$$A = \begin{pmatrix} 2 & -2 \\ -2 & -1 \end{pmatrix} \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\Phi_A(\lambda) = \left| \lambda I_2 - A \right|$$

$$= \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right|$$

$$= \left| \begin{pmatrix} \lambda - a & -b \\ -c & \lambda - d \end{pmatrix} \right|$$

$$= (\lambda - a)(\lambda - d) - bc$$

$$= \lambda^2 - (a+d)\lambda + \underbrace{\boxed{(ad - bc)}}_{\det(A)}$$

$\mathrm{tr}(A) = a + d$

trace, 跡.

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$$A^2 - (a+d)A + (ad - bc) I_2 = O_2$$

$$A = \begin{pmatrix} 2 & -2 \\ -2 & -1 \end{pmatrix} \Rightarrow A^2 - A - 6 I_2 = O_2$$

$$\lambda^2 - \lambda - 6 = 0$$

$$(\lambda - 3)(\lambda + 2)$$

$$\lambda = 3, -2$$

$$A^2 - (3 + (-2)) A + 3 \times (-2) I_2 = O_2$$

$$\begin{cases} A^2 - 3A = -2A + 6 I_2 \longrightarrow \\ A^2 + 2A = 3A + 6 I_2 \longrightarrow \end{cases}$$

$$\begin{cases} A(A-3I_2)=(-2)(A-3I_2) \\ A(A+2I_2)=3\ (A+2I_2) \end{cases}$$

$$\begin{cases} A^n(A-3I_2)=(-2)^n(A-3I_2) \\ A^n(A+2I_2)=3^n(A+2I_2) \end{cases}$$

$-)$

$$-5A^n=(-2)^n(A-3I_2)$$
$$-3^n(A+2I_2)$$

$$\lambda^n = q(\lambda)\underbrace{(\lambda^2-\lambda-6)}_{\substack{\shortparallel \\ (\lambda-3)(\lambda+2)}} + a\lambda + b$$

$$\begin{cases} 3^n = 3a+b \\ (-2)^n = -2a+b \end{cases}$$

$-)$

$$3^n-(-2)^n = 5a$$

$$a = \frac{1}{5}(3^n-(-2)^n)$$

$$b = \cdots$$

$$A^n = \underbrace{[\ ]}_{q(A)}\overbrace{(A^2-A-6I_2)}^{\substack{O_2 \\ \shortparallel}} + aA + b.$$

$$= aA + b$$

固有方程式が重根を持つ場合.

$$B = \begin{pmatrix} 1 & -1 \\ 4 & 5 \end{pmatrix}$$

$$|\lambda I_2 - B| = \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ 4 & 5 \end{pmatrix} \right|$$

