04年7月31日 (1回目)

1変数の微分・積分.

時間があれば偏微分.

数学はどのように使う.

http://www.math.hc.keio.ac.jp/tose/

Google で「戸瀬信之」

→ 講義 → 微分積分

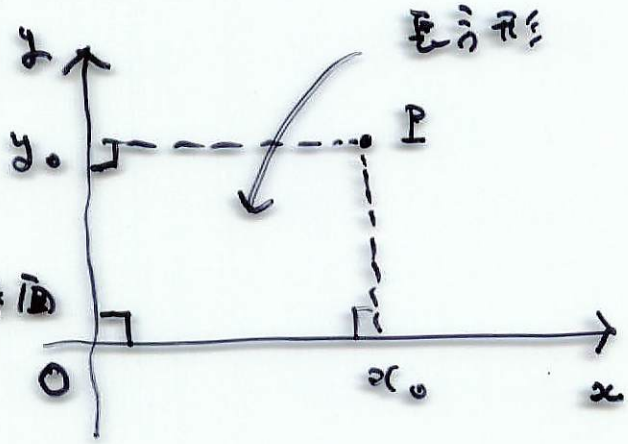
半年分の講義のOHP.

1次関数, 2次関数 + 割り (極限)

(I) 1次関数

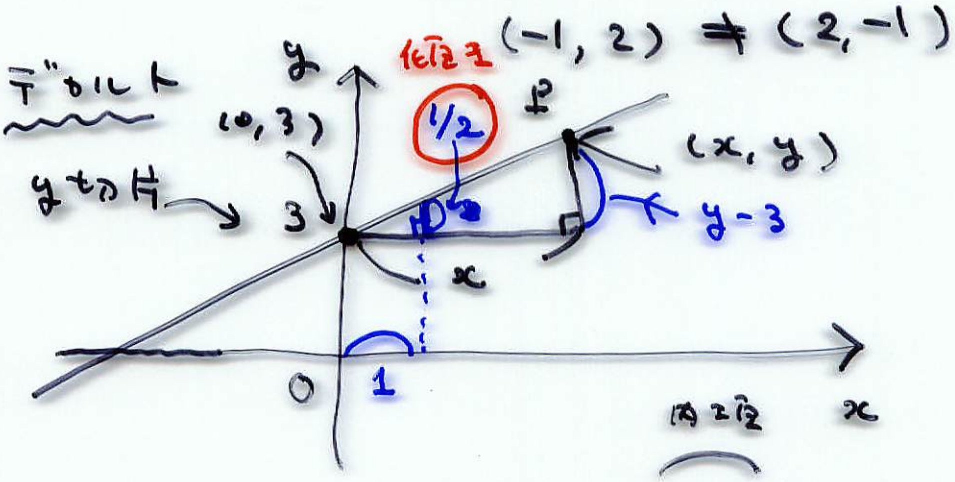
座標平面 = デカルト平面

原点 = Origin



P ⟷ (x_0, y_0) 順序対

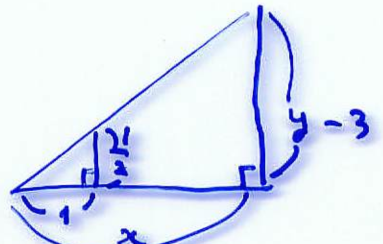
$(-1, 2) \neq (2, -1)$



内項

$$y-3 : x = \frac{1}{2} : 1$$

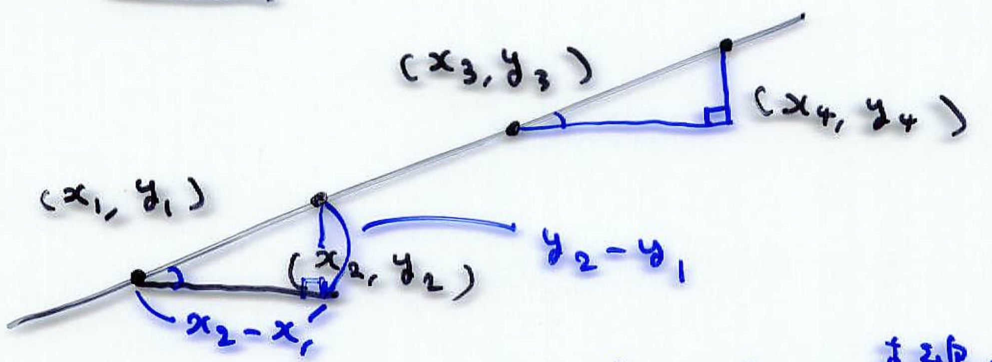
外項



$$\rightsquigarrow\ y-3 = \frac{1}{2}x.$$

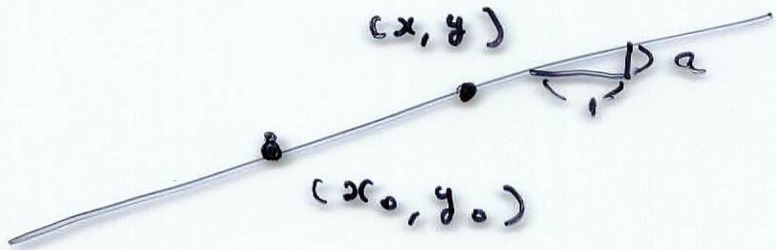
傾き

直線



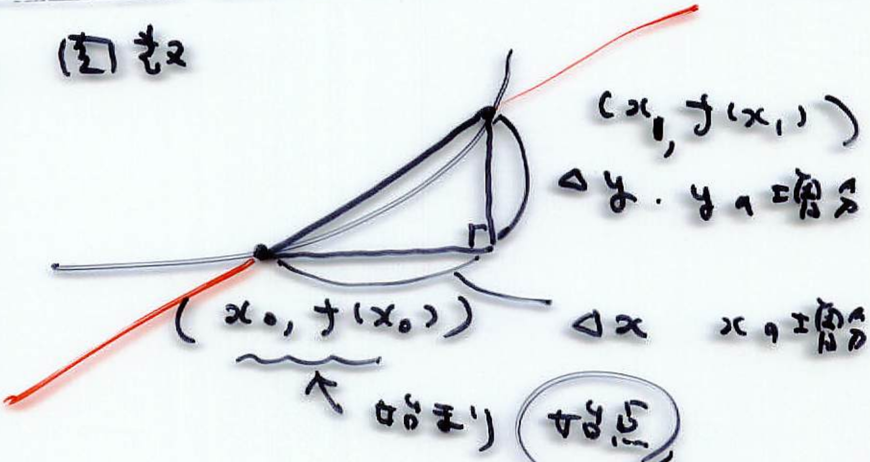
