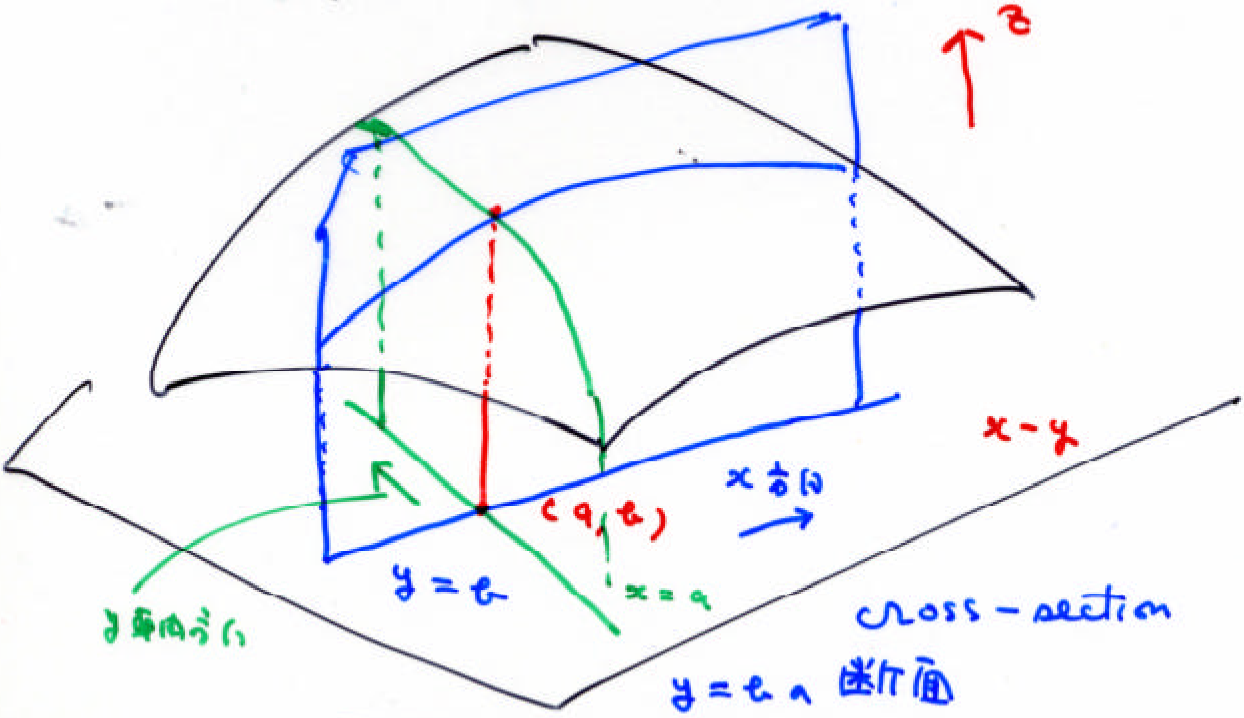
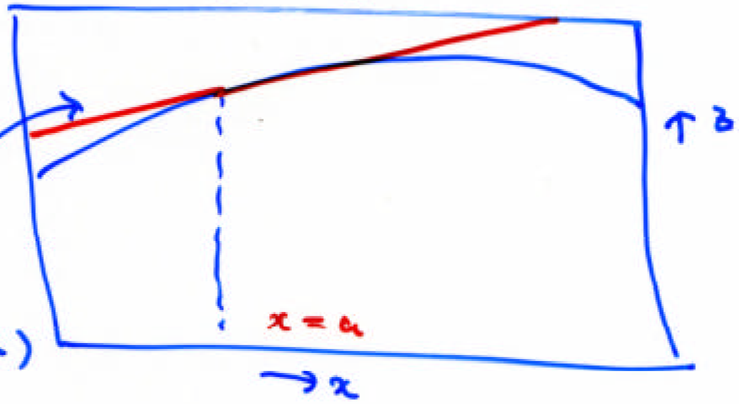
$z = f(x, y)$

