key word　直交行列、対称行列の直交化.
(2次形式)

P : n次正方行列.
　ℝ値

P : 直交 (orthogonal)

$$\Leftrightarrow \quad (P\vec{x}, P\vec{y}) = (\vec{x}, \vec{y})$$

$$\Leftrightarrow \quad \| P\vec{x} \| = \| \vec{x} \|$$

$$\Leftrightarrow \quad {}^tPP = P\,{}^tP = I_n \qquad P = (\vec{p}_1\ \vec{p}_2 \cdots \vec{p}_n)$$

$$\Leftrightarrow \quad (\vec{p}_i, \vec{p}_j) = \begin{cases} 1 & (i = j) \\ 0 & (i \neq j) \end{cases}$$

$${}^tPP = \begin{pmatrix} {}^t\vec{p}_1 \\ {}^t\vec{p}_2 \\ \vdots \\ {}^t\vec{p}_n \end{pmatrix} (\vec{p}_1\ \vec{p}_2 \cdots \boxed{\vec{p}_n})$$

$$= \begin{pmatrix} {}^t\vec{p}_1\vec{p}_1 & \cdots & {}^t\vec{p}_1\vec{p}_n \\ {}^t\vec{p}_2\vec{p}_1 & & {}^t\vec{p}_2\vec{p}_n \\ \vdots & & \vdots \\ {}^t\vec{p}_n\vec{p}_1 & \cdots & {}^t\vec{p}_n\vec{p}_n \end{pmatrix} = I_n$$

$$(\vec{a}, \vec{b}) = {}^t\vec{a}\,\vec{b} = (a_1 \cdots a_n)\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \sum_{i=1}^{n} a_i b_i$$

$$\vec{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

$A : m \times n \qquad A = (\vec{a}_1 \cdots \vec{a}_n)$

$$A\vec{x} = A\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = x_1\vec{a}_1 + x_2\vec{a}_2 + \cdots + x_n\vec{a}_n$$

$$\begin{array}{ccc} \mathbb{R}^n & \longrightarrow & \mathbb{R}^m \\ \vec{x} & \longmapsto & A\vec{x} \end{array} \qquad m\Big\{\overbrace{\begin{array}{|c|}\hline \vec{a}_1 \cdots \vec{a}_n \\ \hline \end{array}}^{n}$$

$\vec{a}_j \in \mathbb{R}^m$

<u>定理</u> $(A\vec{x}, \vec{y}) = (\vec{x}, {}^tA\vec{y})$

$\vec{y} \in \mathbb{R}^m$

${}^tA : n \times m, \quad {}^tA\vec{y} \in \mathbb{R}^n$

$$(P\vec{x}, P\vec{y}) = (\vec{x}, \vec{y}) \qquad ({}^tPP - I_n)\vec{y} = 0$$

$$\| \qquad\qquad\qquad\qquad \downarrow$$

$$(\vec{x}, {}^tPP\vec{y}) \qquad\qquad {}^tPP - I_n = O_n$$

$$\rightsquigarrow (\vec{x}, ({}^tPP - I_n)\vec{y}) = 0 \qquad {}^tPP = I_n$$

$$\boxed{\text{全ての } \vec{x} \text{ に対して } (\vec{x}, \vec{v}) = 0 \iff \vec{v} = \vec{0}}$$

$\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \longrightarrow$ $\vec{x} = \vec{v}$ とおく. $\rightarrow \|\vec{v}\| = 0$

$(\vec{v}, \vec{v}) = 0 \qquad \Updownarrow$

$\| \qquad \vec{v} = 0.$

$\|\vec{v}\|^2$

$\vec{x} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \rightsquigarrow v_1 = 0 \quad \cdots\cdots$

C : n次正方行列

$$\boxed{\begin{gathered} C\vec{x} = \vec{0} \ (全ての\ \vec{x} \in \mathbb{R}^n) \\ \Longleftrightarrow \quad C = O_n. \end{gathered}}$$

$\Rightarrow$ $\quad C = (\vec{c}_1\ \vec{c}_2 \cdots \vec{c}_n) \quad \vec{e}_j = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow j$

前提

$C\vec{e}_j = \vec{c}_j = \vec{0}$ (全てのj)

すべてのj

$\rightsquigarrow C = O_n$

$$\boxed{(\vec{c}_1\ \vec{c}_2 \cdots \vec{c}_n)\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = x_1\vec{c}_1 + \cdots + x_n\vec{c}_n}$$

P : 直交 $\Longleftrightarrow (P\vec{x}, P\vec{y}) = (\vec{x}, \vec{y})$

$\Longleftrightarrow \|P\vec{x}\| = \|\vec{x}\|$

$\Longleftrightarrow {}^tPP = P\,{}^tP = I_n \quad \Longleftrightarrow {}^tPP = I_n$