この値 $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_4 - y_3}{x_4 - x_3}$ = 直線の傾き.

↑ 直角三角形の相似



$$\frac{y - y_0}{x - x_0} = \frac{a}{1} \rightsquigarrow y = y_0 + \overset{\text{傾き}}{a}(x - a)$$

$y = f(x)$ 関数



$\Delta x = x_1 - x_0$, $\Delta y = f(x_1) - f(x_0)$

$$\frac{\Delta y}{\Delta x} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$ 平均変化率

Ex①. (1) (2, 1) を通る 傾き(−2) の直線 (2.5

(2) (2, 1), (−2, 3) を通る直線

$y = a(x - x_0) + y_0$

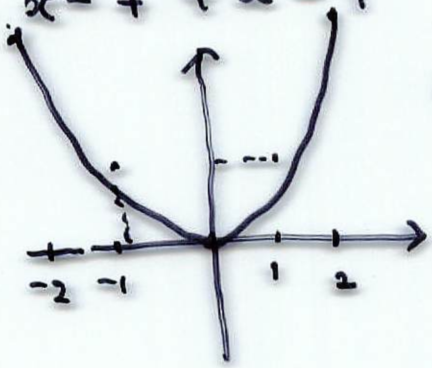
(x_0, y_0) を通り 傾き a の直線

$f(x) = \underbrace{a(x - x_0) + y_0}_{1\text{次式}}$.

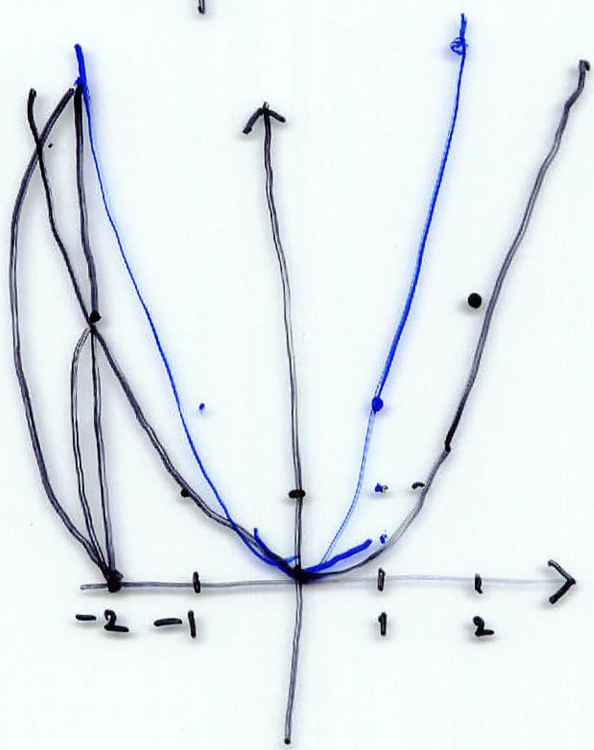
2次式
(3.1)

