(?) $y = e^{-\frac{t^2}{2}} = e^{x}$ $\quad \left(x = -\frac{t^2}{2} \right)$ $\quad e^{t} = \exp(t)$

exponential

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ $\quad (e^{x})' = e^{x}$

$= e^{x} \cdot (-t)$

$= -t\, \boxed{e^{-\frac{t^2}{2}}}$

$\quad\quad\ \ \vee$
$\quad\quad\ \ 0$

$\frac{dy}{dt} \underset{<}{\overset{>}{=}} 0 \iff t \underset{>}{\overset{<}{=}} 0$

$|t| \longrightarrow$ 大 $\rightsquigarrow -\frac{t^2}{2} \longrightarrow$ 小

$y = e^{-\frac{t^2}{2}} \longrightarrow$ 小

$y'' = (-t)' e^{-\frac{t^2}{2}} + (-t) \left(e^{-\frac{t^2}{2}} \right)'$

$= -e^{-\frac{t^2}{2}} + (-t)^2 e^{-\frac{t^2}{2}}$

$= (t^2 - 1)\, e^{-\frac{t^2}{2}}$

$\quad\quad\quad\quad\ \vee$
$\quad\quad\quad\quad\ 0$

t		-1		0		1	
y'	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$+$	0	$-$	$-$	$-$	0	$+$
y	⤴	↑	↱		↴		↳

$e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

$f'' > 0$

下に凸

$f'' < 0$

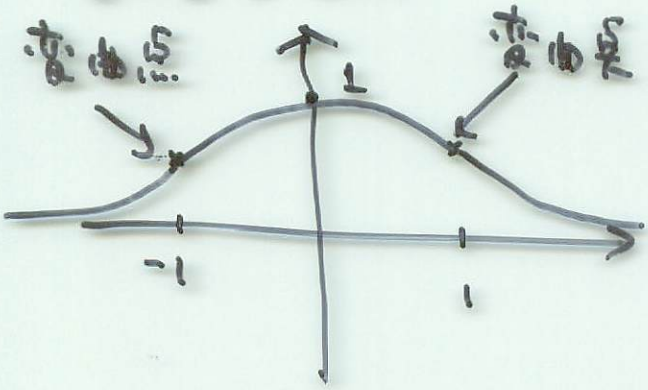
上に凸

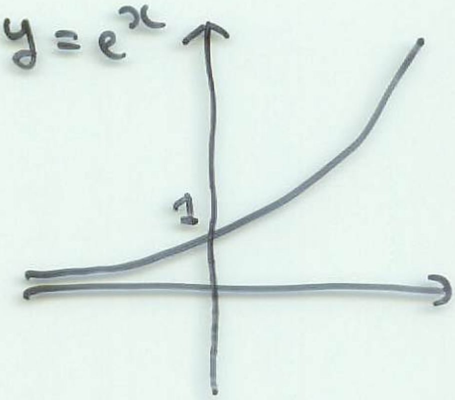
$|t| \longrightarrow +\infty$

$-\frac{t^2}{2} \longrightarrow -\infty$

$e^{-\frac{t^2}{2}} \longrightarrow 0$

$y = e^{-\frac{t^2}{2}} > 0$





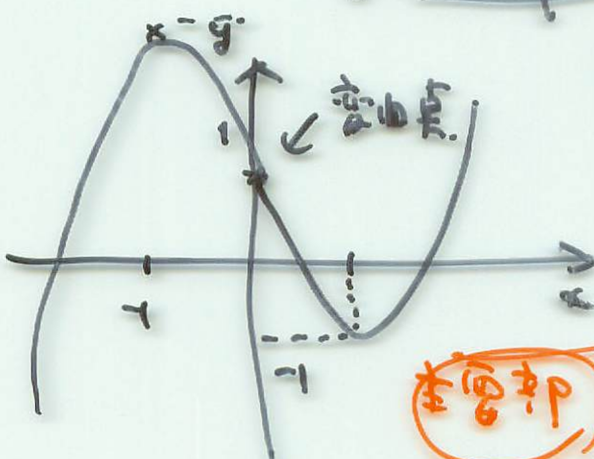
$x \longrightarrow +\infty \quad y = e^{x} \longrightarrow +\infty$

