(1) $y = e^{3t}$

$(e^t)' = e^t$

$y = e^x, \quad x = 3t$

$$\boxed{\begin{array}{l} c:\text{定数} \\ (e^{ct})' = c\,e^{ct} \end{array}}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = e^x \cdot 3 = 3e^{3t}.$$

($1\frac{1}{2}$) $y = e^{-t} \rightsquigarrow y' = -e^{-t}$.

(2) $y = e^{-t^2}$.

$y = e^x, \quad x = -t^2$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = e^x \cdot (-2t) = -2t\,e^{-t^2}.$$

(3) $y = e^{-\frac{t^2}{2}}$

$x = -\frac{t^2}{2}$

$$\frac{dy}{dt} = e^x \cdot (-t) = -t\,e^{-\frac{t^2}{2}}$$

$$y = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

標準正規分布の確率密度
期待0，標準偏差1.

$$y = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-\frac{(t-m)^2}{2\sigma^2}}$$

期待値 m
標準偏差 σ.

($3\frac{1}{2}$) $y = te^t$

$(fg)' = f'g + fg'$

$$y' = (t)'e^t + t \cdot (e^t)' = 1 \cdot e^t + t \cdot e^t = (1+t)e^t$$

(4) $y = t^2 e^{-t}$, $y' = (t^2)'e^{-t} + t^2(e^{-t})'$

$$= 2t\,e^{-t} + t^2(-e^{-t})$$
$$= (2t - t^2)e^{-t}$$

$$0.9999\cdots\cdots = 1.$$

$$(\text{左}) = 9 \times \sum_{n=0}^{+\infty} \frac{1}{10^{n+1}}$$

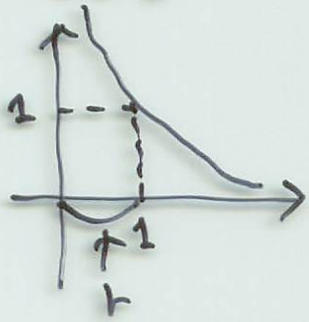
$$= \frac{9}{10} \times \sum_{n=0}^{+\infty} \left(\frac{1}{10}\right)^n = \frac{9}{10} \times \frac{1}{1-\frac{1}{10}} = \frac{9}{10} \times \frac{10}{9} = 1$$

$$\underbrace{0.9,\ 0.99,\ 0.999,\ \cdots\cdots}\quad \leqq 1.$$

単調増加かつ上に有界な数列.

$$\lim_{n\to\infty} n r^n = 0 \qquad (|r|<1)$$

(I) $r>0$ の場合. $\quad 0<r<1 \rightsquigarrow s>1$



$s = 1+\theta$ とおくと $\theta>0$

$$\frac{1}{r^n} = s^n = (1+\theta)^n$$

$$= 1 + n\theta + \frac{n(n-1)}{2}\theta^2 + \boxed{\cdots + \theta^n} > 0$$

$0<$ $\quad 0<$

$${}_nC_k\,\theta^k > 0$$

$$> \frac{n(n-1)}{2}\theta^2$$

$$0 < r^n < \frac{2}{n(n-1)}\cdot\frac{1}{\theta^2}$$

$$0 < nr^n < \frac{n}{n(n-1)}\cdot\frac{2}{\theta^2}$$

$$= \boxed{\frac{1}{n-1}\cdot\frac{1}{\theta^2}} \longrightarrow 0$$

0 $\quad$ はさみうち. $\quad 0$

N.B.

Noter Bien

$$\frac{1}{r^n} = (1+\theta)^n$$

$$= 1 + n\theta + \frac{n(n-1)}{2}\theta^2 + \frac{n(n-1)(n-2)}{3!}\theta^3 + \cdots + \theta^n$$

$$0 < n^2 r^n < n^2 \times \frac{1}{n(n-1)(n-2)} \times \frac{6}{\theta^3}$$

$$\lim_{n\to\infty} n^k r^n \longrightarrow \infty$$

$$(|r|<1,\ k=1,2,3,\cdots)$$

$R > 1.$

$$\lim_{n \to +\infty} R^n = +\infty$$

$\frac{1}{R} = r \rightsquigarrow 0 < r < 1.$

$$\frac{n^k}{R^n} = n^k r^n \longrightarrow 0$$

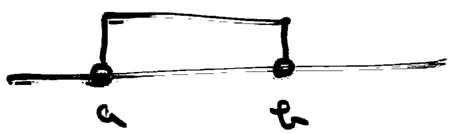
$\rightsquigarrow$ R^n の方が n^k より速く発散

$a_n \longrightarrow +\infty, \quad b_n \longrightarrow +\infty$

a_n の方が速く発散.

