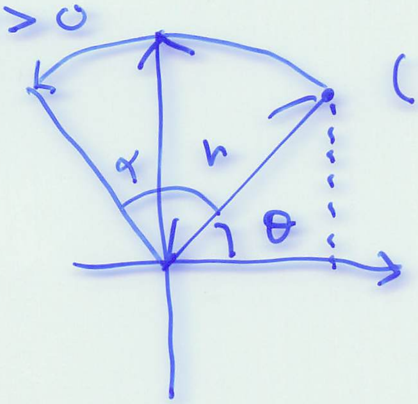
Ⅱ $\begin{pmatrix} \boxed{\cos\alpha \quad -\sin\alpha} \\ \boxed{\sin\alpha \quad \cos\alpha} \end{pmatrix} \begin{pmatrix} \boxed{\begin{matrix}\cos\beta \\ \sin\beta\end{matrix}} & \boxed{\begin{matrix}-\sin\beta \\ \cos\beta\end{matrix}} \end{pmatrix}$

$$= \begin{pmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -\cos\alpha\sin\beta - \sin\alpha\cos\beta \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix}$$

$r > 0$



$(r\cos\theta, r\sin\theta)$

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = \begin{pmatrix} r\cos(\theta+\alpha) \\ r\sin(\theta+\alpha) \end{pmatrix}$$

$$\begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{pmatrix}$$

折り返し

いつか…

$$R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$R(\alpha)\,R(\beta) = R(\alpha+\beta)$$

$$R(\alpha)^{-1} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos(-\alpha) & -\sin(-\alpha) \\ \sin(-\alpha) & \cos(-\alpha) \end{pmatrix} = R(-\alpha)$$

II $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\qquad A^2 - (a+d)A = ?$

$$A^2 - (a+d)A$$

$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \boxed{\begin{matrix} a \\ c \end{matrix}} & \boxed{\begin{matrix} b \\ d \end{matrix}} \end{pmatrix} - (a+d)\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} a^2 + bc & b(a+d) \\ c(a+d) & d^2 + bc \end{pmatrix} - \begin{pmatrix} a^2 + ad & b(a+d) \\ c(a+d) & d^2 + ad \end{pmatrix}$$

$$= \begin{pmatrix} bc - ad & 0 \\ 0 & bc - ad \end{pmatrix}$$

$$= (bc - ad)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ “} I_2$$

$$A^2 - (a+d)A + \det(A)\, I_2 = O_2$$

$$O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

ケーリー・・ハミルトンの定理.

Cayley - Hamilton

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$|A - \lambda I_2| = \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = (a-\lambda)(d-\lambda) - bc$$

$$= \lambda^2 - (a+d)\lambda + (ad - bc)$$

Aの固有多項式.

$$\lambda \longleftrightarrow A, \quad 1 \longleftrightarrow I_2$$

$\rightsquigarrow$ ケーリー・ハミルトンの定理.

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$$

ケーリー・ハミルトンの定理.

$$A^2 - 4A - 5I_2 = O_2$$

$$\boxed{a_{n+2} - 4a_{n+1} - 5a_n = 0 \text{ の解法を思い出す.}}$$

$$\lambda^2 - 4\lambda - 5 = 0 \quad (\lambda - 5)(\lambda + 1) = 0$$

$$\Longleftrightarrow \lambda = 5, -1$$

$$\begin{cases} A^2 - 5A = -(A - 5I_2) \\ A^2 + A = 5(A + I_2) \end{cases}$$

$$\begin{cases} A(A - 5I_2) = -(A - 5I_2) \\ A(A + I_2) = 5(A + I_2) \end{cases}$$