$x \longrightarrow -\infty \quad y = e^{x} \longrightarrow 0$

$y = x^3 - 3x + 1.$ $\quad y' = 3x^2 - 3 = 3(x-1)(x+1)$

$y'' = 6x$

		-1		0		1	
y'	+	0	−	−	−	0	+
y''	−	−	−	0	+	+	+
y	↗	3	↘	1	↘	-1	↗



(I) $x \longrightarrow +\infty$

$x^3 - 3x + 1 \longrightarrow +\infty$

$= \underbrace{x^3}_{\text{主要部}} \boxed{\left(1 - \frac{3}{x^2} + \frac{1}{x^3}\right)}$

$\sim x^3.$ $\quad \downarrow 1$

(II) $x \longrightarrow -\infty$

$x^3 \left(1 - \frac{3}{x^2} + \frac{1}{x^3}\right)$

$\sim x^3 \longrightarrow -\infty$ $\quad \downarrow 1$

(1) $y = \dfrac{x}{x^2+1}$, (2) $y = \dfrac{1}{x^2+x+1}$

(3) $y = t^2 \log t$ (4) $\boxed{y = \dfrac{1}{1+e^{-t}}}$

↑ ロジスティック曲線.

---

(1) $\displaystyle\int_0^1 e^{-2t}\,dt = \left[\frac{1}{-2} e^{-2t}\right]_0^1 = -\frac{1}{2}\left(e^{-2}-1\right)$

$= \dfrac{1}{2}\left(1-\dfrac{1}{e^2}\right)$

$(e^{ct})' = c\,e^{ct}$.

$\left(\dfrac{1}{c} e^{ct}\right)' = e^{ct} \rightsquigarrow \displaystyle\int e^{ct}\,dt = \frac{1}{c} e^{ct}$

(2) $\displaystyle\int_1^e t \log t\,dt = \int_1^e \left(\frac{t^2}{2}\right)' \log t\,dt$

$\displaystyle = \left[\frac{t^2}{2}\log t\right]_1^e - \int_1^e \frac{t^2}{2}\cdot\frac{1}{t}\,dt$

$$\boxed{\int_a^b \underset{\sim}{F'} G\,dt = \left[FG\right]_a^b - \int_a^b F G'\,dt}$$

$\displaystyle = \left(\frac{e^2}{2}\cdot\log e - \frac{1}{2}\cdot 0\right) - \frac{1}{2}\int_1^e t\,dt$

$\displaystyle = \frac{e^2}{2} - \frac{1}{2}\left[\frac{t^2}{2}\right]_1^e = \frac{e^2}{2} - \frac{1}{4}(e^2-1)$

$= \dfrac{1}{4}e^2 + \dfrac{1}{4} = \dfrac{1}{4}(e^2+1)$

(2) $\int_0^1 t e^{-t}\,dt = \int_0^1 t(-e^{-t})'\,dt$

$$\boxed{\begin{aligned}(e^{-t})' &= -e^{-t}\\ (-e^{-t})' &= e^{-t}\end{aligned}}$$

$$= \left[-te^{-t}\right]_0^1 + \int_0^1 \underbrace{(t)'}_{1} e^{-t}\,dt$$

$$= -1\cdot e^{-1} + \int_0^1 e^{-t}\,dt$$

$$= -\frac{1}{e} + \left[-e^{-t}\right]_0^1$$

$$= -\frac{1}{e} + (-e^{-1}+1)$$

$$= 1-\frac{2}{e}$$

(4) $\int_1^e t^2\log t\,dt = \int_1^e \left(\frac{t^3}{3}\right)'\log t\,dt$

$\left(\frac{t^3}{3}\right)' = t^2$

$$= \left[\frac{t^3}{3}\log t\right]_1^e - \int_1^e \frac{t^3}{3}\cdot\frac{1}{t}\,dt$$

$$= \frac{e^3}{3} - \int_1^e \frac{t^2}{3}\,dt = \frac{e^3}{3} - \frac{1}{3}\left[\frac{t^3}{3}\right]_1^e$$

$$= \frac{e^3}{3} - \frac{1}{9}(e^3-1) = \frac{2}{9}e^3 + \frac{1}{9}$$

Ex.