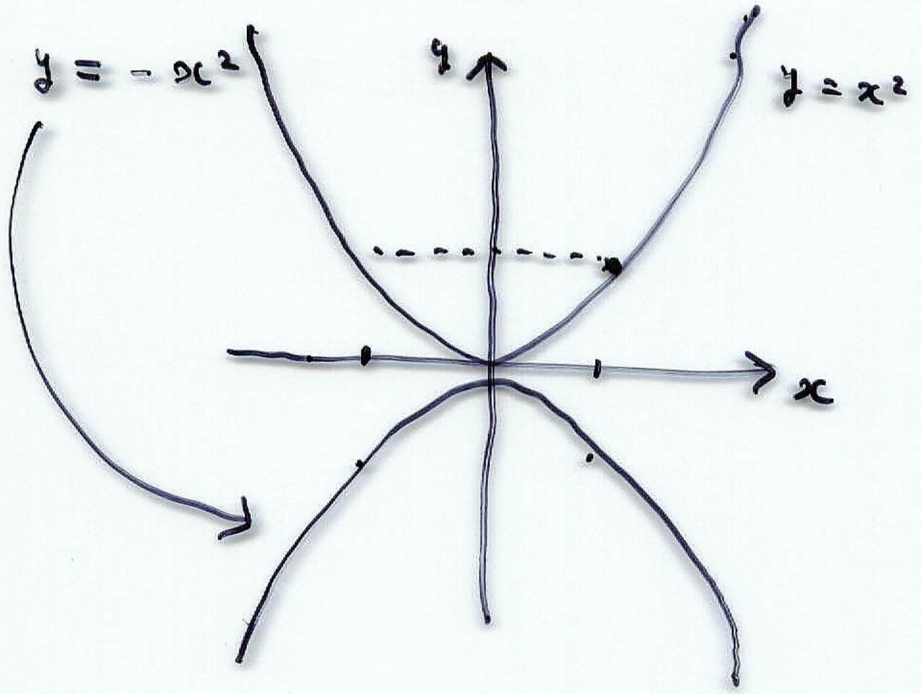
$y = x^2 + 4x - 1$ 放物線.

$y = x^2$



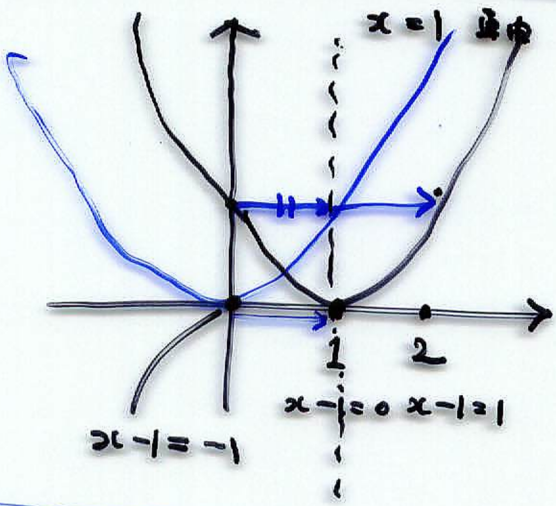
$y = 2x^2$



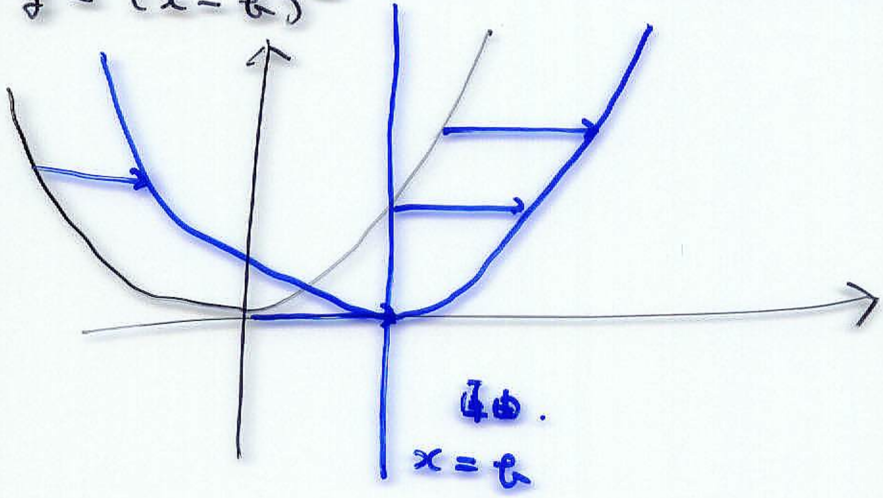


$y = ax^2$ $a \neq 0$ のグラフ OK.

