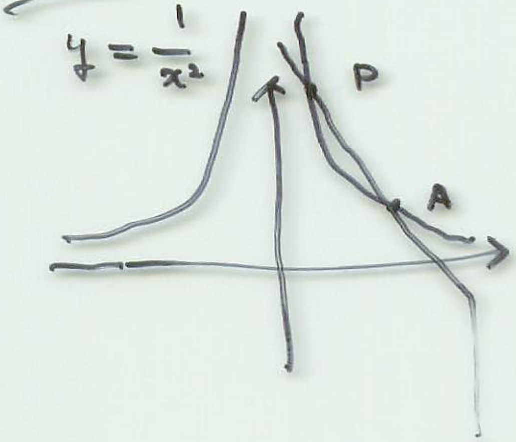
(1

(1) $\dfrac{3+n}{1+2n} = \dfrac{\frac{3}{n}+1}{\frac{1}{n}+2} \longrightarrow \dfrac{0+1}{0+2} = \dfrac{1}{2}$

(2) $\dfrac{2-n+3n^2}{1+n+n^2} = \dfrac{2/n^2 - 1/n + 3}{1/n^2 + 1/n + 1} \longrightarrow \dfrac{0-0+3}{0+0+1}$

$= 3$



$$\frac{\frac{1}{x^2} - \frac{1}{\alpha^2}}{x-\alpha} = \frac{\alpha^2 - x^2}{x^2\alpha^2(x-\alpha)}$$

$$= - \frac{\cancel{(x-\alpha)}(x+\alpha)}{x^2\alpha^2\cancel{(x-\alpha)}}$$

$$= -\frac{1}{\alpha^2}\cdot\frac{1}{x^2}\cdot(x+\alpha)$$

$x_n \longrightarrow \alpha \quad (n \to +\infty)$

をみたす数列をとる.

$$-\frac{1}{\alpha^2}\cdot\frac{1}{x_n^2}\cdot(x_n+\alpha) \longrightarrow -\frac{1}{\alpha^2}\cdot\frac{1}{\alpha^2}\cdot(\alpha+\alpha)$$

$$= -\frac{2\alpha}{\alpha^4} = -\frac{2}{\alpha^3}$$

$\dfrac{1}{x^n} = x^{-n}$. $\quad \boxed{(x^{-n})' = -n\,x^{-n-1}}$

$(x^{-2})' = -2x^{-3}$ を意味する

Exo (1) $y = \dfrac{1}{1+x}$ $\quad \dfrac{\frac{1}{1+x} - \frac{1}{1+\alpha}}{x-\alpha}$

$x \to \alpha$ のとき

$1+x \to 1+\alpha$

$\dfrac{1}{1+x} \to \dfrac{1}{1+\alpha}$

$$= \frac{(1+\alpha)-(1+x)}{(1+x)(1+\alpha)(x-\alpha)} = \frac{\alpha - x}{(1+x)(1+\alpha)(x-\alpha)}$$

$$= -\frac{1}{(1+x)} \times \frac{1}{1+\alpha} \longrightarrow -\frac{1}{(1+\alpha)^2}$$

2) $y = \dfrac{1}{(x-1)^2}$

$$\frac{\dfrac{1}{(x-1)^2} - \dfrac{1}{(\alpha-1)^2}}{x-\alpha} = \frac{(\alpha-1)^2 - (x-1)^2}{(x-1)^2(\alpha-1)^2(x-\alpha)}$$

$$= \frac{\cancel{(\alpha-x)}(\alpha+x-2)}{(x-1)^2(\alpha-1)^2\cancel{(x-\alpha)}}$$

$$= -\frac{\alpha+x-2}{(x-1)^2(\alpha-1)^2}$$

$A^2 - B^2$
$= (A-B)(A+B)$

$$\to -\frac{2\alpha-2}{(\alpha-1)^2\cdot(\alpha-1)^2}$$

$$\boxed{\begin{array}{l} x \to \alpha \text{ のとき} \\ (x-1)^2 \to (\alpha-1)^2 \end{array}}$$