(1) $\int_1^2 x\log(x+1)\,dx$ (2) $\int_0^1 t^2 e^t\,dt$

$$F'(x) = f(x)$$

$$\left(\frac{x^3}{3}\right)' = x^2$$

$$\frac{d}{dt}F(3t+2) = \frac{dF}{dx}\cdot\frac{dx}{dt}$$

$$= f(3t+2)\cdot 3$$

$$\left\{\frac{(3t+2)^3}{3}\right\}'$$

$$= (3t+2)^2\cdot 3.$$

$$\left\{\frac{1}{3}\cdot\frac{(3t+2)^3}{3}\right\}' = (3t+2)^2$$

$$\frac{d}{dt}\left(\frac{1}{3}F(3t+2)\right) = f(3t+2)$$

$$\int (3t+2)^2\,dt = \frac{1}{3}\cdot\frac{(3t+2)^2}{3} + C$$

$$\int f(3t+2)\,dt = \frac{1}{3}F(3t+2) + C$$

$a \neq 0$

$$\left(F(at+b)\right)' = af(at+b)$$

$$\left(\frac{1}{a}F(at+b)\right)' = f(at+b)$$

$$\int (2-3t)^3\,dt =$$

$$\int f(at+b)\,dt$$

$$= \frac{1}{a}F(at+b) + C$$

$$-\frac{1}{3}\cdot\frac{1}{4}(2-3t)^4 + C.$$

$\underset{\sim\sim}{x < 0}$ $\qquad \log|x| = \log(-x)$

$$(\log(-x))' = \frac{1}{-x} \cdot (-1) = \frac{1}{x}$$

$x > 0 \qquad \log|x| = \log x$

$$(\log x)' = \frac{1}{x}$$

$$\boxed{(\log|x|)' = \frac{1}{x}}$$

$$\int \frac{1}{x}\,dx = \log|x| + C$$

$$\int \frac{1}{2t+1}\,dt = \frac{1}{2}\log|2t+1| + C$$

$x = 2t+1$

$$\int \sqrt{t-2}\,dt = \frac{2}{3}(t-2)^{\frac{3}{2}} + C$$

$$\int \sqrt{x}\,dx = \frac{2}{3}x\sqrt{x} + C.$$

$$\left(x^{\frac{3}{2}}\right)' = \frac{3}{2}x^{\frac{1}{2}}$$

$$F'(x) = f(x).$$

$$\frac{d}{dt} F(x(t)) = \frac{dF}{dx} \cdot \frac{dx}{dt} = f(x(t)) \cdot x'(t)$$

$$\int_\alpha^\beta f(x(t))\, x'(t)\, dt = \Big[F(x(t)) \Big]_\alpha^\beta$$

$$= F(x(\beta)) - F(x(\alpha))$$

$a = x(\alpha)$

$$= F(b) - F(a)$$

$b = x(\beta)$

$$= \int_a^b f(x)\, dx.$$

$$\int_\alpha^\beta f(x(t))\, x'(t)\, dt = \int_a^b f(x)\, dx$$

$\longrightarrow$

$$\int_0^1 \frac{t}{1+t^2}\, dt = \int_0^1 \frac{\frac{1}{2}(1+t^2)'}{1+t^2}\, dt$$

$(1+t^2)' = 2t$

$$= \frac{1}{2} \int_0^1 \frac{(1+t^2)'}{1+t^2}\, dt$$

$x = 1+t^2$

t	$0 \nearrow 1$
x	$1 \nearrow 2$

$$= \frac{1}{2} \int_1^2 \frac{1}{x}\, dx.$$

$$= \frac{1}{2} \Big[\log x \Big]_1^2$$

$$= \frac{1}{2} \log 2.$$

$$\int_0^1 (2t+1)^3\,dt = \frac{1}{2}\int_0^1 (2t+1)^3 (2t+1)'\,dt$$

$x = 2t+1$

$(2t+1)'$

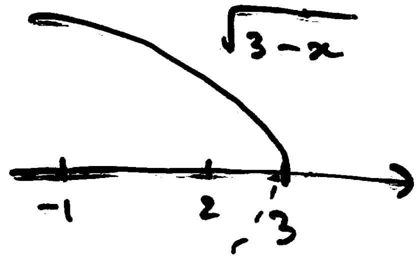
$$= \frac{1}{2}\int_1^3 x^3\,dx$$

t	$0 \nearrow 1$
x	$1 \nearrow 3$

$$= \frac{1}{2}\left[\frac{x^4}{4}\right]_1^3$$

$$= \frac{1}{8}(81-1) = 10$$

$$\int_{-1}^{2} \frac{t}{\sqrt{3-t}}\,dt$$



$x = 3-t \qquad \frac{dx}{dt} = (-1)$

$$\frac{t}{\sqrt{3-t}} = \frac{3-x}{\sqrt{x}}$$

t	$-1 \to 2$
x	$4 \to 1$

$$= -\int_{-1}^{2} \frac{t}{\sqrt{3-t}} \underbrace{\frac{dx}{dt}}_{=-1}\,dt$$

$$= -\int_4^1 \frac{3-x}{\sqrt{x}}\,dx$$

$$= \int_1^4 \frac{3-x}{\sqrt{x}}\,dx$$

$$= 3\int_1^4 \frac{1}{\sqrt{x}}\,dx - \int_1^4 \sqrt{x}\,dx$$

$$= 3\left[2\sqrt{x}\right]_1^4 - \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^4$$