$$= \begin{vmatrix} \lambda - 1 & 1 \\ -4 & \lambda - 5 \end{vmatrix}$$

$$= (\lambda - 1)(\lambda - 5) + 4$$

$$= \lambda^2 - 6\lambda + 9 = (\lambda - 3)^2$$

$$a_{n+2} - 6a_{n+1} + 9a_n = 0$$

$$a_{n+2} - 3a_{n+1} = 3(a_{n+1} - 3a_n)$$

$$b_{n+1} = 3b_n$$

$$a_{n+1} - 3a_n = 3^{n-1}(a_2 - 3a_1)$$

$$\boxed{b_n = a_{n+1} - 3a_n}$$

$$\frac{1}{3^{n+1}}$$

$$\frac{1}{3^{n+1}} a_{n+1} - \frac{1}{3^n} a_n = \frac{1}{3^2}(a_2 - 3a_1)$$

$\boxed{c_n = \frac{1}{3^n} a_n}$ とおく $c_{n+1} - c_n = \bigcirc$

$$\frac{1}{3^n} a_n = (n-1)\frac{1}{3^2}(a_2 - 3a_1) + \frac{1}{3} a_1$$

$$a_n = (n-1)3^{n-2}(a_2 - 3a_1) + 3^{n-1} a_1$$

$$A^2 - 6A + 9I_2 = 0$$

$$A^2 - 3A = 3A - 9I_2$$

$$A(A - 3I_2) = 3(A - 3I_2)$$

$$A^n(A - 3I_2) = 3^n(A - 3I_2)$$

$$A^{n+1} - 3A^n = 3^n(A - 3I_2)$$

$\frac{1}{3^{n+1}}$

$$\frac{1}{3^{n+1}} A^{n+1} - \frac{1}{3^n} A^n = \frac{1}{3}(A - 3I_2)$$

$$\boxed{B_n = \frac{1}{3^n} A^n} \qquad B_{n+1} - B_n =$$

$$B_0 = \frac{1}{3^0} I_2 = \frac{1}{1} I_2$$

$$\frac{1}{3^n} A^n = B_n = n \times \frac{1}{3}(A - 3I_2) + I_2$$

$$\boxed{A^n = n\, 3^{n-1}(A - 3I_2) + 3^n I_2}$$

$$(fg)' = f'g + fg'$$

$$\begin{cases} \lambda^n = q(\lambda)(\lambda - 3)^2 + a\lambda + b \\ n\lambda^{n-1} = q'(\lambda)(\lambda - 3)^2 + q(\lambda) \times 2(\lambda - 3)^2 + a \end{cases}$$

$$a = n\, 3^{n-1}, \qquad 3a + b = 3^n.$$

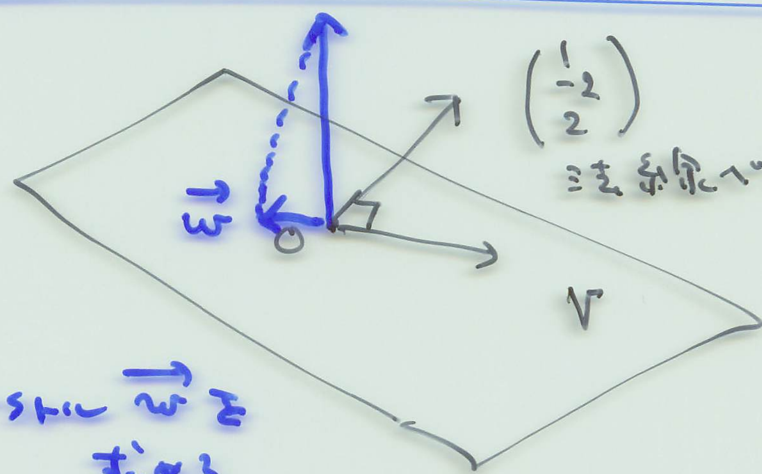
$$b = 3^n - 3a = 3^n - n\, 3^n = (1 - n)3^n$$

$$A^n = q(A)\overbrace{(A - 3I_2)^2}^{0_2} + aA + b$$

$$= aA + bI_2$$

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} ;\ x - 2y + 2z = 0 \right\}$$

