逆行列

A : n次正方行列.

X : n次正方行列.

$$AX = XA = I_n$$

をみたす X が存在するとき. A : 正則

$$\left.\begin{array}{l} AX = XA = I_n \\ AY = YA = I_n \end{array}\right\}$$

$\rightsquigarrow$ $AX = I_n$ $\overset{Y\cdot}{\rightsquigarrow}$ $Y(AX) = YI_n = Y$

$Y(AX)$ = (← 結合則) $(YA)X = I_n X = X$

$\rightsquigarrow$ $X = Y$. 逆行列は存在すれば唯一つ.

n = 2

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$: 正則 $\Longleftrightarrow$ $|A| = ad - bc \neq 0$

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

(⟺ について) 2週後に n一般

(A^{-1} について) n = 3 (小テ)

~~AX~~ XA $= I_n$ ⟶ $XA = AX = I_n$.

↑ (保留)

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \quad a\ t \equiv \quad A^{-1} = ?$$

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{1r \times = \frac{1}{2}} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$2r + = 1r \times (-1)$$
$$3r + = 1r \times (-1)$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{3}{2} & \frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & 0 & 1 \end{array}\right) \xrightarrow{2r \times = \frac{2}{3}} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & 0 & 1 \end{array}\right)$$

$$3r + = 2r \times \left(-\frac{1}{2}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & \frac{4}{3} & -\frac{1}{3} & -\frac{1}{3} & 1 \end{array}\right) \xrightarrow{3r \times = \frac{3}{4}} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{array}\right)$$

$$\frac{3}{2} - \frac{1}{6} = \frac{8}{6} = \frac{4}{3}$$

$$1r \times = 2r \times \left(-\frac{1}{2}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{array}\right) \longleftarrow \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{array}\right)$$

$= A^{-1}$

$$\frac{9}{12} = \frac{3}{4}$$

$$\begin{array}{rcccccc} & 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ - & 0 & 0 & \frac{1}{3} & -\frac{1}{12} & -\frac{1}{12} & \frac{1}{4} \\ \hline & 0 & 1 & 0 & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \end{array}$$

$$\begin{array}{rcccccc} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ - & 0 & 0 & \frac{1}{3} & -\frac{1}{12} & -\frac{1}{12} & \frac{1}{4} \\ \hline & 1 & 0 & 0 & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}$$

$$A\vec{x}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{x}_1 = \begin{pmatrix} \frac{3}{4} \\ -\frac{1}{4} \\ -\frac{1}{4} \end{pmatrix}$$

$$A\vec{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \vec{x}_2 = \begin{pmatrix} -\frac{1}{4} \\ \frac{3}{4} \\ -\frac{1}{4} \end{pmatrix}$$

$$A\vec{x}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \vec{x}_3 = \begin{pmatrix} -\frac{1}{4} \\ -\frac{1}{4} \\ \frac{3}{4} \end{pmatrix}$$

$$X = (\vec{x}_1\ \vec{x}_2\ \vec{x}_3)$$

$$AX = A(\vec{x}_1\ \vec{x}_2\ \vec{x}_3) = (A\vec{x}_1\ A\vec{x}_2\ A\vec{x}_3)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I_3$$

実は $AX = I_3 \rightsquigarrow XA = I_3$ も出て $X = A^{-1}$

基本行列

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \boxed{\frac{1}{2} x_1} \\ x_2 \\ x_3 \end{pmatrix}$$

1行 $\times = \frac{1}{2}$.

$$\begin{pmatrix} 1 & 0 & 0 \\ \overset{\lambda}{-1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ \overset{\lambda}{-}x_1 + x_2 \\ x_3 \end{pmatrix}$$

2行 $+=$ 1行 $\times (-1)$

$\times \lambda$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix}$$

1行 ↔ 3行.

(i) $i \neq j$ i行とj行の交換

(ii) $i \neq j$ i行のλ倍をj行に加える.

(iii) $\lambda \neq 0$ i行をλ倍.

行基本変形 $\Longleftrightarrow$ 基本行列をかける.

基本行列は正則

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = I_3$$

$$(A \mid I_3) \to \cdots \to (I_3 \mid X)$$

正則 P ‖

$$(P_\ell \cdots P_2 P_1)(A \mid I_3) = (I_3 \mid X)$$

$$P(A \mid I_3) = (I_3 \mid X)$$

$$\parallel$$

$$(PA \mid PI_3) = (PA \mid P)$$

注 正則×正則＝正則

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$\left.\begin{aligned} PA &= I_3 \\ X &= P \end{aligned}\right\} \to XA = I_3$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} = (b_2 c_3 - b_3 c_2)$

I

(1) $\begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & b_2 & c_2 \\ b_3 & b_3 & c_3 \end{vmatrix} = 0$

(2) $\begin{vmatrix} c_1 & b_1 & c_1 \\ c_2 & b_2 & c_2 \\ c_3 & b_3 & c_3 \end{vmatrix} = 0$

II

$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 0 & 8 \\ 2 & 3 & 3 \end{pmatrix}$ に対して A^{-1} を計算.

III

$$\left(\begin{array}{ccc|c} x & y & z & \\ 1 & 1 & a & 1 \\ 1 & a & 1 & 3 \\ a & 1 & 1 & 2a \end{array}\right)$$ に解が無い a を求めよ.

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 0 & 8 & 0 & 1 & 0 \\ 2 & 3 & 3 & 0 & 0 & 1 \end{array}\right) \to \cdots \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & \end{array}\right)$$