

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$



$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

1311の展開
1312の展開

$$= \alpha \cdot \begin{vmatrix} \beta & 0 \\ \varepsilon & \gamma \end{vmatrix} - 0 \cdot * + 0 \cdot *$$

$$\begin{vmatrix} \alpha & 0 & 0 \\ \delta & \beta & 0 \\ \gamma & \varepsilon & \gamma \end{vmatrix}$$

$$= \alpha \begin{vmatrix} \beta & 0 \\ \varepsilon & \gamma \end{vmatrix} - \delta \begin{vmatrix} 0 & 0 \\ \varepsilon & \gamma \end{vmatrix} + 0 \begin{vmatrix} 0 & 0 \\ \beta & 0 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 \\ \varepsilon & \gamma \end{vmatrix} = 0$$

$$+ \gamma \begin{vmatrix} 0 & 0 \\ \beta & 0 \end{vmatrix} = 0$$

$$= \alpha \beta \gamma$$

$$\begin{vmatrix} a & d & e \\ 0 & b & f \\ 0 & c & g \end{vmatrix} = a \begin{vmatrix} b & f \\ c & g \end{vmatrix} - 0 \cdot * + 0 \cdot *$$

$$= a b c$$

$$\begin{vmatrix} x & a & b \\ 0 & z & y \\ 0 & w & u \end{vmatrix} = x \begin{vmatrix} z & y \\ w & u \end{vmatrix} - 0 \cdot * + 0 \cdot *$$

$$= x(zu - wy)$$

$$\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = a \begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} + 1 \begin{vmatrix} 1 & a \\ a & 1 \end{vmatrix}$$

$$= a(a^2 - 1) - (a - 1) + (a^2 - a)$$

$$= (a-1)^2(a+2)$$

$$= (a-1) \{ a(a+1) - 1 + (a-1) \} = (a-1)(a^2 + a - 2)$$

$$\begin{aligned}
 \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} &= a \begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ a & 1 \end{vmatrix} \\
 &= a(a^2 - 1) - (a - 1) + (1 - a) \\
 &= (a - 1) \{a(a + 1) - 1 - 1\} \\
 &= (a - 1)(a^2 + a - 2) \\
 &= (a - 1)(a - 1)(a + 2) \\
 &= (a - 1)^2(a + 2)
 \end{aligned}$$

$$\left(\begin{array}{ccc|c} a & 1 & 1 & * \\ 1 & a & 1 & * \\ 1 & 1 & a & * \end{array} \right) \rightarrow \dots$$

x, y, z



条件は

$$A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix} \text{ 行列}$$

$$\Leftrightarrow |A| \neq 0$$

$$\Leftrightarrow a \neq 1, -2.$$

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} * \\ * \\ * \end{pmatrix}$$

$$a \neq 1, -2 \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} * \\ * \\ * \end{pmatrix}$$

$n=2$ のときは
2.2.3.

$$\begin{vmatrix} x & 0 & a \\ y & 0 & b \\ z & 1 & c \end{vmatrix} = x \begin{vmatrix} 0 & b \\ 1 & c \end{vmatrix} - y \begin{vmatrix} 0 & a \\ 1 & c \end{vmatrix} + z \begin{vmatrix} 0 & a \\ 0 & b \end{vmatrix}$$

(2,3)

$$= x(-b) - y(-a)$$

0

$$= ay - bx$$

$$= -1 \cdot \begin{vmatrix} y & b \\ z & c \end{vmatrix} + 0 \cdot \begin{vmatrix} x & a \\ z & c \end{vmatrix} - 1 \cdot \begin{vmatrix} x & a \\ y & b \end{vmatrix}$$

$$= -1 \cdot \begin{vmatrix} x & a \\ y & b \end{vmatrix} = ay - bx$$

$$\begin{vmatrix} x & y & z \\ 1 & 0 & 0 \\ a & b & c \end{vmatrix} = x \begin{vmatrix} 0 & 0 \\ b & c \end{vmatrix} - 1 \cdot \begin{vmatrix} y & z \\ b & c \end{vmatrix} + a \begin{vmatrix} y & z \\ 0 & 0 \end{vmatrix}$$

$$= -(yc - zb)$$

(2,1)

$$= -1 \cdot \begin{vmatrix} y & z \\ b & c \end{vmatrix} + 0 \cdot \begin{vmatrix} x & z \\ a & c \end{vmatrix} - 0 \cdot \begin{vmatrix} x & y \\ a & b \end{vmatrix}$$

$$= -1 \cdot \begin{vmatrix} y & z \\ b & c \end{vmatrix} = -(yc - zb)$$

$$n=3$$

余因+展開.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

odd impair
even pair

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$= -b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + b_2 \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} - b_3 \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$= c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} - c_2 \begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} + c_3 \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

③

$$\mathbb{R}^3 \xrightarrow{F} \mathbb{R}.$$

定数項なし
"定数"

$$\vec{a} \mapsto c_1 a_1 + c_2 a_2 + c_3 a_3$$

$$F(\lambda \vec{a} + \mu \vec{d}) = \lambda F(\vec{a}) + \mu F(\vec{d})$$

c_1, c_2, c_3 は定数.

