dfrac{g}{f}\right)' = \dfrac{g'f - gf'}{f^2}$

$$y' = \frac{(x)'(x^2+1) - x(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{\boxed{(x^2+1)^2}} > 0$$

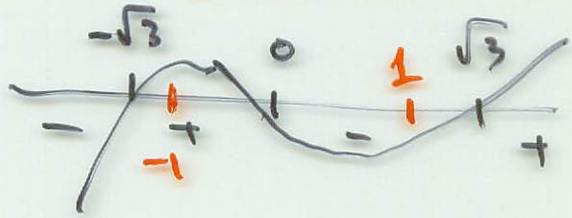
$$y'' = \frac{(1-x^2)'(x^2+1)^2 - (1-x^2)\{(x^2+1)^2\}'}{(x^2+1)^4}$$


$$= \frac{-2x\,(x^2+1)^2 - (1-x^2)\,2(1+x^2)\cdot 2x}{(x^2+1)^4}$$

$$\frac{du^2}{dx} = \frac{du^2}{du}\frac{du}{dx} = 2u\frac{du}{dx}$$

$$= \frac{2x\{-(x^2+1) - 2(1-x^2)\}}{(x^2+1)^3}$$

$$= \frac{2x(x^2-3)}{\boxed{(x^2+1)^3}} > 0.$$




$\alpha < \beta < \gamma$

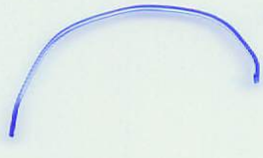
$(x-\alpha)(x-\beta)(x-\gamma)$


$y = \dfrac{x}{x^2+1}$

| $x$ | | $-\sqrt{3}$ | | $-1$ | | $0$ | | $1$ | | $\sqrt{3}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y'$ | $-$ | $-$ | $-$ | $0$ | $+$ | $+$ | $+$ | $0$ | $-$ | $-$ | $-$ |
| $y''$ | $-$ | $0$ | $+$ | $+$ | $+$ | $0$ | $-$ | $-$ | $-$ | $0$ | $+$ |
| $y$ | ↘ | $-\frac{\sqrt{3}}{4}$ | ↘ | $-\frac{1}{2}$ | ↗ | $0$ | ↗ | $\frac{1}{2}$ | ↘ | $\frac{\sqrt{3}}{4}$ | ↘ |

$y'' > 0$ $\qquad\qquad$ $y'' < 0$

下に凸

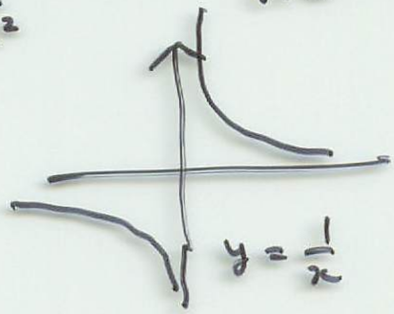


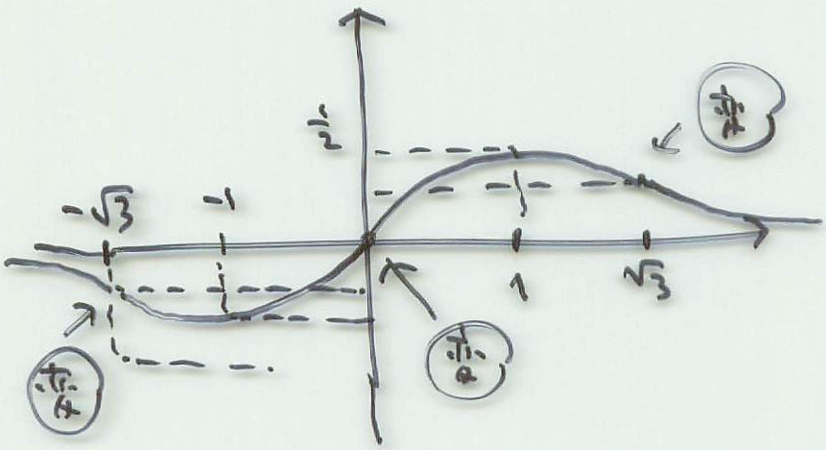
上に凸

$x \longrightarrow \pm\infty$

$$\frac{x}{x^2+1} = \frac{1}{x}\cdot\frac{x^2}{x^2+1} = \frac{1}{x}\cdot\frac{1}{1+\frac{1}{x^2}} \longrightarrow 0\times\frac{1}{1+0} = 0$$