$$= 3(2\cdot 2-2) - \frac{2}{3}(8-1)$$

$$= 6 - \frac{14}{3} = \frac{4}{3}$$

$$\left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\left(x^{\frac{3}{2}}\right)' = \frac{3}{2}\sqrt{x}$$

$$\int_1^2 e^{-2t}\,dt = \int_{-2}^{-4} e^{x}\left(-\frac{1}{2}\,dx\right)$$

$x = -2t$

$dx = -2\,dt.$

$dt = -\frac{1}{2}\,dx$

t	$1 \to 2$
x	$-2 \to -4$

$$= -\frac{1}{2}\int_{-2}^{-4} e^{x}\,dx.$$

$$= -\frac{1}{2}\left[e^{x}\right]_{-2}^{-4} \cdots\cdots$$

$$\int_0^1 \frac{x-1}{(2-x)^2}\,dx = \int_2^1 \frac{1-u}{u^2}\,(-du)$$

$u = 2-x. \rightsquigarrow x = 2-u$

$du = -dx. \rightsquigarrow dx = -du$

x	$0 \to 1$
u	$2 \to 1$

$$= \int_1^2 \left(\frac{1}{u^2} - \frac{1}{u}\right)du$$

$\left(\frac{1}{u}\right)' = -\frac{1}{u^2}$

$$= \left[-\frac{1}{u} - \log u\right]_1^2$$

$\left(-\frac{1}{u}\right)' = \frac{1}{u^2}$

$$= \left(-\frac{1}{2} - \log 2\right) - (-1 - 0)$$

$$= \frac{1}{2} - \log 2.$$

$$\int_1^2 x\sqrt{2-x}\,dx. = \int_1^0 (2-u^2)u(-2u\,du)$$

$u=\sqrt{2-x}$

$u^2 = 2-x.$

$x = 2-u^2$

$dx = -2u\,du$

$$\begin{array}{c|c} x & 1 \to 2 \\ \hline u & 1 \to 0 \end{array}$$

$$= 2\int_0^1 (2u^2 - u^4)\,du$$

$$= 2\left[\frac{2}{3}u^3 - \frac{u^5}{5}\right]_0^1$$

$$= 2\left(\frac{2}{3} - \frac{1}{5}\right) = \frac{14}{15}$$

$$S_N = \sum_{n=1}^{N} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{N} \longrightarrow +\infty$$

$$(N \to +\infty)$$

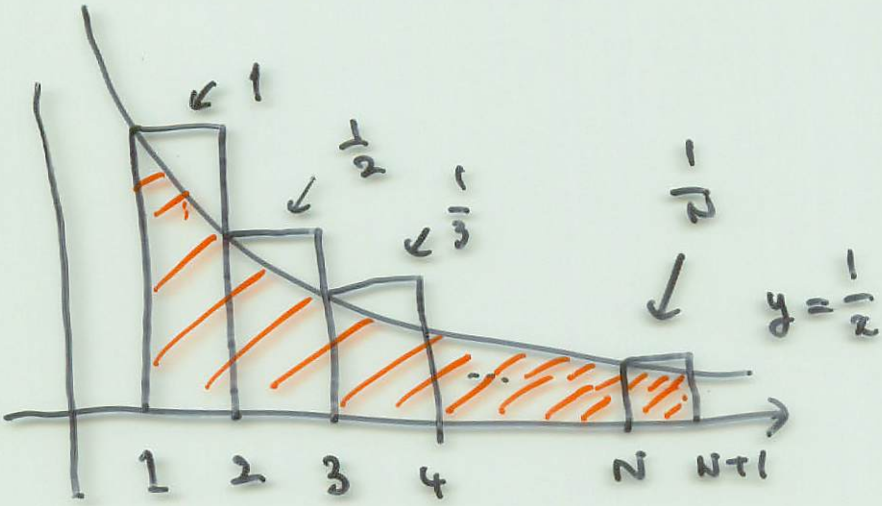
偶関数. 奇関数.

正規分布.

