$w = x^3 + y^3 + z^3 - 3xz - 3y^2$

$$\begin{cases} w_x = 3x^2 - 3z = 3(x^2 - z) = 0 & \cdots ① \\ w_y = 3y^2 - 6y = 3(y-2)y = 0 \\ w_z = 3z^2 - 3x = 3(z^2 - x) = 0 & \cdots ② \end{cases}$$

$$\left.\begin{array}{l} ① \Leftrightarrow z = x^2 \\ ② \Leftrightarrow x = z^2 \end{array}\right\} \longrightarrow \left[\begin{array}{l} x = x^4 \\ x = 0 \text{ or } x^3 = 1 \end{array}\right]$$

$x = 0$ or $x = 1$ が必要.

$z = 0, 1$

$③ \Leftrightarrow y = 0$ or $y = 2$

$(x, y, z) = (0,0,0), (1,0,1), (0,2,0), (1,2,1)$

$H_{xx} = 6x$, $H_{xy} = 0$, $H_{xz} = -3$

$H_{yx} = 0$, $H_{yy} = 6y - 6$, $H_{yz} = 0$

$H_{zx} = -3$, $H_{zy} = 0$, $H_{zz} = 6z$

$$\begin{bmatrix} 6x & 0 & -3 \\ 0 & 6y-6 & 0 \\ -3 & 0 & 6z \end{bmatrix}$$

$(0,0,0)$

~~$F(x, z$~~

$f(x, y, z)$ が極小. (仮)

$\Downarrow$

$F(x, z) = f(x, 0, z)$

$$\begin{bmatrix} 6x & -3 \\ -3 & 6 \end{bmatrix} = \begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix}$$

$\det(\quad) = -9 < 0$

$F(x, z)$ を $(0, 0)$ で 極小. (正)

$\to$ 矛盾.

$F(x, y)$, $F_x(a, b) = F_y(a, b) = 0$.

$\det(H(F)(a, b)) < 0 \Longrightarrow (a, b)$ は 極小でも 極大でもない.

$H(F)(a,b)$ の固有値

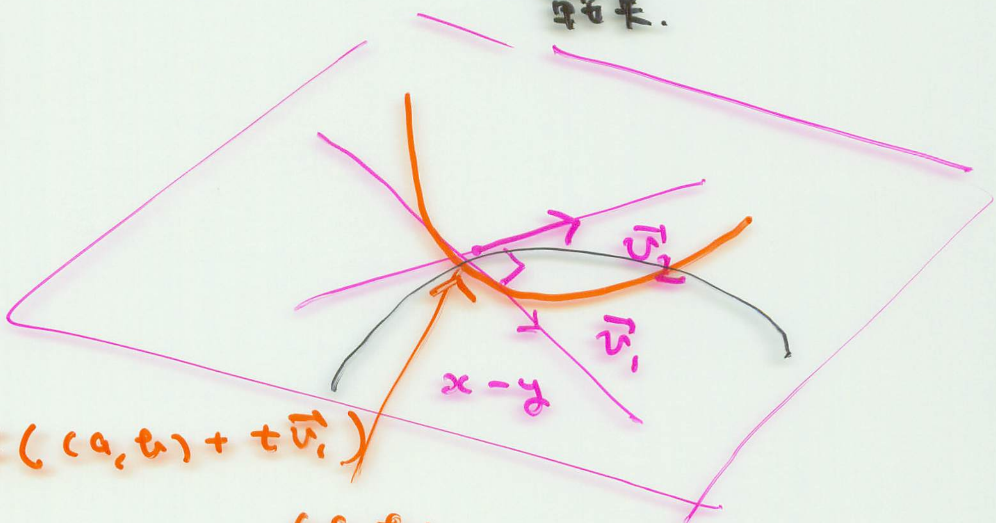
$\alpha > 0$, $\beta < 0$

$\downarrow \qquad \downarrow$

$\vec{v}_1 \perp \vec{v}_2$

$\det(H(F)(a,b)) < 0$ のとき

鞍点.



(a,b)

$G_1(t) = F((a,b) + t\vec{v}_1)$

$G_1'(0) = 0$

$G_1''(0) = (H(F)(a,b)\vec{v}_1, \vec{v}_1)$

$= \alpha \|\vec{v}_1\|^2 > 0$

$G_2(t) = F((a,b) + t\vec{v}_2)$

$G_2'(0) = 0$, $G_2''(0) = \beta \|\vec{v}_2\|^2 < 0$

$(1,0,1)$ $\begin{bmatrix} \boxed{\begin{matrix} 6 & 0 \\ 0 & -6 \end{matrix}} & \begin{matrix} -3 \\ 0 \end{matrix} \\ \begin{matrix} -3 & 0 \end{matrix} & 6 \end{bmatrix}$ $F(x,y) = f(x,y,1)$

$\det(H(F)(1,0)) = \begin{vmatrix} 6 & 0 \\ 0 & -6 \end{vmatrix} = -36 < 0$

⇝ 極大でも極小でもない.