$$\frac{1}{x} \longrightarrow 0 \quad (x \to \pm\infty)$$




$y = \frac{x}{1+x^2}$, $\quad 1+x^2 > 0$

$y \gtreqless 0$

$\iff x \gtreqless 0$

(3) $y = t^2 \log t$ $\quad (\log t)' = \frac{1}{t}$

$(t > 0)$ $\quad (fg)' = f'g + fg'$

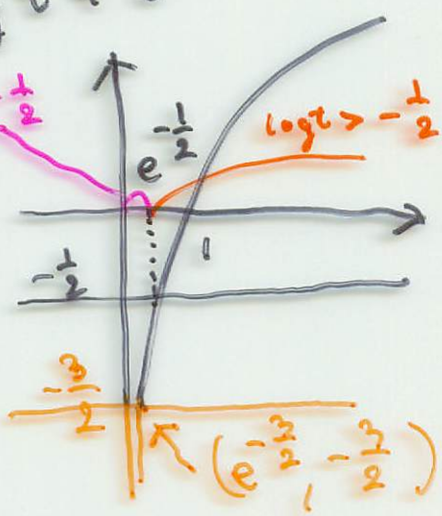
$$y' = 2t\log t + t^2\cdot\frac{1}{t} = 2t\log t + t$$

$$= 2t\left(\log t + \frac{1}{2}\right)$$

$0 < 2t$

$$y'' = 2\left(\log t + \frac{1}{2}\right) + 2t\cdot\frac{1}{t}$$

$$= 2\left(\log t + \frac{3}{2}\right)$$




| $t$ | (0) | | $e^{-\frac{3}{2}}$ | | $e^{-\frac{1}{2}}$ | |
|---|---|---|---|---|---|---|
| $y'$ | | − | − | − | 0 | + |
| $y''$ | | − | 0 | + | + | + |
| $y$ | (0) | ↘ | | ↘ | | ↗ |

$$y = e^{-3}\cdot\left(-\frac{3}{2}\right) = \frac{-3}{2e^3}$$

$$y = e^{-1}\cdot\left(-\frac{1}{2}\right) = -\frac{1}{2e}$$

$y = t^2 \log t \longrightarrow +\infty$

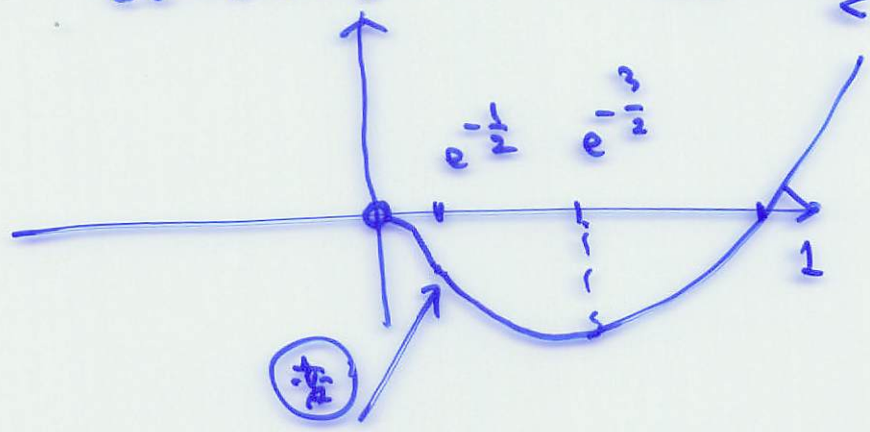
$(t \to +\infty)$

$y \Rightarrow t \cdot t \log t \longrightarrow 0$

$(t \to +0)$

$t^2 \log t \gtreqless 0$

$\Longleftrightarrow t \gtreqless 1$

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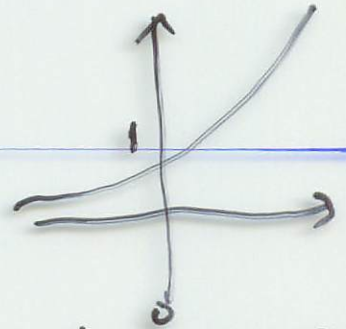
(4) $y = \dfrac{1}{1+e^{-t}}$