$\Longleftrightarrow \quad \frac{b_n}{a_n} \longrightarrow 0 \quad (n \to \infty)$

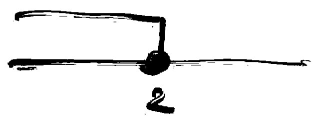
$(a, b) = \{ x ; \ a < x < b \}$ 開区間.



$[1, 2] = \{ x ; \ 1 \leqq x \leqq 2 \}$ 閉区間.

$(1, +\infty) = \{ 1 < x \}$

$(-\infty, 2] = \{ x \leqq 2 \}$



$\mathbb{R}$: 実数全体の集合.

X, Y : 集合.

$$f : \begin{array}{ccc} X & \longrightarrow & Y \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ x & \longmapsto & f(x) \end{array}$$

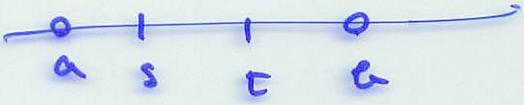
写像.

$Y = \mathbb{R}$ のとき 関数.

(a, b) 上 $f'(x) > 0$

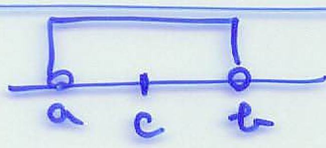
$\longrightarrow$ (a, b) 上 f は単調増加.

$s < t \Longrightarrow$



$f(s) < f(t)$

準備



定理 $f : (a, b) \longrightarrow \mathbb{R}$

$a < c < b$ とする.

$f(c)$ が極小値.
(大)
このよで最大・最小.

$\Longrightarrow f'(c) = 0$

$\delta > 0$ が存在して

小 $f(t) \geqq f(c)$

$(t \in (c-\delta, c+\delta))$

大 $f(t) \leqq f(c)$

$(t \in (c-\delta, c+\delta))$

極小の場合

$$\frac{f(t)-f(c)}{t-c} \longrightarrow f'(c)$$

$$(t \longrightarrow c)$$

① $t_n \longrightarrow c$ $c \longleftarrow t_n$

$c < t_n < b$

$$0 \leq \frac{\boxed{f(t_n)-f(c)}^{\geq 0}}{\underbrace{\boxed{t_n - c}}_{>0}} \qquad n \longrightarrow +\infty$$

$$\boxed{0 \leq f'(c)}$$

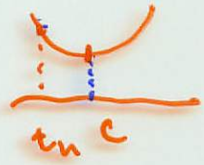
基本定理.

$$\boxed{\begin{array}{l} a_n \leq b_n. \\ \left.\begin{array}{l} a_n \longrightarrow \alpha \\ b_n \longrightarrow \beta \end{array}\right\} \Rightarrow \alpha \leq \beta \end{array}}$$

② $t_n \longrightarrow c$ とする.

$a < t_n < c$

$$\boxed{\frac{\boxed{f(t_n)-f(c)}^{\geq 0}}{\underbrace{\boxed{t_n - c}}_{<0}}} \leq 0$$



$n \longrightarrow +\infty$

$$\downarrow \qquad\qquad \downarrow$$

$$f'(c) \qquad\qquad 0$$

$$\boxed{f'(c) \leq 0}$$

ℝ IR IR

ℂ ℂ ℂ Black Board

ℤ ℤ Bold face

$\vec{a}$ $\mathbf{a}$

$$\boxed{\begin{array}{l} a_n \leq b_n \\ a_n \to \alpha,\ b_n \to \beta. \\ \Rightarrow \quad \alpha \leqq \beta \end{array}}$$

$$\uparrow$$

$$\boxed{\begin{array}{l} 0 \leq c_n \\ c_n \to \gamma \end{array} \Rightarrow \gamma \geqq 0}$$