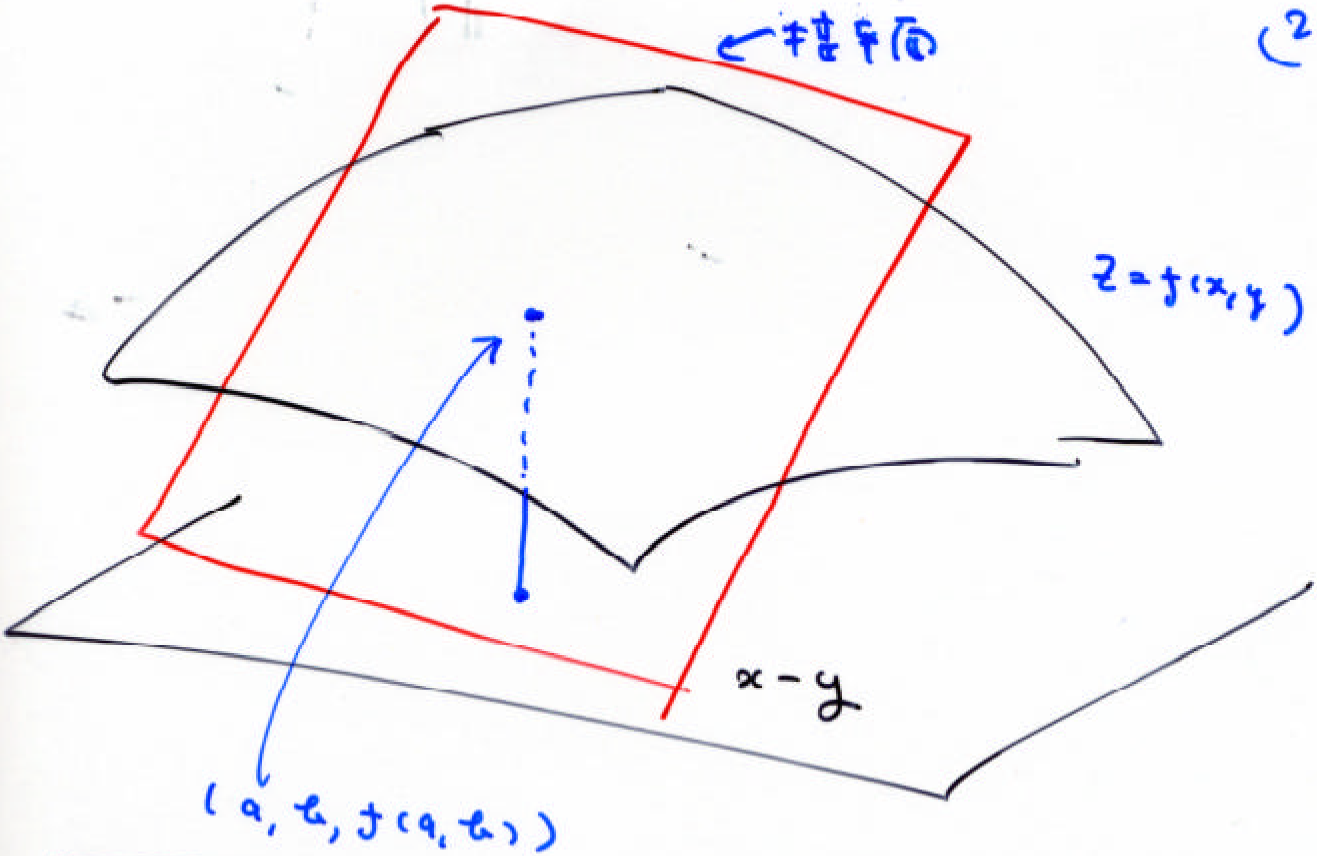


cross-section

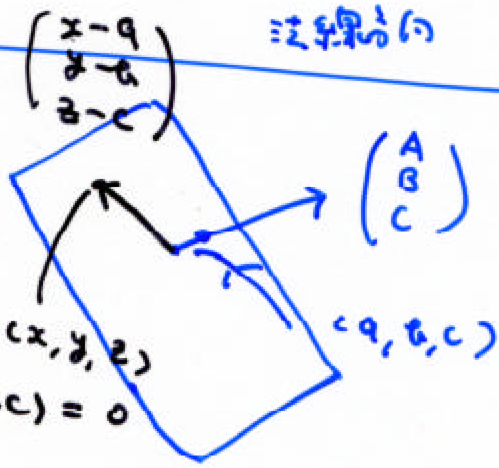
$y = b$ の断面

$z = f(x, b)$
$= F(x)$
傾き
$F'(a)$
$\parallel$
$\frac{\partial f}{\partial x}(a, b)$





平面の方程式



$$\begin{pmatrix}A\\B\\C\end{pmatrix}\cdot\begin{pmatrix}x-a\\y-b\\z-c\end{pmatrix}=0$$