$$= -\frac{2}{(\alpha-1)^3}$$

---

3) $y = \dfrac{1}{1+x^2}$

$$\frac{\dfrac{1}{1+x^2} - \dfrac{1}{1+\alpha^2}}{x-\alpha} = \frac{(1+\alpha^2)-(1+x^2)}{(1+x^2)(1+\alpha^2)(x-\alpha)}$$

$$= \frac{(\alpha-x)(\alpha+x)}{(1+x^2)(1+\alpha^2)(x-\alpha)} = -\frac{\alpha+x}{(1+x^2)(1+\alpha^2)}$$

$$\to -\frac{2\alpha}{(1+\alpha^2)(1+\alpha^2)}$$

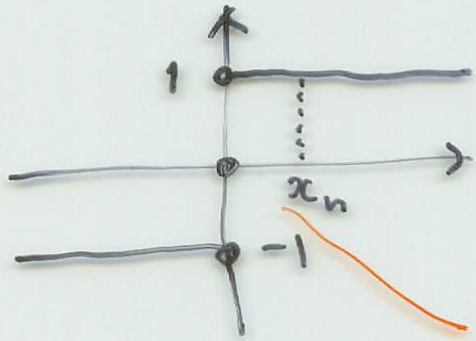
$$= -\frac{2\alpha}{(1+\alpha^2)^2}$$

Leibnitz の公式.　　Euler …　　3

$$(fg)' = f'g + fg'$$

準備　$f(x) \longrightarrow f(a) \quad (x \to a)$

$\overset{\Longleftrightarrow}{\text{定ぎ}}$　x は a で連続.



$x_n = \frac{1}{n} \to 0$

$f(x_n) = 1 \to 1$

$\nparallel$

$f(0) = 1$

$x = 0$ で不連続.

定理　$f(x)$ が $x = a$ で微分できる.

$\left(\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a} \text{ が存在} \right)$

$\Rightarrow$ $f(x)$ は $x = a$ で連続

$f(x) \to f(a) \quad (x \to a)$

$$f(x) - f(a) = \frac{f(x) - f(a)}{x - a} \times (x - a)$$

$x \to a$ とする

$$f(x) = f(a) + \boxed{\frac{f(x) - f(a)}{x - a}} \times \boxed{(x - a)}$$

$$\longrightarrow f(a) + f'(a) \times 0 = f(a)$$

$$\mathrm{I}\quad (f(x) \pm g(x))' = f'(x) \pm g'(x)$$

$$\mathrm{II}\quad (f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

$$\mathrm{III}\quad \left(\frac{1}{f(x)}\right)' = \frac{-f'(x)}{(f(x))^2}$$

IIの証明.

$$\frac{f(x)g(x) - f(\alpha)g(\alpha)}{x - \alpha} = \cancel{f(x)g(x) - f(\alpha)g(x)}$$

$$= \frac{f(x)g(x) - f(\alpha)g(x) + f(\alpha)g(x) - f(\alpha)g(\alpha)}{x - \alpha}$$

$$= g(x)\,\frac{f(x) - f(\alpha)}{x - \alpha} + f(\alpha)\,\frac{g(x) - g(\alpha)}{x - \alpha}$$

$$\Longrightarrow g(\alpha)\cdot f'(\alpha) + f(\alpha)g'(\alpha)$$

III.

$$\frac{\dfrac{1}{f(x)} - \dfrac{1}{f(\alpha)}}{x - \alpha} = \frac{f(\alpha) - f(x)}{f(x)f(\alpha)(x - \alpha)}$$

$$= -\frac{1}{f(\alpha)}\cdot\frac{1}{f(x)}\cdot\frac{f(x) - f(\alpha)}{x - \alpha}$$

$$\rightarrow -\frac{1}{f(\alpha)}\cdot\frac{1}{f(\alpha)}\cdot f'(\alpha) = -\frac{f'(\alpha)}{(f(\alpha))^2}$$

5

$$(x^n)' = n x^{n-1} \quad (n = 1, 2, 3, \cdots)$$

$$(c)' = 0$$

$$(a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0)'$$

$$= a_n \cdot n x^{n-1} + a_{n-1}(n-1)x^{n-1} + \cdots + a_1 + 0$$

$$\boxed{\begin{aligned}(c f(x))' &= \overset{=0}{\boxed{(c)'}} f(x) + c f'(x) \\ &= c f'(x)\end{aligned}}$$

$$\left(\frac{1}{x^n}\right)' = \frac{-(x^n)'}{(x^n)^2} = \frac{-n x^{n-1}}{x^{2n}}$$

$$= -\frac{n}{x^{n+1}}$$

$$\boxed{(x^{-n})' = -n x^{-n-1}}$$

$$(x^n)' = n x^{n-1}. \quad (n = 1, 2, 3, \cdots)$$

数学的帰納法で

$n=1$ ok. $(x)' = 1$.