$(0,2,0)$ $\begin{bmatrix} 0 & 0 & -3 \\ 0 & 6 & 0 \\ -3 & 0 & 0 \end{bmatrix}$ $F(x,z) = f(x,2,z)$

$\begin{vmatrix} 0 & -3 \\ -3 & 0 \end{vmatrix} = -9 < 0.$

極大でも極小でもない.

$(1,2,1)$

極小.

$\begin{bmatrix} 6 & 0 & -3 \\ 0 & 6 & 0 \\ -3 & 0 & 6 \end{bmatrix}$

$\det(3) = 3 > 0$

$\det\begin{pmatrix} 3 & 0 \\ 0 & 6 \end{pmatrix} = 18 > 0$

$$\begin{vmatrix} 6 & 0 & -3 \\ 0 & 6 & 0 \\ -3 & 0 & 6 \end{vmatrix} = \cancel{\begin{vmatrix} 6 & 0 & -3 \\ 0 & 6 & 0 \end{vmatrix}} = 6 \begin{vmatrix} 6 & -3 \\ -3 & 6 \end{vmatrix}$$

$$= 6 \times 27 > 0$$

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$$\vec{y}(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$$

$$\frac{d}{dt}\vec{y}(t) = \begin{pmatrix} y_1'(t) \\ y_2'(t) \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$$

$$\boxed{\frac{d}{dt}\vec{y}(t) = A\vec{y}(t)}$$

Aの固有値. $\begin{vmatrix} \lambda-1 & -12 \\ -3 & \lambda-1 \end{vmatrix} = (\lambda-1)^2 - 36 = 0$

$$\lambda = 1 \pm 6 \rightsquigarrow 7, -5.$$

$\underline{\lambda = 7}$

$$\begin{pmatrix} 6 & -12 \\ -3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \Leftrightarrow x - 2y = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2y \\ y \end{pmatrix} = y\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\underline{\lambda = -5}$

$$\begin{bmatrix} -6 & -12 \\ -3 & -6 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \Leftrightarrow x + 2y = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2y \\ y \end{pmatrix} = y\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\vec{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \qquad P = (\vec{v}_1\ \vec{v}_2)$$

$$AP = P\begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}$$

$$\begin{cases} \dfrac{d}{dt}\vec{y}(t) = A\,PP^{-1}\,\vec{y}(t) \\ \end{cases} \qquad AP = P\begin{pmatrix}7 & 0\\ 0 & -5\end{pmatrix}$$

$\vec{z}(t) = P^{-1}\vec{y}(t)$ とする.

$A(t),\ B(t)$

$m\times n \quad n\times l.$

$$\boxed{\begin{aligned}&\frac{d}{dt}\big(A(t)B(t)\big)\\ &= \frac{d}{dt}A(t)\cdot B(t)\\ &\quad + A(t)\frac{d}{dt}B(t)\end{aligned}}$$

$(fg)' = f'g + fg'$

$$\begin{aligned}
\frac{d}{dt}\vec{z}(t) &= \underbrace{(P^{-1})'}_{O_2}\vec{y}(t) \\
&\quad + P^{-1}\frac{d}{dt}\vec{y}(t) \\
&= P^{-1}\frac{d}{dt}\vec{y}(t) \\
&= P^{-1}A\,\vec{y}(t) \\
&= P^{-1}A\underbrace{P\cdot P^{-1}}_{I_2}\underbrace{\vec{y}(t)}_{} \qquad (P^{-1}\vec{y}(t) = \vec{z}(t)) \\
&= \begin{pmatrix}7 & 0\\ 0 & -5\end{pmatrix}\vec{z}(t) \rightsquigarrow \frac{d}{dt}\vec{z}(t) = \begin{pmatrix}7 & 0\\ 0 & -5\end{pmatrix}\vec{z}(t)
\end{aligned}$$

$$\vec{z}(t) = \begin{pmatrix} z_1(t)\\ z_2(t)\end{pmatrix}$$

$\Updownarrow$

$$\begin{cases} z_1'(t) = 7z_1(t) \\ z_2'(t) = -5z_2(t)\end{cases}$$

$F'(t) = cF(t) \qquad c$: tに依らない定数.