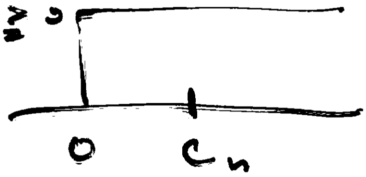
$c_n = b_n - a_n \geqq 0$

$\swarrow$

$\beta - \alpha = \gamma \geqq 0$

$\Downarrow$

$\beta \geqq \alpha$



$x'(t) = c\,x(t)$ をみたす [illegible]

$x(t) = C e^{ct}.$

Rolleの定理

$f:[a,b]\longrightarrow\mathbb{R}$

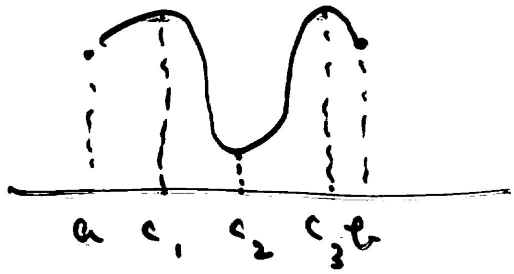
各点で連続, (a,b)の各点で微分可能.

$f(a)=f(b)$

$\Longrightarrow$ $f'(c)=0$ を満たす c が

(a,b) に存在する.

論理の[illegible]



Ⅰ $f(t)\equiv f(a)=f(b)$

$f'(t)=0$ ok.

Ⅱ fには最大値 M, 最小値 m がある.

(最大値の定理)

Ⅰ が成立しないとする.

(i) $M>f(a)=f(b)$ OR

(ii) $m<f(a)=f(b)$

が成立

(i)の場合. $M=f(c_1)\rightsquigarrow a<c_1<b$. となる

$\rightsquigarrow$ Mは極大値

$f'(c_1)=0$

(ii)の場合.

$m=f(c_2)\rightsquigarrow a<c_2<b$

$\rightsquigarrow$ mは極小値

$f'(c_2)=0$

定理（平均値の定理）

$f: [a, b] \longrightarrow \mathbb{R}$

各点で連続，(a, b) の各点で微分可能.

$$\Longrightarrow \frac{f(b)-f(a)}{b-a} = f'(c)$$

をみたす c が

(a, b) に存在.

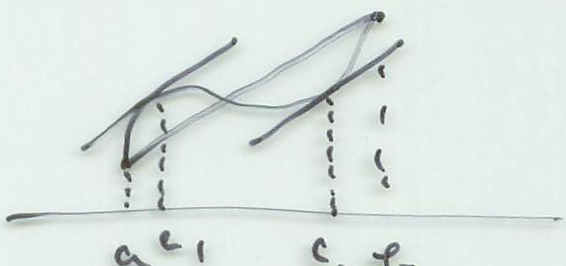
$$\varphi(x) = f(x) - \frac{f(b)-f(a)}{b-a}(\underset{\substack{\uparrow \\ a}}{x}-a)$$

$$\begin{aligned} &\varphi(a) = f(a) \\ &\ \ \parallel \\ &\varphi(b) = f(b) - (f(b)-f(a)) = f(a) \end{aligned}$$

$$\varphi'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$$

$\varphi'(c) = f'(c) - \dfrac{f(b)-f(a)}{b-a} = 0$ を満たす c がある.

(by Rolle)



a c_1 c_2 b

$f: (a, b) \longrightarrow \mathbb{R}$

(狭) 単調増加.　$a < s < t < b$

減少

$\Longrightarrow f(s) < f(t)$

$>$

(広) ———— $\Longrightarrow f(s) \leqq f(t)$

$\geqq$.

定理

$f'(t) > 0$ （$\geqq$） $t \in (a, b)$

$\Longrightarrow$ 狭義の単調増加

（広義）

$s < t$

$a \quad s \quad t \quad b$

$$\frac{f(t) - f(s)}{t - s} = f'(c) > 0 \ (\geqq)$$

$t - s > 0$

をみたす c がある $s < c < t$

$f(t) - f(s) > 0 \ (\geqq) \rightarrow f(t) > f(s) \ (\geqq)$

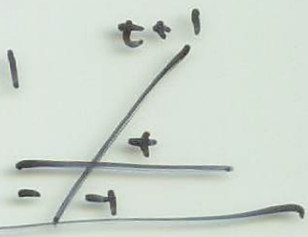
$y = e^t \qquad (e^t)' = e^t > 0.$

→ 狭義の単調増加.