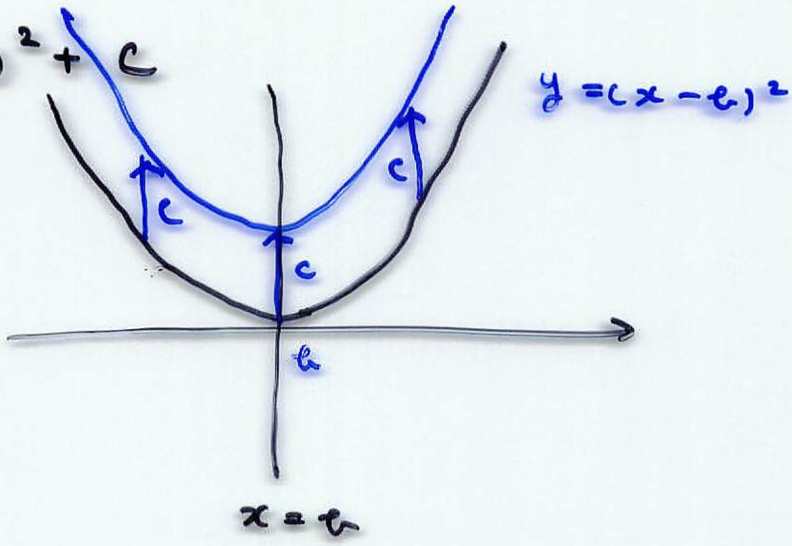
$y = (x-1)^2$



$y = (x-b)^2$



$y = (x-b)^2 + c$



$y = x^2 + \underset{\sim}{4}x - 1$

$4 = 2A \rightsquigarrow A = 2$

$(x+A)^2 = x^2 + \underset{\sim}{2A}x + A^2$

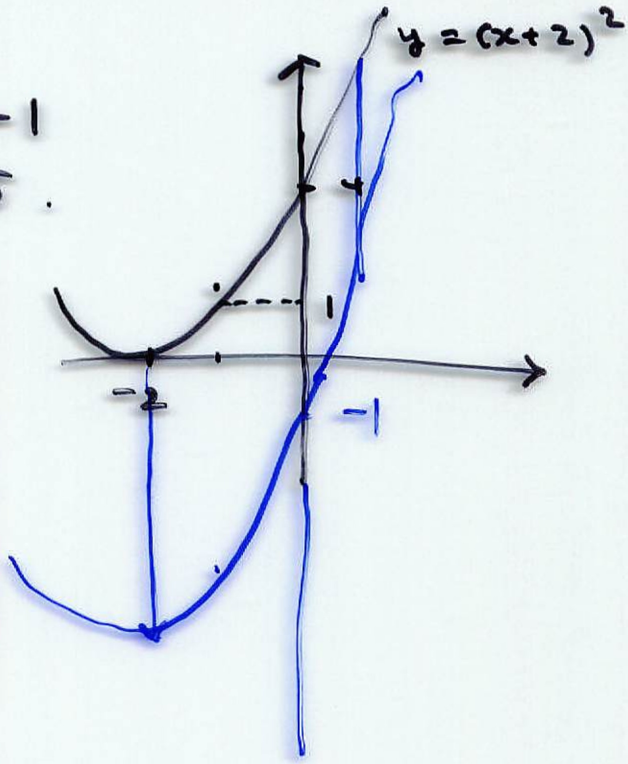
$(x+2)^2 = \underset{\sim\sim}{x^2 + 4x} + 4$

$x^2 + 4x = (x+2)^2 - 4$

$$\boxed{\begin{aligned}&(x+A)^2 - A^2 \\ &= x^2 + 2Ax.\end{aligned}}$$

平方完成.

$$\begin{aligned} y &= (x+2)^2 - 4 - 1 \\ &= (x+2)^2 - 5. \end{aligned}$$



$\boxed{\begin{array}{l} y = x^2 \text{ のグラフを} \\ x\text{軸方向に } -2 \\ \quad + \\ y\text{軸方向に } -5 \\ \quad \text{平行移動} \end{array}}$