Taylor展開.

積分判定法　$\alpha > 0$

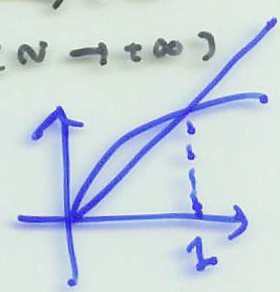
$$\sum_{n=1}^{+\infty} \frac{1}{n^\alpha} = \begin{cases} +\infty & (\alpha \leqq 1) \\ \text{収束} & (\alpha > 1) \end{cases}$$



$$1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{N} > \int_1^{N+1}\frac{1}{x}\,dx$$
$$=\Big[\log x\Big]_1^N$$

追い出しの原理.

$$=\log N \longrightarrow +\infty \quad (N\to+\infty)$$



定理 $a_n \leqq b_n$ (全てのn)

$n\to+\infty$ のとき $a_n\to+\infty$

$\Longrightarrow b_n \to +\infty$

n 自然数. $0<\alpha\leqq 1$ $\quad \frac{1}{n^\alpha} \geqq \frac{1}{n}$

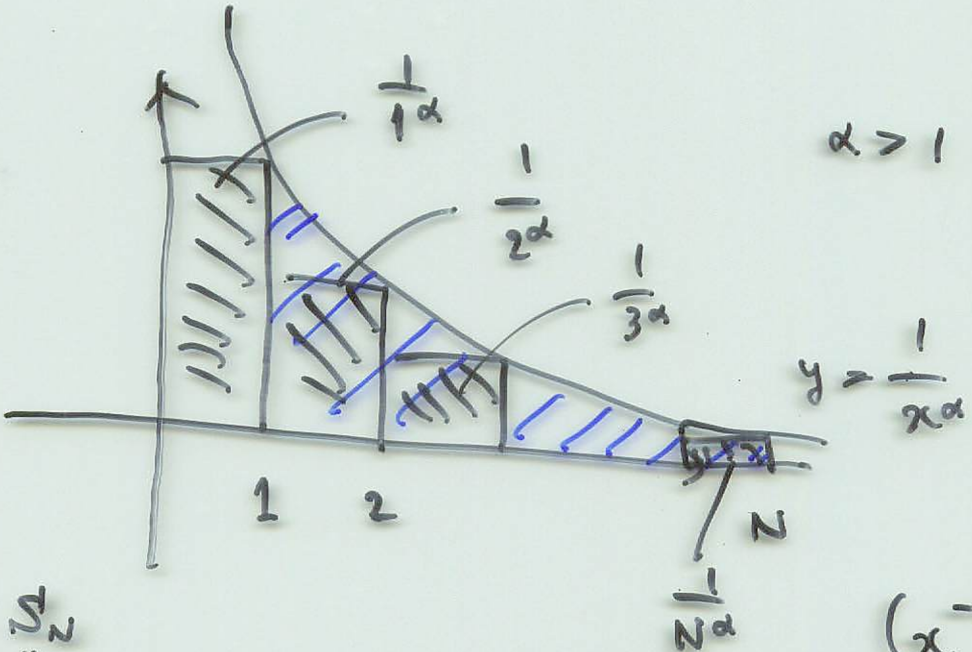
$x>1$ $\quad \sqrt{x} < x$

$\quad \frac{1}{x} < \frac{1}{\sqrt{x}}$

$0<\alpha\leqq 1$ のとき.

$$\underset{\downarrow\ +\infty}{\sum_{n=1}^{N}\frac{1}{n^\alpha}} \geqq \sum_{n=1}^{N}\frac{1}{n} \longrightarrow +\infty\ (N\to+\infty)$$

$\alpha > 1$



$$(x^{-\alpha+1})' = (-\alpha+1)x^{-\alpha}$$

$$S_N = \boxed{1 + \frac{1}{2^\alpha} + \cdots + \frac{1}{N^\alpha}} = 1 + \int_1^N \frac{1}{x^\alpha}\,dx$$

$$= 1 + \left[\frac{1}{1-\alpha}x^{-\alpha+1}\right]_1^N \qquad \alpha > 1$$

$$= 1 + \underset{<0}{\frac{1}{1-\alpha}}N^{-\alpha+1} - \frac{1}{1-\alpha}$$

$$= 1 + \frac{1}{\alpha-1} \underbrace{- \frac{1}{\alpha-1}N^{-\alpha+1}}_{<0} < 1 + \frac{1}{\alpha-1}$$

$$S_{N+1} = S_N + \underbrace{\frac{1}{(N+1)^\alpha}}_{>0} \qquad S_N < S_{N+1}$$

$\{S_N\}$ 収束 $\longleftarrow$ $\{S_N\}$：単調増加.