I) 3y = 123 系に於て.

15)

$$|\lambda \vec{a} + \mu \vec{b}| \vec{c} = \lambda |\vec{a} \vec{b} \vec{c}| + \mu |\vec{a} \vec{b} \vec{c}|$$

$$|\vec{a} (\lambda \vec{b} + \mu \vec{c})| = \lambda |\vec{a} \vec{b} \vec{c}| + \mu |\vec{a} \vec{b} \vec{c}|$$

25).
334 は 123 系に於て.

II) 234 系に於て $\vec{c} = 0$

$$\begin{vmatrix} a_1 & a_1 & c_1 \\ a_2 & a_2 & c_2 \\ a_3 & a_3 & c_3 \end{vmatrix}$$

$$|\vec{a} \vec{a} \vec{c}| = 0$$

$$= c_1 \begin{vmatrix} a_2 & a_2 \\ a_3 & a_3 \end{vmatrix} - c_2 \begin{vmatrix} a_1 & a_1 \\ a_3 & a_3 \end{vmatrix} + c_3 \begin{vmatrix} a_1 & a_1 \\ a_2 & a_2 \end{vmatrix}$$

334 系に於て 3 次元空間

2x2

$$|\vec{a} \vec{a}| = 0$$

III) $(I_3) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

正交性.

(IV) $i \neq j$ 234 is a cyclic permutation $\Rightarrow (-1)^{i-1}$ times.

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \end{vmatrix} = - \begin{vmatrix} \vec{c} & \vec{b} & \vec{a} \end{vmatrix}$$

134 is 331, a cyclic permutation

$n=2$ is 123. $\begin{vmatrix} \vec{a} + \vec{c} & \vec{b} & \vec{a} + \vec{c} \end{vmatrix} = \dots$

exactly in the same way...

234 is a cyclic permutation.

$$\begin{aligned} & \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + b_2 \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} - b_3 \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \\ & = - \left(-b_1 \begin{vmatrix} c_2 & a_2 \\ c_3 & c_3 \end{vmatrix} + b_2 \begin{vmatrix} c_1 & a_1 \\ c_3 & a_3 \end{vmatrix} - b_3 \begin{vmatrix} c_1 & a_1 \\ c_2 & c_2 \end{vmatrix} \right) \\ & = - \begin{vmatrix} \vec{c} & \vec{b} & \vec{a} \end{vmatrix} \end{aligned}$$

(V) $i \neq j$ $i \neq j$ is a cyclic permutation \Rightarrow add 2 times

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \end{vmatrix} = \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} + \lambda \vec{a} \end{vmatrix}$$

is zero.

$$\begin{aligned} \text{RHS } (T_0) &= \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \end{vmatrix} + \lambda \begin{vmatrix} \vec{a} & \vec{b} & \vec{a} \end{vmatrix} \\ &= \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \end{vmatrix} = (T_2) = 0 \\ & \text{LHS} \end{aligned}$$

$$\begin{aligned}
 & | \vec{a} + \vec{b} \quad \vec{b} \quad \vec{a} + \vec{c} | = 0 \\
 & = | \vec{a} \quad \vec{b} \quad \vec{a} | + | \vec{a} \quad \vec{b} \quad \vec{c} | \\
 & \quad + | \vec{c} \quad \vec{b} \quad \vec{a} | + \underbrace{| \vec{c} \quad \vec{b} \quad \vec{c} |}_{=0}
 \end{aligned}$$

$$\leadsto | \vec{a} \quad \vec{b} \quad \vec{c} | = - | \vec{c} \quad \vec{b} \quad \vec{a} |$$

(14)

$$n = 2$$

$$| \vec{a} \quad \vec{b} | = - | \vec{b} \quad \vec{a} |$$

ⅢⅥ

$$| {}^t A | = | A | \quad | \vec{a} \vec{b} \vec{c} | = \begin{vmatrix} {}^t a_1 & {}^t a_2 & {}^t a_3 \\ {}^t b_1 & {}^t b_2 & {}^t b_3 \\ {}^t c_1 & {}^t c_2 & {}^t c_3 \end{vmatrix}$$

$$\vec{r} \leftrightarrow \vec{r}'$$

$$\begin{vmatrix} a_1 \\ b_1 \\ c_1 \end{vmatrix} = \begin{vmatrix} {}^t a_1 & {}^t b_1 & {}^t c_1 \end{vmatrix}$$