(131) ①

$$y = x^2 - 2x + 3$$

$$= (x-1)^2 - 1 + 3$$

$$= (x-1)^2 + 2.$$

$(x-1)^2$

$= x^2 - 2x + 1$

② $y = x^2 - 6x + 3$

$$= (x-3)^2 - 9 + 3$$

$$= (x-3)^2 - 6.$$

$(x-3)^2$

$= x^2 - 6x + 9$

$x^2 - 6x$

$= (x-3)^2 - 9$

③ $y = 3x^2 + 4x - 1$

$$= 3\left(x + \frac{2}{3}\right)^2 - 3 \times \left(\frac{2}{3}\right)^2 - 1$$

$$= 3\left(x + \frac{2}{3}\right)^2 - \frac{7}{3}$$

$3\left(x^2 + \frac{4}{3}x\right)$

$\parallel$

$3\left(x + \frac{2}{3}\right)^2 - 3 \times \left(\frac{2}{3}\right)^2$

平方完成

EX0

(1) $y = x^2 + 2x - 3$

(2) $y = x^2 - 8x + 2$

(3) $y = 4x^2 - 6x + 1$

数列

国債. 利子率 2%. (年利)

0年度. 500 単位. $b_0 = 500$ (6

毎年 T 単位 返す.

1年後の残高

$$b_1 = (500 - T) \times 1.02$$

$$= (b_0 - T) \times 1.02$$

$$b_2 = (b_1 - T) \times 1.02$$

漸化式.

差分方程式

$$b_{n+1} = (b_n - T) \times 1.02 \quad (n = 0, 1, 2, \cdots)$$

$$b_n = \boxed{} \ ?$$

$b_{n+1} \leftarrow b_0, b_1, \cdots, b_n$ の式で表す.

帰納的定義.

recursive.

等差数列.

$$a_{n+1} - a_n = d \quad (一定)$$

$a_0 = 3$, $d = 2$

0	1	2	3	...
3	5	7	9	...

$$\boxed{a_n = a_0 + d \times n.}$$

等比数列　公比 2

3　6　12　24 …… 3×2^n

$b_0 = 3 \to \to \to \cdots \to b_n = 3\times 2^n$　(nステップ)

(7)

$$a_{n+1} = \alpha a_n \quad (n=0,1,2,\cdots)$$

α : 公比.

$$\boxed{a_n = \alpha^n a_0}$$

$$\begin{cases} a_{n+1} = 3a_n + 4 \\ a_0 = c \end{cases}$$

ガンマ
γ gamma.

$$a_{n+1} - \gamma = 3(a_n - \gamma)$$

$\{a_n - \gamma\}$ は公比3の等比数列.

$$a_{n+1} = 3a_n + \boxed{(\gamma - 3\gamma)}$$

$$\gamma - 3\gamma = 4 \rightsquigarrow \gamma = -2$$

$$\boxed{a_{n+1} + 2 = 3(a_n + 2)}$$

$\{a_n + 2\}$ は公比3の等比数列.

$$a_n + 2 = 3^n(a_0 + 2)$$

$$\boxed{a_n = 3^n(c+2) - 2.}$$

$$a_{n+1} = 3a_n + 4$$

$$\gamma = 3\gamma + 4$$

例

$$a_{n+1} = 3a_n + 2$$

$$\lambda = 3\lambda + 2$$ 特性方程式.

$$-2\lambda = 2$$

$$\lambda = -1$$

$$\begin{aligned} a_{n+1} &= 3a_n + 2 \\ -)\ (-1) &= 3\times(-1) + 2 \\ \hline a_{n+1}+1 &= 3(a_n+1) \end{aligned}$$

$\{a_n+1\}$ は公比 3 の等比数列.

$$a_n + 1 = 3^n (a_0 + 1)$$

$$\boxed{a_n = 3^n (a_0 + 1) - 1}$$

$$\begin{cases} b_{n+1} = (b_n - T)(1+r) \\ b_0 = M \end{cases}$$

$$\lambda = (\lambda - T)(1+r) \quad \lambda = \cdots$$

$$T(1+r) = r\lambda \rightsquigarrow \lambda = \frac{T(1+r)}{r} = \lambda_0$$

$$b_{n+1} - \lambda_0 = (1+r)(b_n - \lambda_0)$$

より $\{b_n - \lambda_0\}$ は等比数列 公比 $(1+r)$

$$b_n = \lambda_0 + (1+r)^n (M - \lambda_0)$$

$M - \lambda_0 < 0$

$M - \dfrac{T(1+r)}{r} \leqq 0$ でないと増加.

$$\frac{1+r}{r} = 1+\frac{1}{r} \qquad r>0.$$

$r \to$ 大. $\quad \frac{1+r}{r} \to$ 小

$$M = T\times\frac{r}{1+r} \longrightarrow \text{大}.$$

$M = 500, \quad r = 0.02.$

$M - T\frac{1+r}{r} \leqq 0 \iff T$ の条件.