$$A^2(A - 5I_2) = A \cdot \overbrace{A(A - 5I_2)}^{=(-1)(A-5I_2)}$$

$$= (-1)A(A - 5I_2)$$

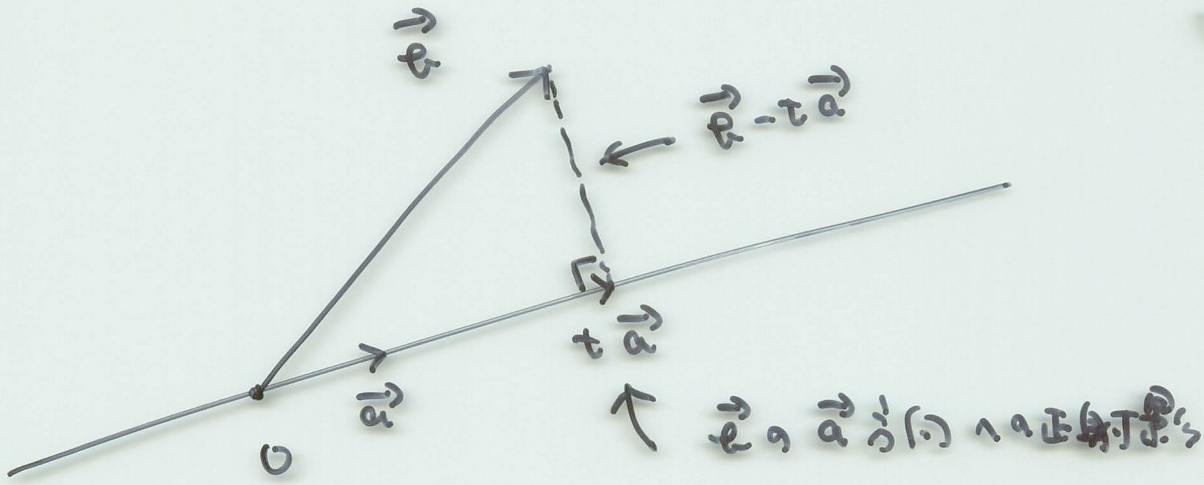
$$= (-1)^2(A - 5I_2)$$

$$\begin{cases} A^n(A-5I_2) = (-1)^n(A-5I_2) \\ A^n(A+I_2) = 5^n(A+I_2) \end{cases}$$

$$\begin{array}{rrcl} & A^{n+1} - 5A^n & = & (-1)^n(A-5I_2) \\ -) & A^{n+1} + A^n & = & 5^n(A+I_2) \\ \hline & -6A^n & = & (-1)^n(A-5I_2) \\ & & & -5^n(A+I_2) \end{array}$$

$$A^n = -\frac{1}{6}\left\{(-1)^n(A-5I_2) - 5^n(A+I_2)\right\}$$

別の解法もあり.



$$(\vec{b} - t\vec{a}, \vec{a}) = 0$$

$$\parallel$$

$$(\vec{b}, \vec{a}) - t(\vec{a}, \vec{a})$$

$$\parallel$$

$$(\vec{a}, \vec{b}) - t\|\vec{a}\|^2$$

$$t = \frac{(\vec{a}, \vec{b})\,\vec{a}}{\|\vec{a}\|^2}$$

$$\vec{a} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$$\vec{v} = \frac{4}{5}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4/5 \\ 8/5 \end{pmatrix}$$

$\frac{a}{b}$

a over b

a sur b

V

$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 2 & 1 & -1 & 1 \\ 3 & -2 & 1 & c \end{array}\right)$$ が解を持つ c を求める.

$2r \mathrel{+}= 1r \times (-2)$

$3r \mathrel{+}= 1r \times (-3)$

$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 7 & -5 & -7 \\ 0 & 7 & -5 & c-12 \end{array}\right)$$

$\downarrow$ $3r \mathrel{+}= 2r \times (-1)$

$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 7 & -5 & -7 \\ 0 & 0 & 0 & c-5 \end{array}\right)$$

$c \neq 5$ とすると

$$0x + 0y + 0z = c-5 \neq 0$$

には解なし.

解がある $\Longrightarrow$ $c = 5$ (必要条件)

<u>$c=5$ のとき</u>

$2r \times = \frac{1}{7}$

$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 1 & -\frac{5}{7} & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{1r \mathrel{+}= 2r \times 3 \text{ で}} \left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{7} & 1 \\ 0 & 1 & -\frac{5}{7} & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

z にはpivotなし

$z = \alpha$

$$\begin{cases} x - \frac{1}{7}z = 1 \\ y - \frac{5}{7}z = -1 \end{cases}$$

$$\begin{pmatrix} x = \frac{1}{7}\alpha + 1 \\ y = \frac{5}{7}\alpha - 1 \\ z = \alpha \end{pmatrix}$$

I　$C = \begin{pmatrix} 5 & 2 \\ -3 & 0 \end{pmatrix}$ に対して ケーリー・ハミルトンの定理を用いて C^n を求めよ.

II　I の C に対して

(1) $|C - \lambda I_2| = 0$ の解 λ を求めよ.

$\begin{pmatrix} x \\ y \end{pmatrix}$

(2) (1)の λ に対して

$(C - \lambda I_2)\vec{v} = 0$ の解を求めよ.

III

$$\begin{cases} x - 3y + 2z - w = 0 \\ -2x + 6y - 3z + w = 0 \\ 3x - 9y + 5z - 2w = 0 \\ 2x - 6y + z + w = 0 \end{cases}$$

を解け.