$y = t e^t \qquad y' = (t)' e^t + t (e^t)'$

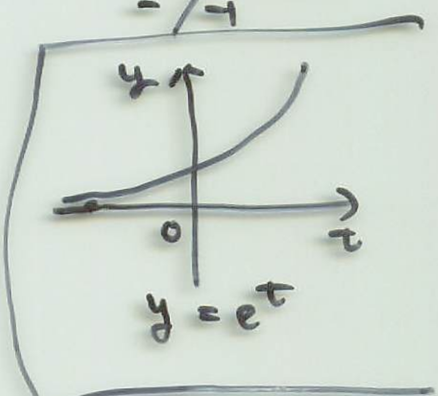
$\qquad\qquad\qquad = e^t + t e^t = (1 + t) e^t$ 　$e^t > 0$

$y' \gtreqless 0 \iff t + 1 \gtreqless 0 \iff t \gtreqless -1$



t		-1	
y'	$-$	0	$+$
y	↘	$-\frac{1}{e}$	↗

増減表.



$\lim_{t \to +\infty} t e^t = +\infty$

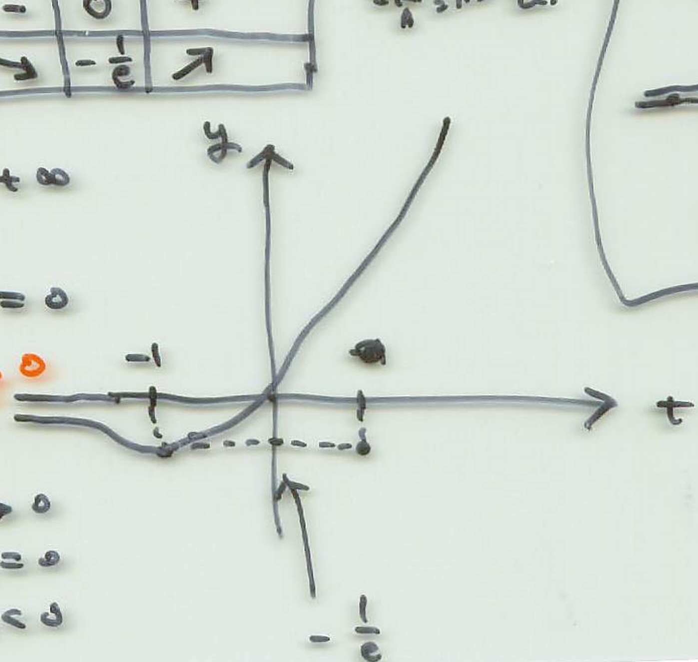
$\lim_{t \to -\infty} t e^t = 0$

$y = t e^t$ 　$e^t > 0$

$t > 0 \quad y > 0$

$t = 0 \quad y = 0$

$t < 0 \quad y < 0$



$1)\quad y = \dfrac{t}{1+t^2}$ $\qquad 2)\quad y = \dfrac{1}{1+t^2}$

$$\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$$

$$(2)\quad y' = \frac{-(1+t^2)'}{(1+t^2)^2} = \frac{-2t}{\boxed{(1+t^2)^2}} > 0$$

$$y' \gtreqless 0 \iff t \lesseqgtr 0$$

t		0	
y'	$+$	0	$-$
y	$\nearrow$	1	$\searrow$

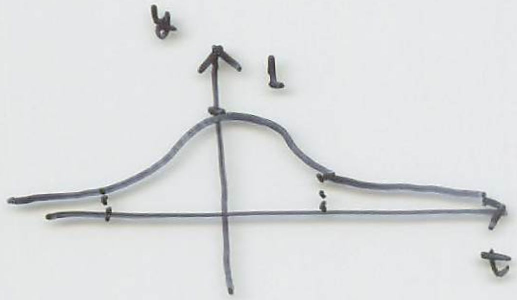
$t \to +\infty$ $\quad \dfrac{1}{t} \to 0$ $\qquad \dfrac{1}{1+t^2} = \dfrac{1}{t^2} \times \dfrac{1}{\frac{1}{t^2}+1} \to 0 \times \dfrac{1}{0+1}$