$n=k$ ok とする. $(x^k)' = k x^{k-1}$ を仮定.

$$(x^{k+1})' = (x^k \cdot x)' = (x^k)' x + x^k \cdot (x)'$$

$$= k x^{k-1} \cdot x + x^k \cdot 1$$

$$= k x^k + x^k = (k+1) x^k$$

$x^{-\frac{1}{2}}$

$$(x^\alpha)' = \alpha x^{\alpha - 1}$$

9 13 ちくま新書

1) $y = \dfrac{1}{1+x} \qquad y' = \dfrac{-(1+x)'}{(1+x)^2} = -\dfrac{1}{(1+x^{\cancel{2}})^2}$

2) $y = \dfrac{1}{1+x^2} \qquad y' = \dfrac{-(1+x^2)'}{(1+x^2)^2} = -\dfrac{2x}{(1+x^2)^2}$

$(\sqrt{x})' = \dfrac{1}{2}\dfrac{1}{\sqrt{x}}.$ $\quad (\sqrt{x}) = x^{\frac{1}{2}},\ \dfrac{1}{\sqrt{x}} = x^{-\frac{1}{2}},\ x\sqrt{x} = x^{\frac{3}{2}}$

1) $\left(\dfrac{1}{\sqrt{x}}\right)' = \dfrac{-(\sqrt{x})'}{(\sqrt{x})^2} = \dfrac{-\frac{1}{2}\frac{1}{\sqrt{x}}}{x} = -\dfrac{1}{2}\cdot\dfrac{1}{x\sqrt{x}}$

$\left(x^{-\frac{1}{2}}\right)' = -\dfrac{1}{2}\cdot x^{-\frac{3}{2}}$

$-\dfrac{3}{2} = -\dfrac{1}{2}-1$

2) $(x\sqrt{x})' = (x)'\sqrt{x} + x(\sqrt{x})' = \sqrt{x} + x\cdot\dfrac{1}{2}\dfrac{1}{\sqrt{x}}$

$= \sqrt{x} + \dfrac{1}{2}\sqrt{x} = \dfrac{3}{2}\sqrt{x}$

3) $\dfrac{1}{(x\sqrt{x})'} =$

$\left(x^{\frac{3}{2}}\right)' = \dfrac{3}{2}x^{\frac{1}{2}}$

4) $\left(\dfrac{1}{3+x}\right)' = \dfrac{-(3+x)'}{(3+x)^2} = -\dfrac{1}{(3+x)^2}$

5) $\left(\dfrac{1}{1+x+x^2}\right)' = \dfrac{-(1+x+x^2)'}{(1+x+x^2)^2} = \dfrac{-1-2x}{(1+x+x^2)^2}$

6) $\left(\dfrac{g}{f}\right)' = \left(g\cdot\dfrac{1}{f}\right)' =$

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3) $\left(\dfrac{1}{x\sqrt{x}}\right)' = \dfrac{-(x\sqrt{x})'}{(x\sqrt{x})^2} = \dfrac{-\frac{3}{2}\sqrt{x}}{x^3}$

$= -\dfrac{3}{2}\cdot x^{-3}x^{\frac{1}{2}}$

$= -\dfrac{3}{2}x^{-3+\frac{1}{2}}$

$= -\dfrac{3}{2}x^{-\frac{5}{2}}$

$\left(x^{-\frac{3}{2}}\right)' = -\dfrac{3}{2}x^{-\frac{5}{2}}$

6) $\left(\frac{g}{f}\right)' = \left(g \times \frac{1}{f}\right)' = g' \cdot \frac{1}{f} + g \cdot \left(\frac{1}{f}\right)'$

$$= \frac{g'}{f} + g \cdot \frac{-f'}{f^2} = \frac{fg' - gf'}{f^2}$$

<u>公式</u> $\left(\frac{g}{f}\right)' = \frac{g'f - gf'}{f^2}$

$y = (x-1)^{10}$ $t = x-1,\ \beta = \alpha - 1$ とおく

$$\frac{(x-1)^{10} - (\alpha-1)^{10}}{x - \alpha} = \frac{t^{10} - \beta^{10}}{(t+1) - (\beta+1)}$$