Ⅰ'

各 \vec{r} の系を \vec{r}' とし

$$\begin{vmatrix} \lambda a_1 + \mu d_1 \\ b_1 \\ c_1 \end{vmatrix} = \lambda \begin{vmatrix} a_1 \\ b_1 \\ c_1 \end{vmatrix} + \mu \begin{vmatrix} d_1 \\ b_1 \\ c_1 \end{vmatrix}$$

Ⅳ Ⅵ

$$= \begin{vmatrix} {}^t (\lambda a_1 + \mu d_1) & {}^t b_1 & {}^t c_1 \end{vmatrix}$$

311の系を \vec{r}' とし

$$\lambda {}^t a_1 + \mu {}^t d_1$$

Ⅰ

$$= \lambda \begin{vmatrix} {}^t a_1 & {}^t b_1 & {}^t c_1 \end{vmatrix}$$

Ⅵ

$$+ \mu \begin{vmatrix} {}^t d_1 & {}^t b_1 & {}^t c_1 \end{vmatrix}$$

$$= \lambda \begin{vmatrix} a_1 \\ b_1 \\ c_1 \end{vmatrix} + \mu \begin{vmatrix} d_1 \\ b_1 \\ c_1 \end{vmatrix}$$

Ⅱ'

各 \vec{r} の系を \vec{r}' とし $\vec{r}' = 0$

$$\begin{vmatrix} a_1 \\ b_1 \\ a_1 \end{vmatrix} = 0$$

$$= \begin{vmatrix} {}^t a_1 & {}^t b_1 & {}^t a_1 \end{vmatrix} = 0$$

Ⅱ

IV

$$\begin{vmatrix} a_1 \\ b \\ c \end{vmatrix} = - \begin{vmatrix} b \\ a_1 \\ c \end{vmatrix}$$
$$= - \begin{vmatrix} b \\ c \\ a_1 \end{vmatrix}$$
$$= - \begin{vmatrix} a_1 \\ b \\ c \end{vmatrix}$$

⑤ 大事. $i \neq j$ i 行 a 与 b 互 j 行 c 与 d 互 c 不变.

$$\begin{vmatrix} a_1 \\ b \\ c \end{vmatrix} = \begin{vmatrix} a_1 \\ \lambda a_1 + b \\ c \end{vmatrix}$$

24312712 糸孔型

$$\textcircled{T_0} = \lambda \begin{vmatrix} a & 1 \\ a & 1 \\ a & a \end{vmatrix} + \begin{vmatrix} a & 1 \\ 1b & 1 \\ a & a \end{vmatrix} = \begin{vmatrix} a & 1 \\ 1b & 1 \\ a & a \end{vmatrix} = \textcircled{T_2}$$

determinant
(le déterminant)

$$\begin{aligned} \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} &= - \begin{vmatrix} 1 & a \\ 1 & a \\ a & 1 \end{vmatrix} = - \begin{vmatrix} 1 & a \\ 0 & a-1 & 1-a \\ 0 & 1-a & 1-a^2 \end{vmatrix} \\ &= -1 \cdot \begin{vmatrix} a-1 & 1-a \\ 1-a & 1-a^2 \end{vmatrix} \quad (a-1) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ &= -(a-1)(1-a) \begin{vmatrix} 1 & 1 \\ -1 & 1+a \end{vmatrix} \\ &= (a-1)^2 (a+2) \end{aligned}$$

Vous pouvez me poser des question.

$$\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = - \begin{vmatrix} 1 & a & 1 \\ a & 1 & 1 \\ 1 & 1 & a \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & a & 1 \\ 0 & 1-a^2 & 1-a \\ 0 & \cancel{1-a} & \cancel{1-a} \\ & 1-a & a-1 \end{vmatrix}$$

$$2r + t = 1r \times (-a)$$

$$3r + t = 1r \times (-1)$$

$$(1-a) \begin{pmatrix} 1+a \\ 1 \end{pmatrix} = - \begin{vmatrix} 1-a^2 & 1-a \\ 1-a & a-1 \end{vmatrix} \quad (1-a) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= - (1-a)^2 \begin{vmatrix} 1+a & 1 \\ 1 & -1 \end{vmatrix}$$

$$= - (a-1)^2 (-a-2)$$

$$= (a-1)^2 (a+2)$$

$$n=2 \text{ 9 1 2 3}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = (\vec{a}_1, \vec{a}_2)$$