内積

$$A(x-a)+B(y-b)+C(z-c)=0$$

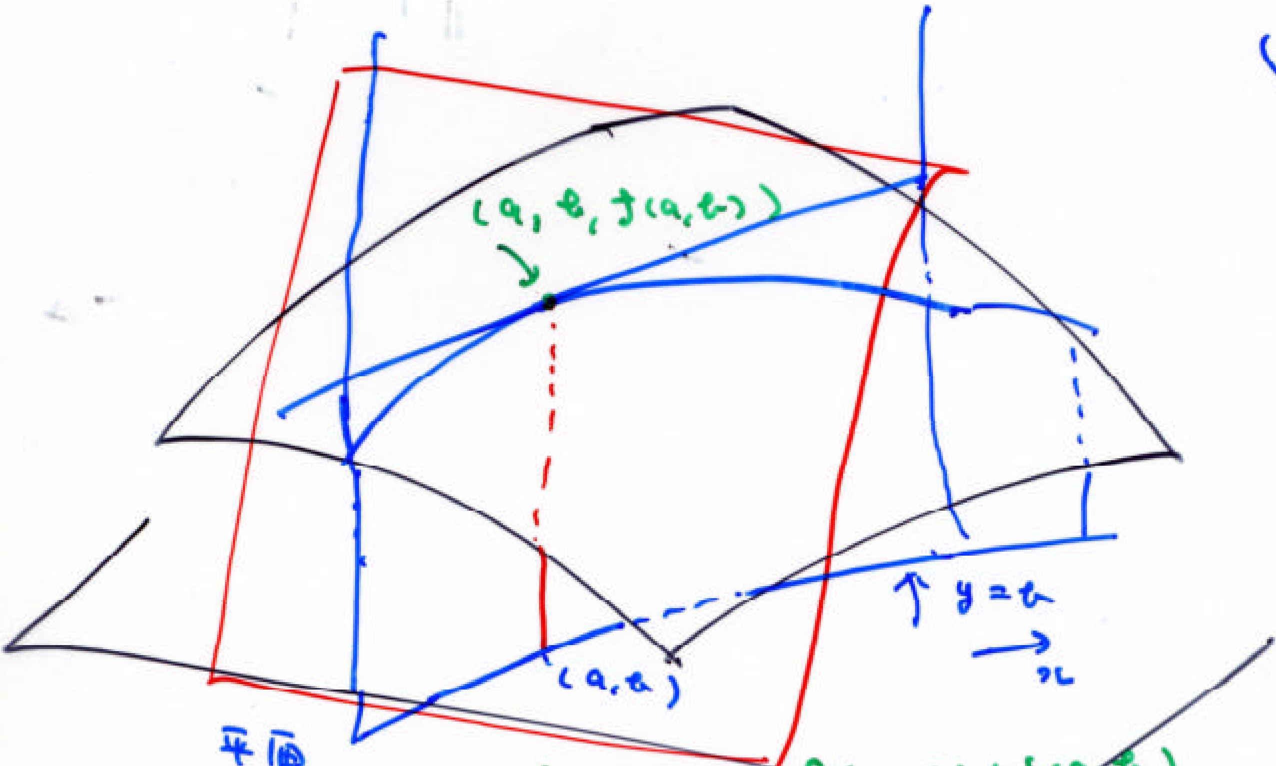
接平面を考えるには $C\neq 0$ としてよい。

$C=0 \rightsquigarrow \begin{pmatrix}A\\B\\0\end{pmatrix} /\!/$ x-y平面の
法線方向

$\rightsquigarrow$ 平面 ⊥ x-y平面 ※

$C\neq 0$ $\quad z=\boxed{-\dfrac{A}{C}}^{\,=\alpha}(x-a)\boxed{-\dfrac{B}{C}}^{\,=\beta}(y-b)+c$

$$\boxed{z=\alpha(x-a)+\beta(y-b)+c}$$

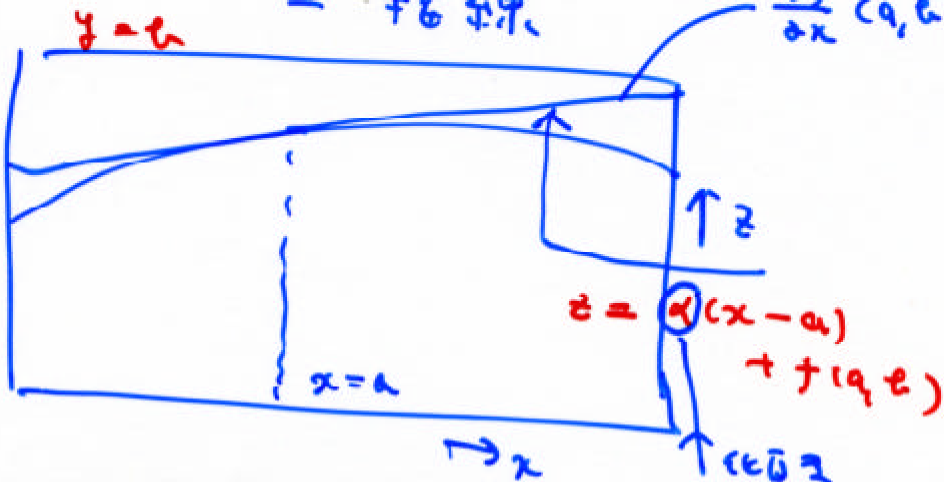


$$z = \alpha(x-a) + \beta\underbrace{(y-b)}_{=0} + f(a,b)$$

slope

[y = b] と 接平面の交わり　傾き

= 接線



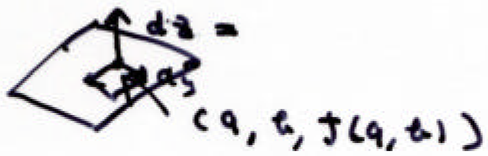
$$\boxed{\alpha = \frac{\partial f}{\partial x}(a,b)}$$

同様に　$\beta = \frac{\partial f}{\partial y}(a,b)$

公式　接線の方程式

$$z = \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b) + f(a,b)$$

$$\underbrace{z - f(a,b)}_{= dz} = \frac{\partial f}{\partial x}(a,b)\boxed{(x-a)}_{= dx} + \frac{\partial f}{\partial y}(a,b)\boxed{(y-b)}_{= dy}$$