$x \to \alpha$ のとき
$t \to \alpha - 1 = \beta$.

$$= \underbrace{\frac{t^{10} - \beta^{10}}{t - \beta}}_{t^{10}\text{の平均変化率}} \longrightarrow 10\beta^9 = 10(\alpha-1)^9$$

$$\{(x-1)^{10}\}' = 10(x-1)^9$$

変数変換の公式 + e の話 $e^t \cdots \log_e t$
Nepier の数

$y = f(x)$, $x = x(t)$.

$y = f(x(t))$ 合成関数

$y = \sqrt{x}$, $x = 1+t^2 \rightsquigarrow y = \sqrt{1+t^2}$

$\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt}$ と示す.

$\dfrac{\Delta y}{\Delta t}$

$y = \sqrt{1+t^2}$ $\quad \dfrac{dy}{dt} = \dfrac{d\sqrt{x}}{dx} \cdot \dfrac{d(1+t^2)}{dt}$

$= \dfrac{1}{2}\dfrac{1}{\sqrt{x}} \cdot 2t$

$= \dfrac{1}{2}\dfrac{1}{\sqrt{1+t^2}} \cdot 2t = \dfrac{t}{\sqrt{1+t^2}}$

$y = \sqrt{t+1}$ $\qquad s = t+1,\ \beta = \alpha+1$

$$\frac{\sqrt{t+1} - \sqrt{\alpha+1}}{t-\alpha} = \frac{\sqrt{s} - \sqrt{\beta}}{s-\beta} \cdot \overset{〃\ 1}{\boxed{\frac{(t+1)-(\alpha+1)}{t-\alpha}}}$$

$t \to \alpha$ のとき.

$s \to \alpha+1 = \beta$

$\longrightarrow$ $\sqrt{s}$ の $s=\beta$ の微分係数

$= \dfrac{1}{2}\dfrac{1}{\sqrt{\beta}} = \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{\alpha+1}}$.

$(\sqrt{s})' = \dfrac{1}{2}\dfrac{1}{\sqrt{s}}$

$$\boxed{(\sqrt{t+1})' = \frac{1}{2}\frac{1}{\sqrt{t+1}}}$$

$$y = \frac{1}{(2x-1)^3}$$

$u = 2x-1$

$x \to \alpha$ のとき

$u \to 2\alpha - 1 = \beta$

$$\frac{\dfrac{1}{(2x-1)^3} - \dfrac{1}{(2\alpha-1)^3}}{x-\alpha}$$

$$= \frac{\dfrac{1}{u^3} - \dfrac{1}{\beta^3}}{u-\beta} \cdot \frac{(2x-1)-(2\alpha-1)}{x-\alpha}$$

$$\left(\frac{1}{u^3}\right)' = -\frac{3}{u^4}$$

$\longrightarrow$ ($\dfrac{1}{u^3}$ の $u=\beta$ における微分係数) $\times$ ($(2x-1)$ の $x=\alpha$ における微分係数……)

$$= -\frac{3}{\beta^4} \times 2 = -\frac{6}{(2\alpha-1)^4}$$

$$y = \frac{1}{(1+t^2)^4}$$

$u = 1+t^2 \longrightarrow 1+\alpha^2 = \beta$

$t \to \alpha$ のとき

$$\frac{\dfrac{1}{(1+t^2)^4} - \dfrac{1}{(1+\alpha^2)^4}}{t-\alpha} = \frac{\dfrac{1}{u^4} - \dfrac{1}{\beta^4}}{u-\beta} \cdot \frac{(1+t^2)-(1+\alpha^2)}{t-\alpha}$$

$$\longrightarrow \left(\frac{1}{u^4}\right)'\Bigg|_{u=\beta} \cdot (1+t^2)'\Big|_{t=\alpha}$$

$$= -\frac{4}{u^5}\Bigg|_{u=\beta} \cdot 2t\Big|_{t=\alpha}$$

$$= -\frac{4}{\beta^5} \times 2\alpha = -\frac{8\alpha}{(1+\alpha^2)^5}$$

$y = f(x), \quad x = x(t)$

$$\frac{f(x(t)) - f(x(\alpha))}{t - \alpha} = \frac{f(x) - f(A)}{x - A} \cdot \frac{x(t) - x(\alpha)}{t - \alpha}$$