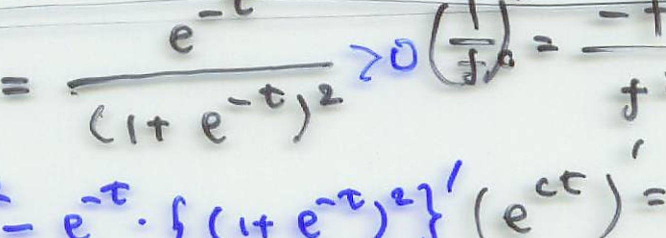



① $t \longrightarrow -\infty$ $\quad y = \dfrac{e^t}{e^t+1} \longrightarrow \dfrac{0}{0+1} = 0$

$e^t \longrightarrow 0$

② $t \longrightarrow \infty$ $\quad e^{-t} \longrightarrow 0$ $\quad y = \dfrac{1}{1+e^{-t}} \longrightarrow \dfrac{1}{1+0} = 1$

$y = \dfrac{D}{1+C e^{-t}}$

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$y' = \dfrac{-(1+e^{-t})'}{(1+e^{-t})^2} = \dfrac{e^{-t}}{(1+e^{-t})^2} > 0$ $\quad \left(\dfrac{1}{f}\right)' = \dfrac{-f'}{f^2}$

$y'' = \dfrac{(e^{-t})'(1+e^{-t})^2 - e^{-t}\cdot\{(1+e^{-t})^2\}'}{(1+e^{-t})^4}$ $\quad (e^{ct})' = c e^{ct}$

$= \dfrac{-e^{-t}(1+e^{-t})^2 - e^{-t}\cdot 2(1+e^{-t})(-e^{-t})}{(1+e^{-t})^4}$ $\quad \dfrac{du^2}{dx} = 2u\dfrac{du}{dx}$

$= \dfrac{-e^{-t}\{(1+e^{-t}) - 2e^{-t}\}}{(1+e^{-t})^3} = \dfrac{-e^{-t}(1-e^{-t}}{\bigcirc}$

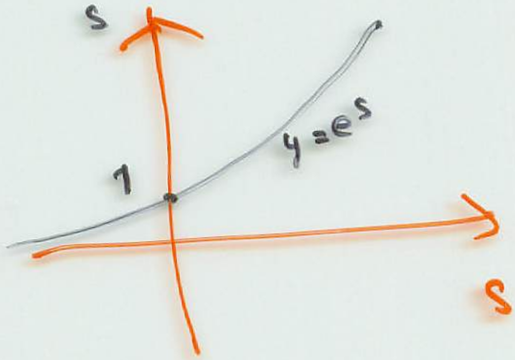
$$y'' = \boxed{\frac{e^{-t}(e^{-t}-1)}{(1+e^{-t})^3}}$$

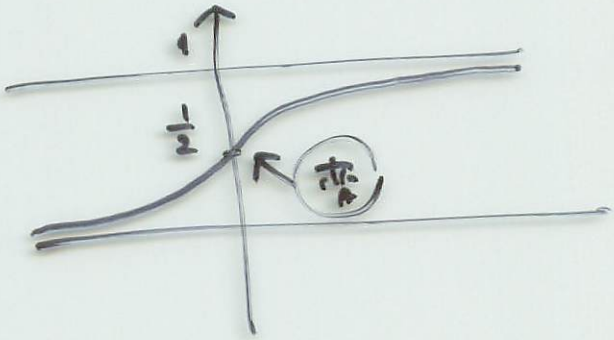
$\underset{0}{\vee}$

$e^{-t} \gtreqless 1 \iff y'' \gtreqless 0$ (4

$\iff (-t) \gtreqless 0 \iff t \lesseqgtr 0$

| | | 0 | |
|---|---|---|---|
| $y'$ | + | + | + |
| $y''$ | + | 0 | − |
| $y$ | ↗ | $\frac{1}{2}$ | ↗ |




$$y = A + \frac{D}{1+ce^{-t}}$$

$t \sim a \longleftarrow t$ が $a$ に近い.