$b_{50} \leqq 0$ の条件

sigma $\qquad$ sum = summa.

Σ 記号. $\qquad \Sigma,\ \sigma \iff S,\ s$

$$\sum_{n=0}^{10} a_n = a_0 + a_1 + a_2 + \cdots + a_{10}$$

$$\prod_{n=0}^{10} a_n = a_0 a_1 a_2 \cdots a_{10}$$

$\Pi,\ \pi \iff P,\ p.$ (pi)

product. 積.

公式

(I) $$\sum_{k=0}^{N}(a_k+b_k) = \sum_{k=0}^{N} a_k + \sum_{k=0}^{N} b_k$$

$$\{(a_0+b_0)+(a_1+b_1)+\cdots+(a_N+b_N)\}$$
$$= (a_0+a_1+\cdots+a_N)+(b_0+b_1+\cdots+b_N)$$

(II) $$\sum_{k=0}^{N}(c\,a_k) = c\sum_{k=0}^{N} a_k$$

$$c a_0 + c a_1 + c a_2 + \cdots + c a_N$$
$$= c(a_0+a_1+a_2+\cdots+a_N)$$

等差数列の和.

$$a_n = c + nd \iff \begin{cases} a_{n+1} - a_n = d \\ a_0 = c \end{cases}$$

$a_0 = c$, 公差 d.

$$\sum_{n=0}^{N} a_n = \sum_{n=0}^{N} c + d \boxed{\sum_{n=0}^{N} n} = \sum_{n=1}^{N} n$$

$$= (N+1)c + d \times \frac{N(N+1)}{2}$$

$$= \frac{(N+1)(2c+Nd)}{2}$$

$$= \frac{(N+1)(a_0 + a_N)}{2}$$

$$\begin{array}{rl} S = & \underset{a_0}{c} + (c+d) + (c+2d) + \cdots + \underset{a_N}{(c+Nd)} \\ +) & (c+Nd) + (c+(N-1)d) + c+(N-2)d + \cdots + c \\ \hline & (2c+Nd) + \cdots\cdots + (2c+Nd) \end{array}$$

$a_N = c + Nd$

$a_0 = c$

$$2S = (N+1) \times (2c+Nd)$$

$$S = \frac{(N+1)(2c+Nd)}{2}$$

$$(P_n)\quad \boxed{\sum_{k=1}^{n} k = \frac{n(n+1)}{2}}$$

$$\begin{array}{rcccccccccccccccc}
S = & 1 & + & 2 & + & 3 & + & 4 & + & 5 & + & 6 & + & 7 & + & 8 \\
+) & 8 & + & 7 & + & 6 & + & 5 & + & 4 & + & 3 & + & 2 & + & 1 \\
\hline
 & 9 & + & 9 & + & 9 & + & 9 & + & 9 & + & 9 & + & 9 & + & 9
\end{array}$$

$$= 8 \times 9$$

$$\therefore \quad S = \frac{8 \times 9}{2}$$

数学的帰納法.

(P_n) が全ての自然数 n で真.

$\equiv$ (P_1 は真) かつ ($P_n \Rightarrow P_{n+1}$ (全ての n))

$$\underset{\text{OK}}{P_1} \rightarrow \underset{\text{OK}}{P_2} \rightarrow P_3 \rightarrow \cdots \rightarrow P_{100}$$

$$(P_n)\quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

(P_1) $\quad 1 = \dfrac{1 \times 2}{2}$ $\quad$ 真

(P_n) 真とする $\qquad$ (P_n) を使う.

$$\sum_{k=1}^{n+1} k = \sum_{k=1}^{n} k + (n+1)$$

$$= \frac{n(n+1)}{2} + (n+1) \quad \boxed{\begin{array}{c}\text{ok}\\ (P_{n+1})\end{array}}$$

$$= (n+1)\left(\frac{n}{2} + 1\right)$$

$$= \frac{(n+1)(n+2)}{2}$$

等比数列の和.