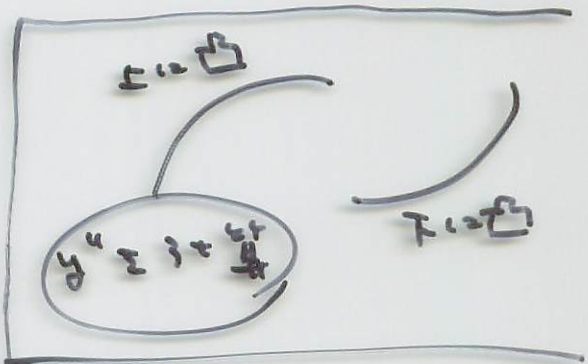
$a \geq 2$ $\quad \dfrac{1}{t^2} \to 0$ $\qquad = 0$

$\vdots$

$t \to -\infty$ $\quad \dfrac{1}{t} \to 0$ $\qquad \dfrac{1}{1+t^2} \to \dfrac{1}{t^2} \times \dfrac{1}{\frac{1}{t^2}+1} \to 0 \times \dfrac{1}{0+1}$

$= 0$





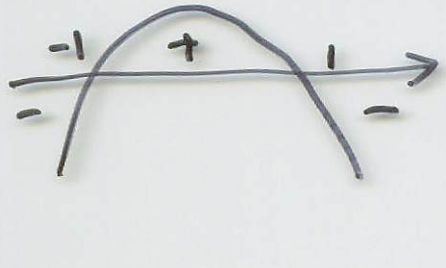
$$(1)\quad y = \frac{t}{1+t^2} \qquad y' = \frac{(t)'(1+t^2) - t(1+t^2)'}{(1+t^2)^2}$$

$$\left(\frac{g}{f}\right)' = \frac{g'f - gf'}{f^2}$$

$$= \frac{1\cdot(1+t^2) - t\cdot 2t}{(1+t^2)^2}$$

$1-t^2$ の符号.

$$= \frac{1-t^2}{\boxed{(1+t^2)^2}} > 0$$



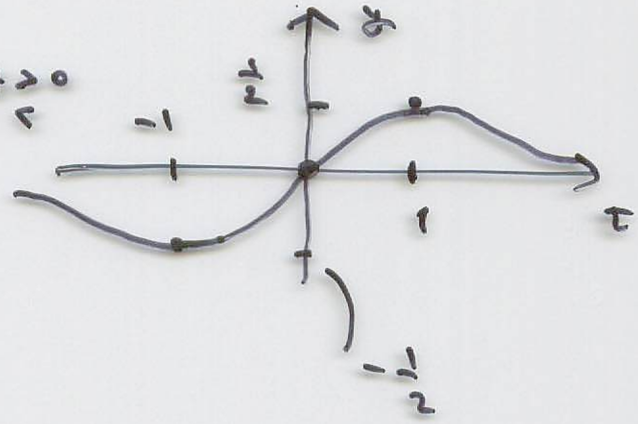
t		-1		1	
y'	$-$	0	$+$	0	$-$
y	$\searrow$	$-\frac{1}{2}$	$\nearrow$	$\frac{1}{2}$	$\searrow$

$$y = \frac{t}{1+t^2}$$

$1+t^2 > 0$

$t > 0 \iff y > 0$

$t < 0 \iff y < 0$



$$\frac{t}{1+t^2} = \frac{1}{t} \times \frac{t^2}{1+t^2} = \frac{1}{\frac{1}{t^2}+1} \times \frac{1}{t}$$

$t \longrightarrow \pm\infty$ のとき

$\frac{1}{t^2} \longrightarrow 0$

$$\longrightarrow \frac{1}{0+1} \times 0 = 0$$

$a \neq 1,\ a > 0$

対数関数の微分.

$y = \log_a x \quad (x > 0)$

$\left.\begin{array}{l} y = a^x. \\ y = \log_a x \end{array}\right\}$ $\quad 0 < a^y = x.$

常用対数.

$\log_{10} x.$

連続を仮定. $\boxed{a = e}$

$\log x = \log_e x.$

自然対数.

$\mathrm{Ln}\, x,\ \ln x.$

$\alpha > 0$

$$\frac{\log x - \log\alpha}{x - \alpha} = \frac{y-\beta}{e^y - e^\beta} = \frac{1}{\frac{e^y - e^\beta}{y-\beta}}$$

$\frac{1}{e^\beta} = \frac{1}{\alpha}$

$(e^y)'\Big|_{y=\beta} = e^\beta$

$\beta = \log\alpha \qquad x \longrightarrow \alpha$ のとき.