$f(t) \doteqdot f(a) + f'(a)(t-a)$ 1次近似

$f(t) \doteqdot f(a) + f'(a)(t-a) + \frac{1}{2!} f''(a)(t-a)^2$

2次近似

$f(t) = (1+t)^{10}$ $(1.1)^{10}$

$(1.01)^{10}$

$$\left.\begin{aligned} f'(t) &= 10(1+t)^9 \\ f''(t) &= 10\cdot 9\,(1+t)^8 \end{aligned}\right\} \longrightarrow \begin{aligned} f(0) &= 1 \\ f'(0) &= 10 \\ f''(0) &= 90 \end{aligned}$$

$(1+t)^{10} \doteqdot 1 + 10t$ 1次

$(1+t)^{10} \doteqdot 1 + 10t + 45t^2$ 1.10462 ....

$(1.01)^{10} \doteqdot 1 + 0.1 + 45 \times 0.0001$

$\doteqdot 1.1045$

(1) $\int_0^2 x\,(x^2+1)^3\,dx = \frac{1}{2}\int_0^2 2x\,(x^2+1)^3\,dx$ (5

$x^2+1 = u$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

$$\begin{array}{c|c} x & 0 \to 2 \\ \hline u & 1 \to 5 \end{array}$$

$= \int_1^5 \frac{1}{2} u^3\,du$

$= \frac{1}{2}\left[\frac{1}{4}u^4\right]_1^5 = \cdots\cdots$

$= \frac{1}{2}\int_0^1 (x^2+1)'(x^2+1)^3\,dx.$

$= \frac{1}{2}\int_0^1 \left\{\frac{1}{4}(x^2+1)^4\right\}'$

$= \frac{1}{2}\left[\frac{1}{4}(x^2+1)^4\right]_0^1$

$\shortparallel$

$\vdots$

(2) $\int_e^{e^2} \frac{dx}{x\log x} = \int_e^{e^2} \frac{(\log x)'}{\log x}\,dx.$ $\quad (\log x)' = \frac{1}{x}$

$\log x = u$

$du = \frac{1}{x}dx$

$= \int_1^2 \frac{du}{u}$

$= \left[\log|u|\right]_1^2 = \log 2$

$= \int_e^{e^2} \left(\log|\log x|\right)'\,dx.$

$= \left[\log|\log x|\right]_e^{e^2}$

$\left(\log|g(x)|\right)'$

$= \frac{g'(x)}{g(x)}$

$\int_a^b \frac{g'(x)}{g(x)}\,dx.$

$= \left[\log|g(x)|\right]_a^b$

$$\begin{array}{c|c} x & e \to e^2 \\ \hline u & 1 \to 2 \end{array}$$

(3)

$$\int_0^1 \frac{e^x}{e^x+1}\,dx = \int_0^1 \frac{(e^x+1)'}{(e^x+1)}\,dx$$

$u = e^x + 1$

$du = e^x \cdot dx$

$$= \Big[\log(e^x+1)\Big]_0^1$$

$$= \log(e+1) - \log 2$$

$$= \log \frac{e+1}{2}$$

$$= \int_2^{e+1} \frac{du}{u}$$

$$= \Big[\log u\Big]_2^{e+1}$$

$A, B > 0$

$\log(A \cdot B) = \log A + \log B.$

$\log A^{-1} = -\log A$

(4)

$$\int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\,dx = \int_1^{\frac{3}{4}} \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{2}du\right)$$