$$a_n = a_0 \alpha^n \quad (n = 0, 1, 2, \cdots) \quad (\alpha \neq 1)$$

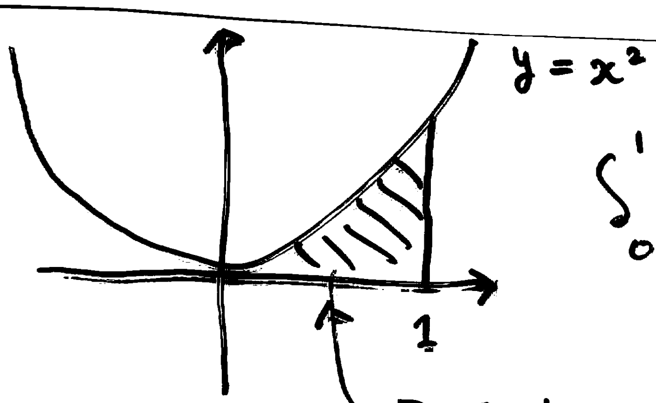
$$S_N = 1 + \alpha + \alpha^2 + \cdots + \alpha^N$$
$$-) \ \alpha S_N = \alpha + \alpha^2 + \cdots + \alpha^N + \alpha^{N+1}$$
$$(1-\alpha) S_N = 1 - \alpha^{N+1}$$

$$\rightsquigarrow \boxed{S_N = \frac{1-\alpha^{N+1}}{1-\alpha}}$$

$$\sum_{n=0}^{N} \alpha^n = \frac{1-\alpha^{N+1}}{1-\alpha}$$

乗数効果
行列累乗.

EXO.
(1) $a_{n+1} = 3a_n - 2$
(2) $a_{n+1} = -2a_n + 1$
(3) $a_{n+1} = 5a_n + 1$



$$\int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}.$$

面積 $\frac{1}{3}$. を示す.

$$\boxed{\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}}$$

$$k^2 + k = k(k+1)$$

$$(k+2)-(k-1) = 3$$

$$= \cancel{(k-1)k(k+}$$

$$= \frac{1}{3}\left\{\underline{k(k+1)}(k+2) - (k-1)\underline{k(k+1)}\right\}$$

$$\sum_{k=1}^{n}(k^2+k) = \frac{1}{3}\sum_{k=1}^{n}\left\{k(k+1)(k+2) - (k-1)k(k+1)\right\}$$

$$\begin{array}{lccc}
k=1 & 1\cdot\cancel{2\cdot 3} & - & 0\cdot 1\cdot 2 \\
k=2 & \cancel{2\cdot 3\cdot 4} & - & \cancel{1\cdot 2\cdot 3} \\
k=3 & \cancel{3\cdot 4\cdot 5} & - & \cancel{2\cdot 3\cdot 4} \\
k=4 & \cancel{4\cdot 5\cdot 6} & - & \cancel{3\cdot 4\cdot 5} \\
\vdots & & & \\
k=n & n(n+1)(n+2) & - & (n-1)n\cancel{(n+1)}
\end{array}$$