$\Longleftrightarrow (\vec{p}_i, \vec{p}_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

${}^tPP = I_n$

$\downarrow \quad \det(I_n) = 1$

$\det({}^tP)\det(P) = 1$

$\parallel$

$\det(P)^2 = 1$

$\rightsquigarrow \det(P) = \pm 1$

A : n次正方行列

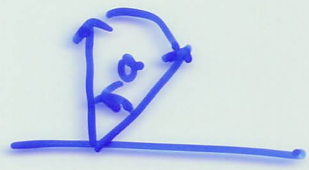
- $\det({}^tA) = \det(A)$
- $\det(AB) = \det(A)\det(B)$

P: $\rightsquigarrow$ 正則 $\quad {}^tPP = I_n \quad {}^tP = P^{-1}$

P $\underline{n=2}$ 　 · $\det(P) = 1$

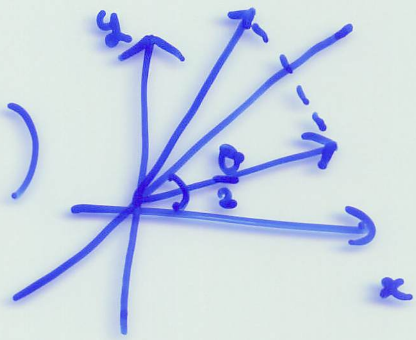
P: 직교행.

$$P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



· $\det(P) = 1$

$$P = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$



前々回のレポートの結果から.

$$A = \begin{pmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{pmatrix}$$

${}^tA = A$

$A = (a_{ij})$　$a_{ij} = a_{ji}$

$$\Phi_A(\lambda) = \det(\lambda I_3 - A)$$
$$= (\lambda - 2)^2(\lambda - 8)$$

$\lambda = 2$　$\vec{v}$: 固有ベクトル　$A\vec{v} = 2\vec{v}$

$$\vec{v} = \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$\rho'' = ?$

γ

$\lambda = 8$

$$A\vec{v} = 8\vec{v}$$

$$\vec{v} = \gamma \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$　$$\vec{p}_3 = \frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

定理 ① A : 対称.

$$\Phi_A(\lambda) = (\lambda - \alpha_1) \cdots (\lambda - \alpha_n)$$

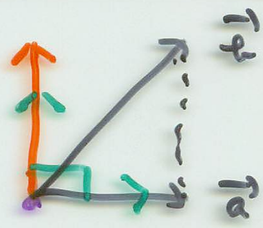
A : 対称.　α_j : 実.

② $\alpha \neq \beta$ 固有値

$$A\vec{v}_1 = \alpha \vec{v}_1, \quad A\vec{v}_2 = \beta \vec{v}_2 \qquad \vec{v}_1 \neq \vec{0}, \ \vec{v}_2 \neq \vec{0}$$

$$(\vec{v}_1, \vec{v}_2) = 0$$

$\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$



$V = \{ \vec{v} \ ; \ (A - 2I_2)\vec{v} = \vec{0} \}$

$$\vec{p_1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\|\vec{a}\|} \vec{a}$$

$\vec{b}$ の $\vec{a}$ 方向への正射影を $\vec{w}$

$$\vec{w} = \frac{(\vec{a}, \vec{b})}{\|\vec{a}\|^2} \vec{a} = \frac{\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right)}{2} \vec{a}$$

$$= \vec{a}$$

$$\vec{b} - \vec{w} = \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{p_2} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$A\vec{p_1} = 2\vec{p_1}$, $A\vec{p_2} = 2\vec{p_2}$

$\|\vec{p_1}\| = \|\vec{p_2}\| = 1$, $(\vec{p_1}, \vec{p_2}) = 0$.

$A\vec{p_3} = 8\vec{p_3}$ 前のページ

$\|\vec{p_3}\| = 1$, 定理② から

$(\vec{p_1}, \vec{p_3}) = (\vec{p_2}, \vec{p_3}) = 0$

$P = (\vec{p_1}\ \vec{p_2}\ \vec{p_3})$ 直交

$$AP = P \begin{pmatrix} 2 & & \\ & 2 & \\ & & 8 \end{pmatrix}$$

$$A = P \begin{pmatrix} 2 & & \\ & 2 & \\ & & 8 \end{pmatrix} {}^tP$$

P : 直交.

${}^tP = P^{-1}$

$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ を直交行列で対角化.

$$\Phi_A(\lambda) = (\lambda+1)^2(\lambda-1)$$

$$-I_3 - A = \begin{pmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & -1 \end{pmatrix} \longrightarrow \cdots \longrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(-I_3 - A)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{0} \iff x + z = 0$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -z \\ y \\ z \end{pmatrix} = z\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$