$y = \log x \qquad y \longrightarrow \log\alpha = \beta$

(対数関数の連続性)

$\alpha = e^\beta$

$$\boxed{(\log x)' = \frac{1}{x} \quad (x > 0)}$$

$$\boxed{(t^{\alpha})' = \alpha t^{\alpha-1}}$$

$t > 0$

$$(t^{\frac{1}{2}})' = \frac{1}{2} t^{-\frac{1}{2}}$$

$$(t^{-\frac{1}{2}})' = -\frac{1}{2} t^{-\frac{3}{2}}$$

$$y = t^{\alpha} = (e^{\log t})^{\alpha} \qquad t = e^{\log t}$$

$$= e^{\alpha \log t} = e^{x} \qquad x = \alpha \log t.$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \qquad (\log t)' = \frac{1}{t}$$

$$= e^{x} \cdot \left(\alpha \frac{1}{t}\right) = e^{\alpha \log t} \cdot \alpha \frac{1}{t}$$

$$= t^{\alpha} \cdot \alpha \frac{1}{t} = \alpha t^{\alpha-1}$$

$$(t^{\frac{1}{3}})' = \frac{1}{3} t^{-\frac{2}{3}}.$$

$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x$$

$$\int_{-\infty}^{+\infty} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi}$$

増減表. y'' を計算.

__Exo.__

(1) $y = t \log t$

(2) $y = \dfrac{\log t}{t}$

(3) $y = \sqrt{t}\,(t-1)$

(4) $y = e^{-\frac{t^2}{2}}$

(5) $y = t^2 e^{-t}$

$(e^{-t})' = -e^{-t}.$

(6) $y = x^3 - 3x + 1$

$$\frac{d}{dt} N(t) = -\gamma N(t)$$

γ : 崩壊係数.

$$N(t) = N(0) e^{-\gamma t}$$

$$e^{-\gamma T} = \frac{1}{2}.$$

$$-\gamma T = -\log 2.$$

$$T = \frac{\log 2}{\gamma}.$$

半減期.

$$N(t+T)$$

$$= N(0) e^{-\gamma (t+T)}$$

$$= \underbrace{N(0) e^{-\gamma t}}_{= N(t)} \times \underbrace{e^{-\gamma T}}_{= \frac{1}{2}}$$

$$= \frac{1}{2} N(t)$$

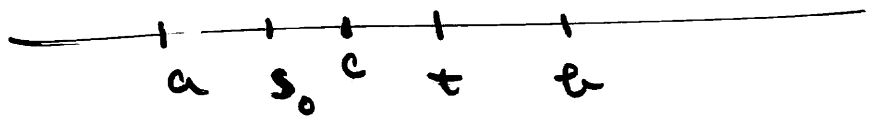
平均値の定理の応用

$f : (a, b) \longrightarrow \mathbb{R}$

各点で微分可能.

$f'(t) \equiv 0$

$\Longrightarrow \quad f(t) \equiv C$



$$\frac{f(t) - f(s_0)}{t - s_0} = f'(c) = 0$$

$$f(t) = f(s_0)$$

応用 $f : \mathbb{R} \longrightarrow \mathbb{R}$ $\quad \alpha \in \mathbb{R}$ 定数

$f'(t) = \alpha f(t)$ が $t \in \mathbb{R}$ の各点で OK.

$\Longrightarrow \quad f(t) = f(0) e^{\alpha t}$

$(e^{ct})' = c e^{ct}$

$$\begin{aligned}
(f(t) e^{-\alpha t})' &= f'(t) \cdot e^{-\alpha t} + f(t) (e^{-\alpha t})' \\
&= \underbrace{f'(t)}_{= \alpha f(t)} e^{-\alpha t} + (-\alpha) f(t) e^{-\alpha t} \\
&= \alpha f(t) e^{-\alpha t} - \alpha f(t) e^{-\alpha t} \\
&\equiv 0
\end{aligned}$$

$$f(t) e^{-\alpha t} = f(0) \cdot e^{-\alpha 0} = f(0)$$

$$\rightsquigarrow \quad f(t) = f(0) e^{\alpha t}$$