（実数の完備性）

$S_N < 1 + \frac{1}{\alpha-1}$ より有界.

準備　Cauchyの平均値の定理.

$f : [a, b] \longrightarrow \mathbb{R}$
$g : [a, b] \longrightarrow \mathbb{R}.$

$f, g : [a, b]$ で連続, (a, b) の各点で微分できる.

$g(b) - g(a) \neq 0$.　仮　$f'(x)$ と $g'(x)$ は同時に 0 になる事ない

$$\Rightarrow \quad \frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

を満たす c が a と b の間に存在.

$A = f(b) - f(a), \quad B = g(b) - g(a)$

$\varphi(x) = B(f(x) - f(a)) - A(g(x) - g(a))$

$\varphi(b) = \varphi(a) = 0.$　Rolleの定理.

$\varphi'(c) = (g(b) - g(a)) f'(c) - (f(b) - f(a)) \times g'(c) = 0$

を満たす c がある.

$$(\underbrace{g(b) - g(a)}_{\neq 0\ \text{仮定}}) f'(c) = (f(b) - f(a)) g'(c)$$

$g'(c) = 0 \Rightarrow f'(c) = 0$ となり
仮定に反する.
$g'(c) \neq 0$

定理 $f : (A, B) \longrightarrow \mathbb{R}$

∞階微分可能.

$$f(t) = f(a) + f'(a)(t-a) + \frac{f''(a)}{2!}(t-a)^2 + \cdots$$

$$\cdots + \frac{f^{(k)}(a)}{k!}(t-a)^k + \cdots + \frac{f^{(n-1)}(a)}{(n-1)!}(t-a)^{n-1}$$

$$+ \boxed{\frac{f^{(n)}(c)}{n!}(t-a)^n} \quad \text{剰余項.}$$

をみたす c が t と a の間に存在.

$$\begin{cases} F(t) = f(a) - f(t) - \displaystyle\sum_{k=1}^{n-1} f^{(k)}(t) \frac{(a-t)^k}{k!} \\ G(t) = (t-a)^n \end{cases}$$

に対して Cauchy の定理を使う.

例 $f(t) = e^t \quad f'(t) = e^t \quad \cdots$

$$f^{(k)}(t) = e^t$$

$a = 0 \qquad f^{(k)}(0) = 1.$

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots + \frac{t^{n-1}}{(n-1)!}$$

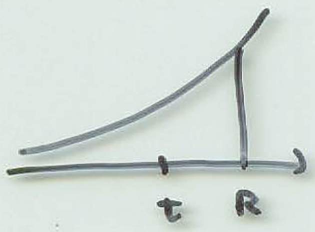
$$+ \frac{e^c}{n!} t^n.$$

をみたす c が 0 と t の間にある.

$$\left|\frac{e^{c_n}}{n!}t^n\right|$$

$$\leqq e^R \times \frac{R^n}{n!} \longrightarrow 0$$

$$(n \to +\infty)$$



$$\boxed{R>0 \text{ のとき } \frac{R^n}{n!} \longrightarrow 0}$$

N > R となる自然数をとる.

$n > N$

$$0 < \frac{R}{1}\cdot\frac{R}{2}\cdot\frac{R}{3}\cdots\cdots\frac{R}{N-1}\cdot\overset{\overset{1}{\vee}}{\frac{R}{N}}\cdot\overset{\overset{\frac{R}{N}}{\vee}}{\frac{R}{N+1}}\cdots\cdots\frac{R}{n}$$

$$< \frac{R^{N-1}}{(N-1)!}\cdot\left(\frac{R}{N}\right)^{n-N+1} \longrightarrow 0 \quad (n \to +\infty)$$

定数 $\qquad \frac{R}{N}$: 定数 $\quad 0 < \frac{R}{N} < 1$

$$e^t = 1 + t + \frac{1}{2!}t^2 + \frac{1}{3!}t^3 + \cdots\cdots$$

$$\boxed{e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots}$$

$e^{i\theta} = \cos\theta + i\sin\theta \quad$ Euler の公式.