$t \to \alpha$ のとき $\longrightarrow f'(A) \cdot x'(\alpha)$

$x(t) \longrightarrow x(\alpha) = A$

$(t \to \alpha)$

公式 $\quad \dfrac{d}{dt} f(x(t)) = f'(x(t)) \cdot x'(t)$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$y = \sqrt{1+t^2} \qquad y = \sqrt{x}. \quad x = 1 + t^2$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{1}{2} \frac{1}{\sqrt{x}} \cdot 2t = \frac{t}{\sqrt{1+t^2}}.$$

$y = \dfrac{1}{(1+t+t^2)^{10}} \qquad y = \dfrac{1}{x^{10}}. \qquad x = 1 + t + t^2$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = -\frac{10}{x^{11}} \cdot (1+2t)$$

$$= -\frac{10(1+2t)}{(1+t+t^2)^{11}}$$

$y = \dfrac{1}{\sqrt{1+t+t^2}} \qquad y = \dfrac{1}{\sqrt{x}}, \quad x = 1 + t + t^2.$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = -\frac{1}{2} x^{-\frac{3}{2}} \cdot (1+2t)$$

$\left(x^{-\frac{1}{2}}\right)' = -\frac{1}{2} x^{-\frac{3}{2}} \qquad = -\dfrac{1+2t}{2(1+t+t^2)^{\frac{3}{2}}}$

1) $y = (3t-1)^{10}$

2) $y = \sqrt{2t + 1}$

3) $y = \dfrac{1}{(2t+1)^3}$

4) $y = \dfrac{1}{(1+t+t^2)^2}$

5) $y = \left(\dfrac{t}{t-1}\right)^5$ $\qquad x = \dfrac{t}{t-1}$

$(t^n)' = n\, t^{n-1}$

$(t^{-n})' = -n\, t^{-n-1}$

$(t^{\frac{1}{2}})' = \frac{1}{2} t^{-\frac{1}{2}}$ $\dashrightarrow (t^{-\frac{1}{2}})'$

$(t^{\frac{3}{2}})'$ etc.

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1) $y = (3t-1)^{10}$ $\qquad y = x^{10}$, $x = 3t-1$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = 10x^9 \cdot 3 = 30(3t-1)^9$$

2) $y = \sqrt{2t+1}$ $\qquad y = \sqrt{x}$, $x = 2t+1$

$$\frac{dy}{dt} = \frac{1}{2}\frac{1}{\sqrt{x}} \cdot 2 = \frac{1}{\sqrt{2t+1}}$$

3) $y = \dfrac{1}{(2t+1)^3}$ $\qquad y = \dfrac{1}{x^3}$, $x = 2t+1$

$$\frac{dy}{dt} = -\frac{3}{x^4} \cdot 2 = -\frac{6}{(2t+1)^4}$$

4) $y = \dfrac{1}{(1+t+t^2)^2}$ $y = \dfrac{1}{x^2}$, $x = 1+t+t^2$

$$\frac{dy}{dt} = -\frac{2}{x^3}\cdot(1+2t) = -\frac{2(2t+1)}{(1+t+t^2)^3}$$

5) $y = \left(\dfrac{t}{t-1}\right)^5$ $y = x^5$, $x = \dfrac{t}{t-1}$

$$\begin{aligned}
\frac{dy}{dt} &= 5\left(\frac{t}{t-1}\right)^4\cdot\left(\frac{t}{t-1}\right)' \\
&= 5\left(\frac{t}{t-1}\right)^4\cdot\frac{(t)'(t-1)-t(t-1)'}{(t-1)^2} \\
&= 5\left(\frac{t}{t-1}\right)^4\cdot\frac{(t-1)-t}{(t-1)^2} \\
&= -5\left(\frac{t}{t-1}\right)^4\cdot\frac{1}{(t-1)^2}
\end{aligned}$$