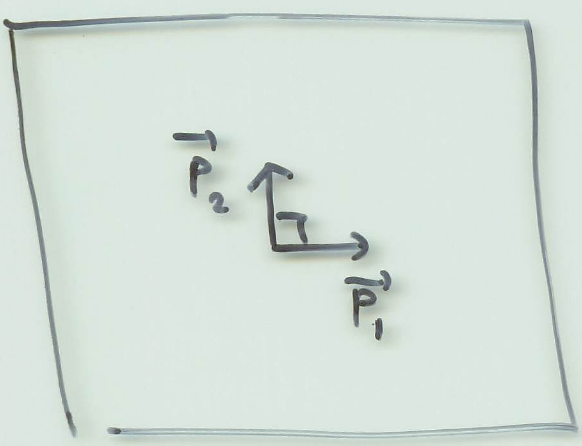
$\vec{v} \in \mathbb{R}^3$

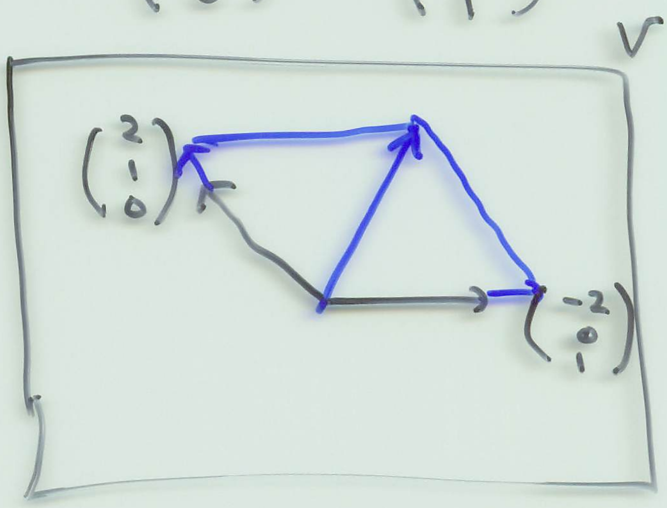


$$\begin{cases} \vec{w} \in V \\ \vec{v} - \vec{w} \perp V \end{cases}$$

をみたすベクトル $\vec{w}$ を求める.

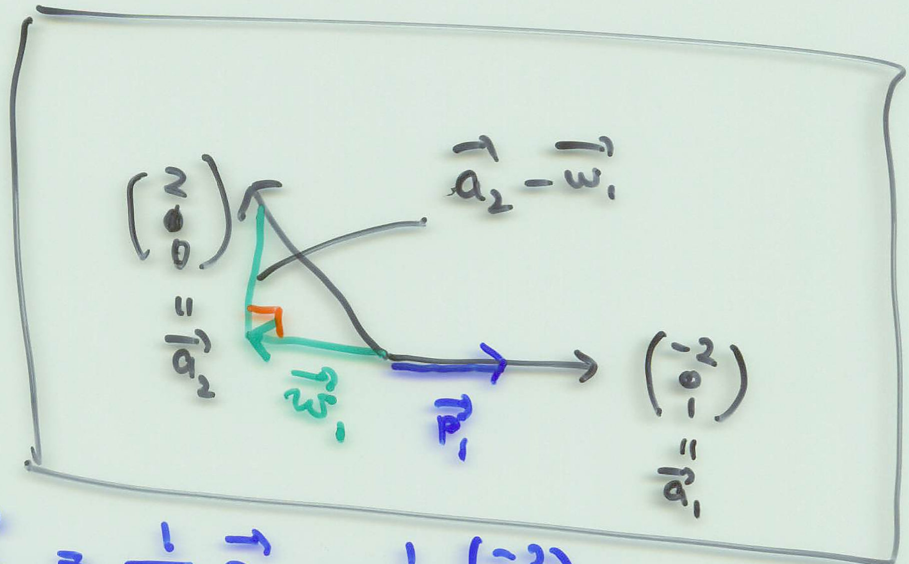
$$V \ni \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2y - 2z \\ y \\ z \end{pmatrix} = y \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$





$$\begin{cases} \|\vec{p}_1\| = \|\vec{p}_2\| = 1 \\ (\vec{p}_1, \vec{p}_2) = 0 \end{cases}$$

正規直交基底



$$\vec{p}_1 = \frac{1}{\|\vec{a}_1\|}\vec{a}_1 = \frac{1}{\sqrt{5}}\vec{a}_1 = \frac{1}{\sqrt{5}}\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$$\boxed{\vec{w}_1 = t\,\vec{p}_1 , \quad \vec{a}_2 - \vec{w}_1 \perp \vec{p}_1}$$

$\vec{w}_1$: $\vec{a}_2$ の $\vec{p}_1$ 方向への正射影.

$$\vec{w}_1 = \frac{1}{\|\vec{p}_1\|^2}(\vec{p}_1 , \vec{a}_2)\,\vec{p}_1$$

$$= \left(\frac{1}{\sqrt{5}}\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right) \times \frac{1}{\sqrt{5}}\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$$= -\frac{4}{5}\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{8}{5} \\ 0 \\ -\frac{4}{5} \end{pmatrix}$$