$$\left(= 1 + i\theta + \frac{1}{2!}(i\theta)^2 + \frac{1}{3!}(i\theta)^3 + \cdots \right.$$

$$\boxed{\lim_{t\to+\infty} \frac{t^k}{e^t} = 0} \qquad k = 1, 2, 3, \cdots$$

$t > 0$

$$e^t = \underbrace{1 + t + \frac{1}{2!}t^2 + \cdots + \frac{t^k}{k!}}_{> 0} + \frac{t^{k+1}}{(k+1)!} + \underbrace{\cdots}_{> 0}$$

$$\longrightarrow \quad e^t > \frac{t^{k+1}}{(k+1)!}$$

$$0 < \frac{t^k}{e^t} < t^k \cdot \frac{(k+1)!}{t^{k+1}} = (k+1)!\,\frac{1}{t}$$

$0 \to 0$, $\frac{t^k}{e^t} \longrightarrow 0$, $\frac{1}{t} \to 0$

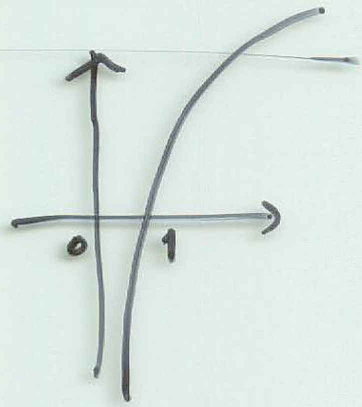
はさみうち.

Ⅰ $t \to +\infty$ $\quad s = \log t \longrightarrow +\infty$

$t = e^s$

$$\frac{\log t}{t} = \frac{s}{e^s} \longrightarrow 0$$

$$\frac{(\log t)^k}{t} = \frac{s^k}{e^s} \longrightarrow 0$$



Ⅱ $t \to +0$ のとき $\quad s = -\log t \longrightarrow +\infty$

$t = e^{-s}$

$$t \log t = e^{-s}(-s)$$

$$= -\frac{s}{e^s} \longrightarrow 0$$

$$\frac{\log t}{\frac{1}{t}}$$

$$t^{\frac{1}{3}} \log t = e^{-3s}(-s) = -\frac{s}{e^{3s}} = -\frac{1}{3} \times \frac{3s}{e^{3s}} \to 0$$

$3s \longrightarrow +\infty$

近似値　$f(t) = \sqrt{1+t}$.

$$f'(t) = \frac{1}{2}\frac{1}{\sqrt{1+t}}, \quad f''(t) = -\frac{1}{4}\frac{1}{(1+t)^{\frac{3}{2}}}$$

$f'(0) = \frac{1}{2}$.

$a = 0$

$$f^{(3)}(t) = \frac{3}{8}\frac{1}{(1+t)^{\frac{5}{2}}}$$

1次

$$\boxed{f(t) \fallingdotseq f(a) + f'(a)(t-a)}$$

$$\sqrt{1+t} \fallingdotseq 1 + \frac{1}{2}t$$

$$\boxed{f(t) \fallingdotseq f(a) + f'(a)(t-a) + \frac{f''(a)}{2!}(t-a)^2}$$

$$\sqrt{1+t} \fallingdotseq 1 + \frac{1}{2}t - \frac{1}{2}\cdot\frac{1}{4}t^2$$
$$= 1 + \frac{1}{2}t - \frac{1}{8}t^2.$$

$$\sqrt{1+t} \fallingdotseq 1 + \frac{1}{2}t - \frac{1}{8}t^2 + \frac{1}{6}\cdot\frac{3}{8}t^3$$
$$= 1 + \frac{1}{2}t - \frac{1}{8}t^2 + \frac{1}{16}t^3$$

Ex. $f(t) = (1+t)^{10}$.

$(1.1)^{10}$ の 1次近似, 2次近似, 3次近似

$(1.01)^{10}$ ______

1) $\int_0^2 x(x^2+1)^3\,dx$ $\qquad (x^2+1)' = 2x$

2) $\int_e^{e^2} \frac{dx}{x\log x}$ $\qquad u = \log x.$

$du = \frac{1}{x}\,dx$

3) $\int_0^1 \frac{e^x}{e^x+1}\,dx$ $\qquad (e^x+1)' = e^x.$

4) $\int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\,dx$ $\qquad (1-x^2)' = -2x.$

$u = 1-x^2$

$du = -2x\,dx.$

5) $\int_1^2 \frac{1}{e^x-1}\,dx$ $\qquad u = e^x-1$ とおく.