$$\sum_{k=1}^{n}(k^2+k) = \frac{1}{3}n(n+1)(n+2)$$

$$\sum_{k=1}^{n} k^2 = \sum_{k=1}^{n}(k^2+k) - \sum_{k=1}^{n} k$$

$$= \frac{1}{3}n(n+1)(n+2) - \frac{1}{2}n(n+1)$$

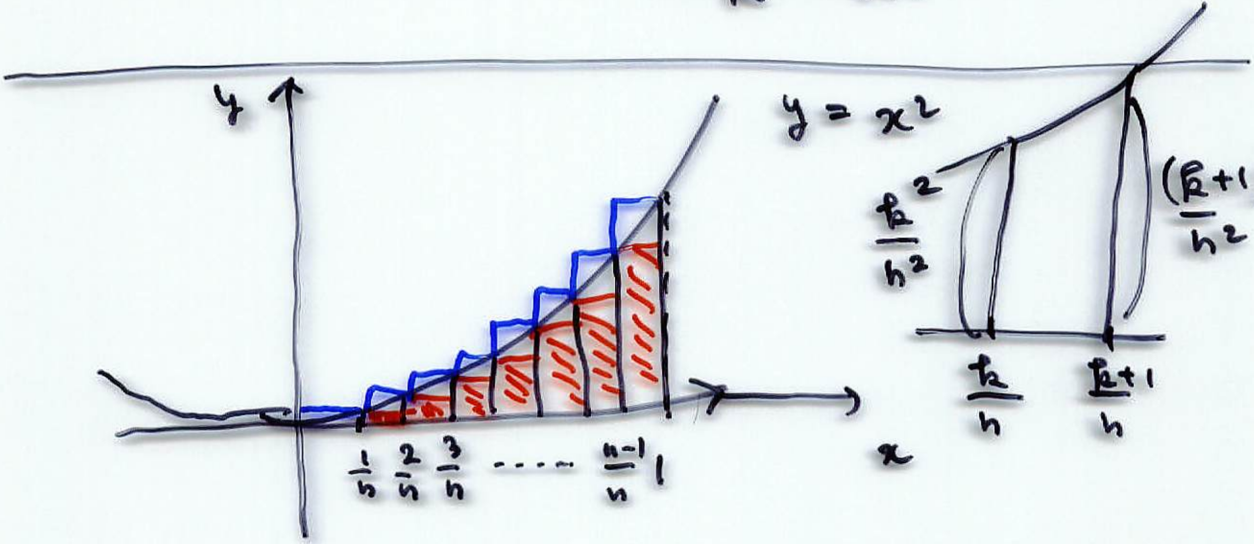
$$= \frac{1}{6}n(n+1)\{2(n+2)-3\}$$

$$= \frac{1}{6}n(n+1)(2n+1)$$

レポート $\displaystyle\sum_{k=1}^{n} k^3 = ?$

ヒント $(k-1)k(k+1) = (k^2-1)k$

$= k^3 - k.$



$$S_n = \frac{1}{n}\times\frac{1^2}{n^2} + \frac{1}{n}\times\frac{2^2}{n^2} + \frac{1}{n}\times\frac{3^2}{n^2} + \cdots + \frac{1}{n}\times\left(\frac{n}{n}\right)^2$$

$$= \sum_{k=1}^{n}\frac{k^2}{n^3} = \frac{1}{n^3}\times\frac{1}{6}n(n+1)(2n+1)$$

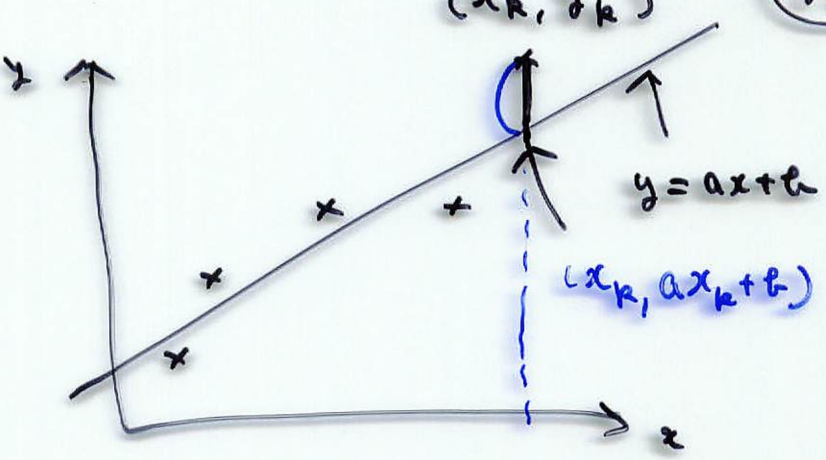
$$s_n = \frac{1}{n}\times\frac{0^2}{n^2} + \frac{1}{n}\times\frac{1^2}{n^2} + \frac{1}{n}\times\frac{2^2}{n^2} + \cdots + \frac{1}{n}\times\left(\frac{n-1}{n}\right)^2$$

$$= \sum_{k=1}^{n}\frac{(k-1)^2}{n^3} = \sum_{k=2}^{n}\frac{(k-1)^2}{n^3} \qquad k-1=l$$

$$= \sum_{l=1}^{n-1}\frac{l^2}{n^3} = \frac{1}{n^3}\times\frac{1}{6}(n-1)n(2n-1)$$

$2(n-1)+1$

x	y
x_1	y_1
x_2	y_2
x_3	y_3
$\vdots$	$\vdots$
x_N	y_N



↑ 実質家計消費支出.

↑ 実質家計可処分所得

相関係数.

$$S = \sum_{k=1}^{n} (y_k - ax_k - b)^2$$

を最小とする a, b を求める.　　回帰直線.

$$y = ax + b$$

乗数効果　　$|r| < 1$.

$$1 + r + r^2 + \cdots = \frac{1}{1-r}$$

$$\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) = s_n \leq S \leq S_n = \frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$n = 10000 \quad \frac{1}{n} = 10^{-4} = 0.0001$

$$S_{10000} = \frac{1}{6} \times (1 + 0.0001)(2 + 0.0001)$$

$$= \frac{1}{6} \times (2 + 0.0003 + 0.00000001)$$

$$= \frac{1}{6} \times 2.00030001$$

$$s_{10000} = \frac{1}{6} \times (2 - 0.0003 + 0.00000001)$$

$\leadsto S = \frac{1}{3}$ はさみうちの原理.

$a_0, a_1, a_2, \cdots \qquad a_n \Longrightarrow \alpha \ (n \Longrightarrow +\infty)$

α に近づく.