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = (\vec{b}_1, \vec{b}_2)$$

$$|AB| = |A\vec{b}_1, A\vec{b}_2| \quad \vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\vec{b}_1 = b_{11}\vec{e}_1 + b_{21}\vec{e}_2$$

$$\vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= |A(b_{11}\vec{e}_1 + b_{21}\vec{e}_2), A\vec{b}_2|$$

$$= b_{11} |A\vec{e}_1, A\vec{b}_2| + b_{21} |A\vec{e}_2, A\vec{b}_2|$$

$$\vec{b}_2 = b_{12}\vec{e}_1 + b_{22}\vec{e}_2$$

$$= b_{11} |\vec{a}_1, A(b_{12}\vec{e}_1 + b_{22}\vec{e}_2)|$$

$$+ b_{21} |\vec{a}_2, A(b_{12}\vec{e}_1 + b_{22}\vec{e}_2)|$$

$$= b_{11} b_{12} |\vec{a}_1, A\vec{e}_1| + b_{11} b_{22} |\vec{a}_1, A\vec{e}_2|$$

$$- |\vec{a}_1, \vec{a}_2| + b_{21} b_{12} |\vec{a}_2, A\vec{e}_1| + b_{21} b_{22} |\vec{a}_2, A\vec{e}_2|$$

$$= (b_{11} b_{22} - b_{12} b_{21}) |\vec{a}_1, \vec{a}_2| = |B| \cdot |A|$$

24 3734 a it 44 34

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

$(A | I_3)$

$$= \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 5 & 0 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \end{array} \right)$$

$$\begin{aligned} 2r + &= 1r \times (-2) \\ 3r + &= 1r \times (-3) \end{aligned}$$

$$\begin{array}{cccccc} 2 & 4 & 5 & 0 & 1 & 0 \\ 2 & 4 & 6 & 2 & 0 & 0 \\ \hline & & & & & \end{array} \quad \begin{array}{cccccc} 3 & 5 & 6 & 0 & 0 & 1 \\ 3 & 6 & 9 & 3 & 0 & 0 \\ \hline & & & & & \end{array}$$

$$-1 \quad -3 \quad -3 \quad 0 \quad 1$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -3 & -5 & 0 & 2 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array} \right)$$

$$1r + = 2r \times (-2)$$

$$\begin{array}{cccc} 1 & 2 & 3 & 1 & 0 & 0 \\ - & 0 & 2 & 6 & 6 & -2 \\ \hline & 1 & 0 & -3 & & \end{array}$$

$$\downarrow$$
~~$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -3 & 0 & 2 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array} \right)$$~~

$$\begin{array}{cccc} -3 & -5 & 0 & 2 \\ + 2 & 3 & 6 & 0 & -3 \\ \hline & 0 & 1 & 0 & -1 \end{array}$$

$$\begin{array}{cccc} 3 & 3 & 0 & -1 \\ - 2 & 3 & 6 & 0 & -3 \\ \hline & 0 & -3 & 0 & 2 \end{array}$$

$(I_3 | A^{-1})$

$$1 \quad 0 \quad -3 \quad -5 \quad 0 \quad 2$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 2 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array} \right) + \begin{array}{cccccc} 0 & 0 & 3 & 6 & -2 & 0 \\ \hline & & & & & \end{array}$$

$$\begin{array}{cccccc} 0 & 1 & 3 & 3 & 0 & -1 \\ - & 0 & 0 & 3 & 6 & -3 & 0 \\ \hline & 0 & 1 & 0 & -3 & 3 & -1 \end{array}$$

$$(A | I_3) \rightarrow \dots \rightarrow (I_3 | B)$$

正
則

$P =$

行基本変形

= 基本変形を I_3 のようにする.

A^{-1}

② 基本変形
は正則.

$$P \dots P_2 P_1 (A | I_3) = (I_3 | B)$$

$P(A | I_3)$

$$(PA | PI_3) = (I_3 | B)$$

P : 正則

$$P(\vec{a}_1, \vec{a}_2, \vec{a}_3 | \vec{e}_1, \vec{e}_2, \vec{e}_3)$$

$$= (P\vec{a}_1, P\vec{a}_2, P\vec{a}_3 | P\vec{e}_1, P\vec{e}_2, P\vec{e}_3) \quad (PA | P)$$

I_3

$$P^{-1}P$$

$$A = P^{-1}I_3$$

$$PA = I_3 \quad \xrightarrow{P^{-1}} \quad A = P^{-1} \quad \text{正則.}$$

$$P = B$$

$$A^{-1} = (P^{-1})^{-1} = P = B \quad (正則)^{-1}: \text{正則}$$

① $\begin{pmatrix} 3 & 6 & -5 \\ 1 & 2 & -2 \\ -2 & -3 & 2 \end{pmatrix}^{-1}$ を求めよ.

② $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = ?$

purser examen.