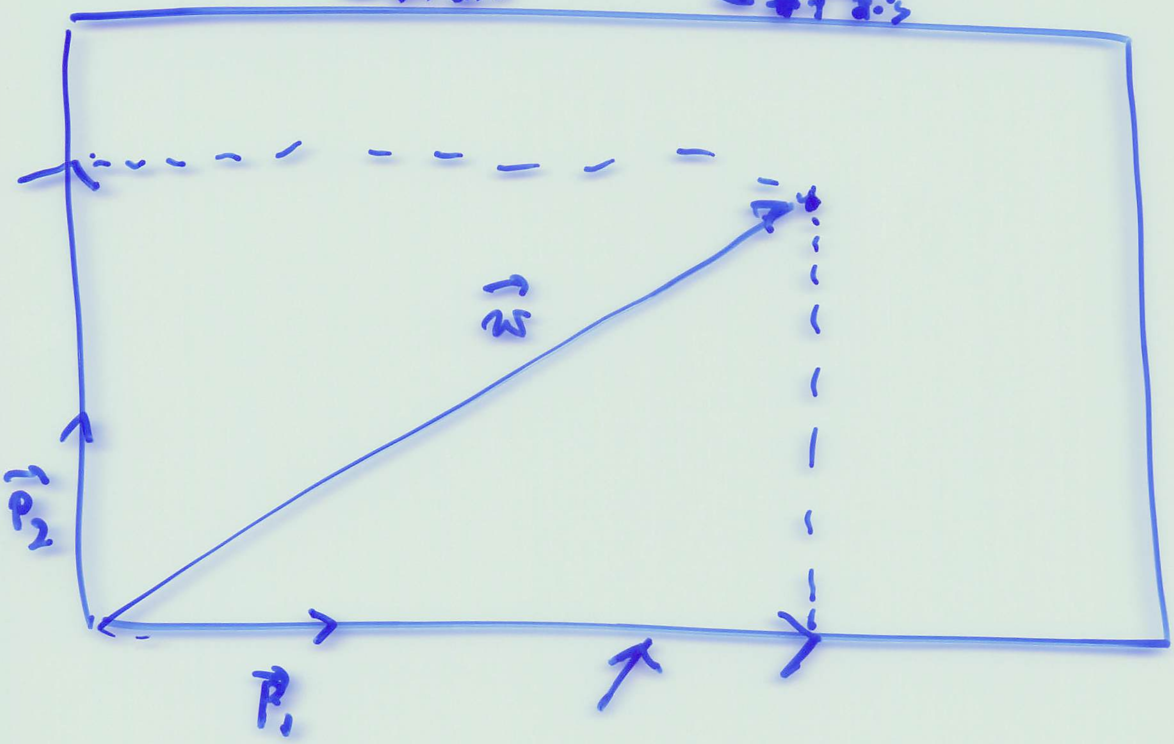
$$\vec{a}_2 - \vec{w}_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{8}{5} \\ 0 \\ -\frac{4}{5} \end{pmatrix} = \begin{pmatrix} \frac{2}{5} \\ \frac{5}{5} \\ \frac{4}{5} \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}$$

$$\vec{p}_2 = \frac{1}{\sqrt{2^2 + 5^2 + 4^2}}\begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix} = \frac{1}{3\sqrt{5}}\begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}$$

$\vec{v} \in \mathbb{R}^3$ の V への正射影を $\vec{w}$

$$\vec{w} = (\vec{p}_1, \vec{v})\,\vec{p}_1 + (\vec{p}_2, \vec{v})\,\vec{p}_2$$

$\vec{v}$ の $\vec{p}_1$ 方向への正射影　　$\vec{v}$ の $\vec{p}_2$ 方向への正射影



I　ケーリー・ハミルトンの定理を用いて

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$$

に対して A^n を求める.

II.　$V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} ;\ x + 3y - z = 0 \right\}$ の

正規直交基底を求める.

ヒント

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3y+z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$\vec{a}$ 方向への $\vec{b}$ の正射影は　$\dfrac{(\vec{a}, \vec{b})}{\|\vec{a}\|^2} \times \vec{a}$