$u = 1 - x^2$

$du = -2x\,dx$

$x\,dx = -\frac{1}{2}du$

| $x$ | $0 \to \frac{1}{2}$ |
|---|---|
| $u$ | $1 \to \frac{3}{4}$ |

$$= -\frac{1}{2}\int_1^{\frac{3}{4}} \frac{1}{\sqrt{u}}\,du$$

$(u^{\frac{1}{2}})' = \frac{1}{2}u^{-\frac{1}{2}}$

$(2u^{\frac{1}{2}})' = u^{-\frac{1}{2}}$

$$= -\frac{1}{2}\Big[2u^{\frac{1}{2}}\Big]_1^{\frac{3}{4}}$$

$$= -\left(\frac{\sqrt{3}}{2} - 1\right)$$

$$= 1 - \frac{\sqrt{3}}{2}.$$

5) $\displaystyle\int_1^2 \frac{dx}{e^x-1}$  $u = e^x - 1 \rightsquigarrow e^x = u+1$

$du = e^x \cdot dx.$

$= (u+1)\,dx$

$$= \int_1^2 \frac{du}{u} \cdot \frac{\cancel{\phantom{x}}}{u+1} \qquad dx = \frac{du}{u+1}$$

$$= \int_{e^{\bullet}-1}^{e^2-1} \frac{du}{u(u+1)}$$

| $x$ | $1 \to 2$ |
|---|---|
| $u$ | $e-1 \to e^2-1$ |

$$= \int_{e-1}^{e^2-1} \left(\frac{1}{u} - \frac{1}{u+1}\right) du \qquad \frac{1}{u(u+1)}$$

$$= \Big[\log u - \log(u+1)\Big]_{e-1}^{e^2-1} \qquad = \frac{1}{u} - \frac{1}{u+1}$$

$$= \ldots \qquad = \frac{(u+1) - u}{u(u+1)}$$

6) $\displaystyle\int_1^2 \frac{dx}{x\sqrt{x+1}} =$

$u = \sqrt{x+1}$

$u^2 = x+1$

$x = u^2 - 1$

$dx = 2u\,du$

$$= \int_{\sqrt{2}}^{\sqrt{3}} \frac{2\cancel{u}\,du}{(u^2-1)\cancel{u}}$$

$$= 2\int_{\sqrt{2}}^{\sqrt{3}} \frac{du}{u^2-1}$$

| $x$ | $1 \to 2$ |
|---|---|
| $u$ | $\sqrt{2} \to \sqrt{3}$ |

$$= \cancel{\frac{1}{2}} \int_{\sqrt{2}}^{\sqrt{3}} \left(\frac{1}{u-1} - \frac{1}{u+1}\right) du \qquad \frac{1}{u^2-1} = \frac{1}{(u-1)(u+1)}$$

$$= \frac{1}{2}\cdot\left(\frac{1}{u-1} - \frac{1}{u+1}\right)$$

$$= \cancel{\frac{1}{2}} \Big[\log(u-1) - \log(u+1)\Big]_{\sqrt{2}}^{\sqrt{3}}$$

$= \ldots$

7) $\int_0^2 (3x-1)^4 \, dx$

$= \left[ \frac{1}{15}(3x-1)^5 \right]_0^2$

$\{(3x-1)^5\}'$

$= 5(3x-1)^4 \cdot 3$

$\left\{ \frac{1}{3\cdot 5}(3x-1)^5 \right\}'$

$= (3x-1)^4.$

$u = 3x-1 \quad du = 3\,dx$

$dx = \frac{1}{3}\,du$

| $x$ | $0 \to 2$ |
|---|---|
| $u$ | $-1 \to 5$ |

$= \int_{-1}^{5} u^4 \cdot \frac{1}{3}\, du$

$= \frac{1}{3}\left[ \frac{1}{5} u^5 \right]_{-1}^{5} = \cdots$

8) $\int_0^2 \sqrt{2-x}\, dx = \int_2^0 \sqrt{u}\,(-du)$

$u = 2-x. \quad du = -dx$

| $x$ | $0 \to 2$ |
|---|---|
| $u$ | $2 \to 0$ |

$dx = -du$

$\| $

$\int_0^2 \sqrt{u}\, du$

$\| $

$\left[ \frac{2}{3} u^{\frac{3}{2}} \right]_0^2$

$\left(u^{\frac{3}{2}}\right)' = \frac{3}{2} u^{\frac{1}{2}}$

$\left(\frac{2}{3} u^{\frac{3}{2}}\right)' = u^{\frac{1}{2}}$

$= \left[ -\frac{2}{3}\sqrt{2-x} \right]_0^2$