$x = \log(u+1)$

$dx = \frac{1}{u+1}\,du.$

6) $\int_1^2 \frac{dx}{x\sqrt{x+1}}$ $\qquad \sqrt{x+1} = u.$

7) $\int_0^2 (3x-1)^4\,dx$ $\qquad u = 3x-1.$

8) $\int_0^2 \sqrt{2-x}\,dx.$ $\qquad u = 2-x$

$v = \sqrt{2-x}.$

偶関数・奇関数．　ℝ上の関数

~~$f(-x) =$~~ 偶　$f(-x) = f(x)$　（全ての x）

奇　$f(-x) = -f(x)$　（ 〃 ）

偶

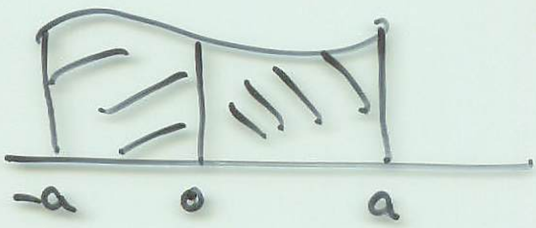
$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

$$= \int_{0}^{a} f(x)\,dx + \boxed{\int_{-a}^{0} f(x)\,dx}$$

$dx = -dy$

$x = -y.$

x	$0 \leftarrow -a$
y	$0 \leftarrow a$



$$\parallel$$

$$\int_{a}^{0} \underbrace{f(-y)}_{= f(y)} (-dy)$$

$$\parallel$$

$$+\int_{0}^{a} f(y)\,dy$$

$$= 2\int_{0}^{a} f(x)\,dx.$$

奇　$$\int_{-a}^{a} f(x)\,dx = 0$$

$$\int_{-a}^{0} f(x)\,dx = \int_{a}^{0} \overset{-f(y)}{\overset{\parallel}{f(-y)}}\,(-dy)$$

$$= -\int_{0}^{a} f(y)\,dy.$$

から分かる．

正規分布

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

$\sigma = 1,\ m = 0$

$$\boxed{\begin{aligned} &\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = 1. \\ &= \lim_{R \to +\infty} \int_{-R}^{R} \underline{\qquad\qquad} \end{aligned}}$$

$$\int_{-R}^{R} f(x)\,dx = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} \int_{-R}^{R} \exp\left(-\frac{(x-m)^2}{2\sigma}\right) dx$$

$$= \int_{\frac{-R-m}{\sigma}}^{\frac{R-m}{\sigma}} \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} e^{-\frac{y^2}{2}} \sigma\, dy.$$

$y = \dfrac{x-m}{\sigma}$

$x = m + \sigma y$

$dx = \sigma\, dy.$

$$= \int_{\frac{-R-m}{\sigma}}^{\frac{R-m}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy.$$

$R \to +\infty$

$$\to \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

$$= 1$$

X: 平均 m，標準偏差 σ の正規分布に従う.

$$P(a \le X \le b) = \int_a^b f(x)\,dx.$$

↑

確率密度関数.

$m=0,\ \sigma=1$ 標準正規分布.

$$\int_{-R}^{R} x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = 0$$

$$\to \quad E(X) = \int_{-\infty}^{+\infty} x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = 0$$

$$\int_{-R}^{R} x^2 \exp\left(-\frac{x^2}{2}\right) dx \qquad \left(e^{-\frac{x^2}{2}}\right)' = -x e^{-\frac{x^2}{2}}$$

$$= \int_{-R}^{R} (-x) \left(e^{-\frac{x^2}{2}}\right)' dx.$$

$$= \left[(-x) e^{-\frac{x^2}{2}} \right]_{-R}^{R} + \int_{-R}^{R} e^{-\frac{x^2}{2}} dx$$

$$= -2 R e^{-\frac{R^2}{2}} + \int_{-R}^{R} e^{-\frac{x^2}{2}} dx$$

$R \to \infty$ $\qquad Re^{-\frac{R^2}{2}} \to 0$

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} x^2 \exp\left(-\frac{x^2}{2}\right) dx = \boxed{\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx} \quad \overset{1}{〃}$$

$$\cancel{V(X) =} \ E(X^2) =$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= 1 - 0 = 1 \leadsto \text{標準偏差} \ 1$$

一般のfに対し

$$E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)\,dx.$$

$$E((X-m)^2) = \int_{-\infty}^{+\infty} (x-m)^2 f(x)\,dx$$

$$\shortparallel$$

$$V(X)$$

$y = \dfrac{x-m}{\sigma}$ と変数変換