$$(1+\theta)^n = 1 + n\theta + \frac{n(n-1)}{2!}\theta^2 + \frac{n(n-1)(n-2)}{3!}\theta^3 + \cdots + \boxed{\frac{n(n-1)\cdots(n-k+1)}{k!}}\,\theta^k + \cdots + \theta^n$$

$$= {}_nC_k = \frac{n!}{(n-k)!\,k!}$$

$$\begin{aligned}
&\frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}\cdot\frac{1}{n^k} \\
&= \frac{1}{k!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{k-1}{n}\right) \\
&< \frac{1}{k!}\left(1-\frac{1}{n+1}\right)\left(1-\frac{2}{n+1}\right)\cdots\left(1-\frac{k-1}{n+1}\right)
\end{aligned}$$

$$a_n = \left(1+\frac{1}{n}\right)^n$$

I $\quad a_1 < a_2 < a_3 < \cdots$

II $\quad a_n \leqq 3.$

$$\boxed{k! \geqq 2^{k-1}}$$

$$k(k-1)(k-2)\cdots 3\cdot 2\cdot 1$$

$$\vee\|$$

$$2\cdot 2\cdot 2\cdot \cdots 2\cdot 2\cdot 1 = 2^{k-1}$$

$$a_n \leqq 1+1+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}$$

$$\leqq 1+1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots+\frac{1}{2^{n-1}}$$

$$\leqq 1+\sum_{n=0}^{+\infty}\left(\frac{1}{2}\right)^n = 1+\frac{1}{1-\frac{1}{2}} = 3.$$

$|r|<1$

$$1+r+r^2+\cdots = \frac{1}{1-r}$$

$$e = \lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n$$

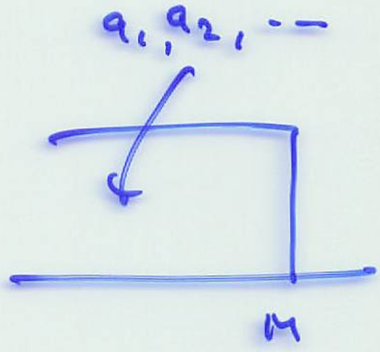
定理 数列 $\{a_n\}$ が

1) $a_1 \leqq a_2 \leqq a_3 \leqq \cdots$

(上に有界)

2) ある M が存在して

$$a_n \leqq M$$



$\Rightarrow$ $\lim_{n \to +\infty} a_n$ が存在する.

$0.9999 \cdots$

$0.\, a_1 a_2 a_3 \cdots$

$a_j = 0, 1, 2, 3, \cdots, 9$

$x_n = 0.\, a_1 a_2 \cdots a_n$

$\{x_n\}$ 単調増加. $\rightsquigarrow$ $\lim_{n \to \infty} x_n$ が存在.

$x_n \leqq 1$ (上に有界)

$$\lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^n = e \rightsquigarrow \lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

↑

ポイント.

$y = e^t$ の微分を.

$x - \alpha = h$ とおく. $x = \alpha + h$

$$\frac{e^x - e^\alpha}{x - \alpha} = \frac{e^{\alpha+h} - e^\alpha}{h}$$

$x \to \alpha$ のとき $h \to 0$

$$= e^\alpha \frac{e^h - 1}{h} \quad \frac{e^h - 1}{h} \to 1$$

$$\to e^\alpha$$

$$(e^t)' = e^t.$$

---

ex. 1) $(e^{3t})'$ $\quad 1\frac{1}{2}$) $(e^{-t})'$

2) $(e^{-t^2})'$ $\quad$ 3) $\left(e^{-\frac{t^2}{2}}\right)'$

$3\frac{1}{2}$) $(t e^t)'$ $\quad$ 4) $(t^2 e^{-t})'$

---

ex. $0.9999999\cdots = 1.$

ex. $9 \times \frac{1}{10} + 9 \times \frac{1}{10^2} + \cdots + 9 \times \frac{1}{10^n} \longrightarrow$ 極限.

$\frac{1}{3} = 0.333\cdots\cdots \rightsquigarrow 1 = 0.999\cdots$