$\downarrow$

$F(t) = F(0)e^{ct}$

$\left(F(t)e^{-ct}\right)' = \cdots \equiv 0$

$F(t)e^{-ct} = K$ (定数)

$t=0$を代入

$F(0) = K$

$\rightsquigarrow F(t) = F(0)e^{ct}$

$$z_1(t) = z_1(0)\, e^{7t}$$
$$z_2(t) = z_2(0)\, e^{-5t}$$

$$\vec{z}(t) = P^{-1}\vec{y}(t)$$

$$P^{-1}\vec{y}(t) = \begin{pmatrix} z_1(0)\, e^{7t} \\ z_2(0)\, e^{-5t} \end{pmatrix}$$
$$= \begin{pmatrix} e^{7t} & 0 \\ 0 & e^{-5t} \end{pmatrix} \begin{pmatrix} z_1(0) \\ z_2(0) \end{pmatrix}$$
$$= \begin{pmatrix} e^{7t} & 0 \\ 0 & e^{-5t} \end{pmatrix} P^{-1}\vec{y}(0)$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$
$$\parallel$$

$$\vec{y}(t) = P \begin{pmatrix} e^{7t} & 0 \\ 0 & e^{-5t} \end{pmatrix} P^{-1}\vec{y}(0)$$

$$= (\vec{v}_1\ \vec{v}_2) \begin{pmatrix} e^{7t} & 0 \\ 0 & e^{-5t} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$
$$= c_1 e^{7t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{-5t} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$\square$

A : 2×2. 固有値. $\alpha, \beta \in \mathbb{R}$

$\alpha \neq \beta$

$$\frac{d}{dt}\vec{y}(t) = A\vec{y}(t) \qquad \alpha \longleftrightarrow \vec{v}_1$$
$$\beta \longleftrightarrow \vec{v}_2$$

$$\vec{y}(t) = c_1 e^{\alpha t}\vec{v}_1 + c_2 e^{\beta t}\vec{v}_2$$

(I) $\beta = 0$. 244 page.

(i) $\alpha < 0$

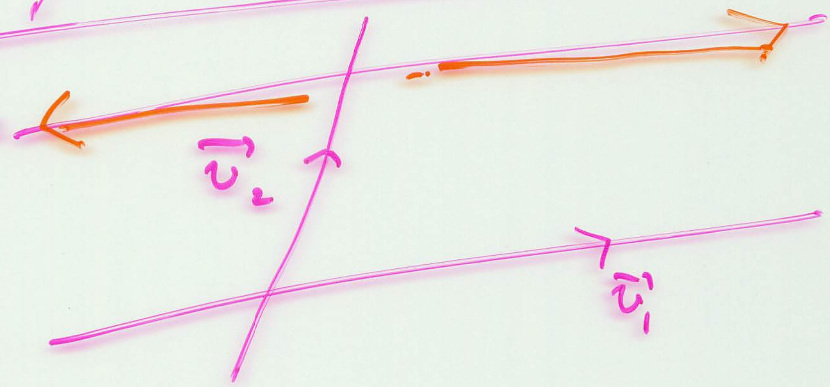
$$\vec{y}(t) = c_1 e^{\alpha t}\vec{v}_1 + c_2\vec{v}_2, \qquad e^{\alpha t} \to 0$$

$\downarrow$

$c_2\vec{v}_2$

$t \to +\infty$

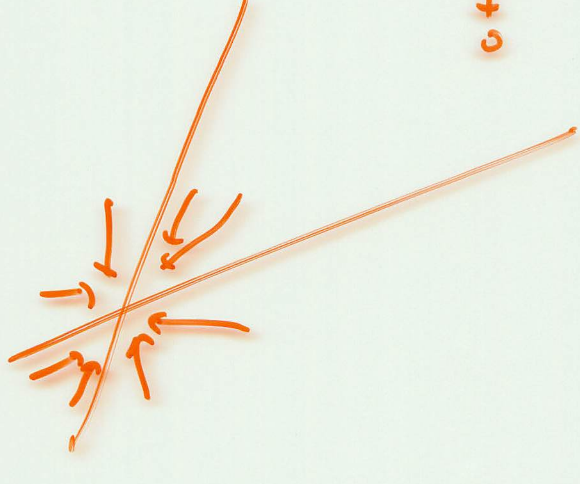
(ii) $\alpha > 0$



(II) $\alpha, \beta < 0$

$$\vec{y}(t) = c_1 \underbrace{e^{\alpha t}}_{\downarrow\, 0}\vec{v}_1 + c_2 \underbrace{e^{\beta t}}_{\downarrow\, 0}\vec{v}_2$$

$\to \vec{0}$



$$A = \begin{pmatrix} 5 & 3 \\ -6 & -4 \end{pmatrix}$$ とする.

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$$

を解け.