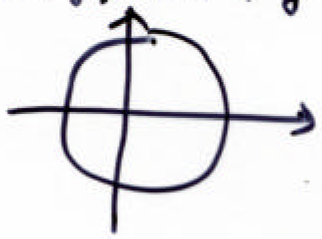


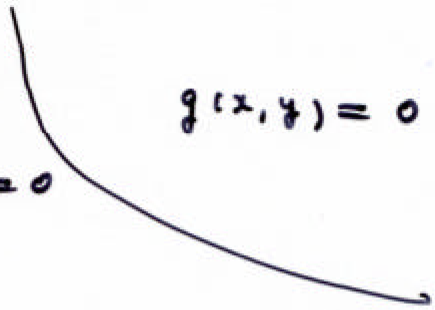
$$dz = \frac{\partial f}{\partial x}(a,b)\,dx + \frac{\partial f}{\partial y}(a,b)\,dy$$

全微分

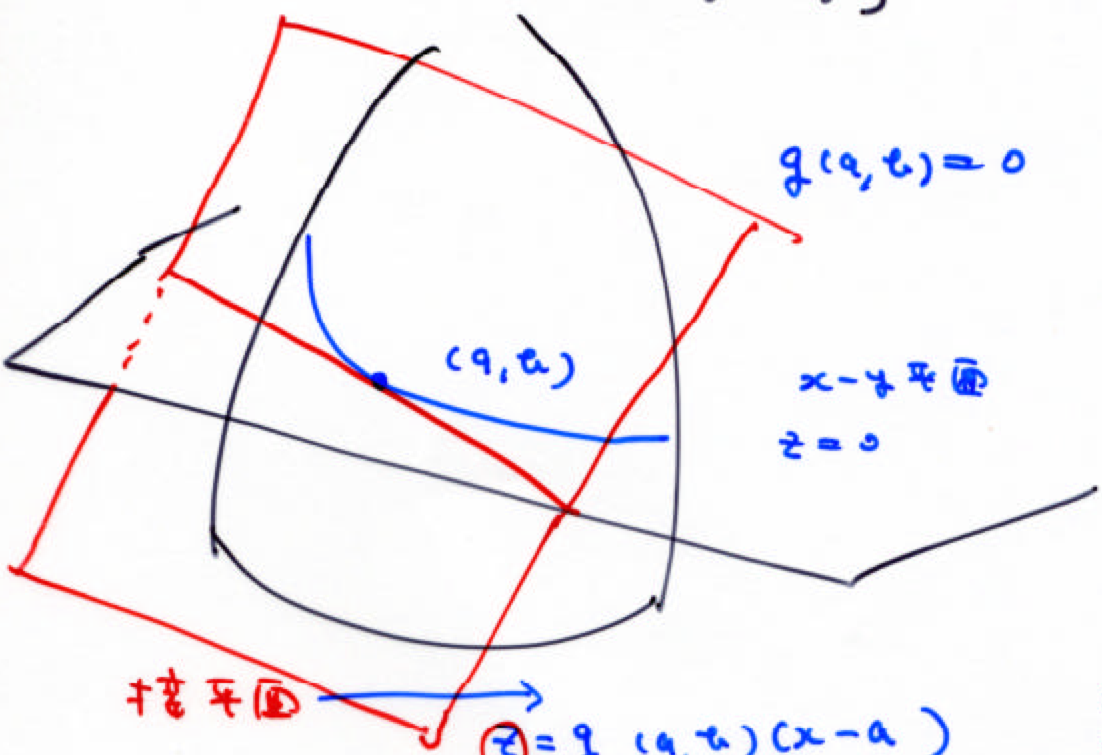
例

$g(x,y) = x^2 + y^2 - 1 = 0$





$z = g(x,y)$



$$\underbrace{z}_{=0} = g_x(a,b)(x-a) + g_y(a,b)(y-b) + \boxed{g(a,b)}^{=0}$$

接線

$$g_x(a,b)(x-a) + g_y(a,b)(y-b) = 0$$

(3)

$$g(x,y) = x^2 + y^2 - 1 = 0$$

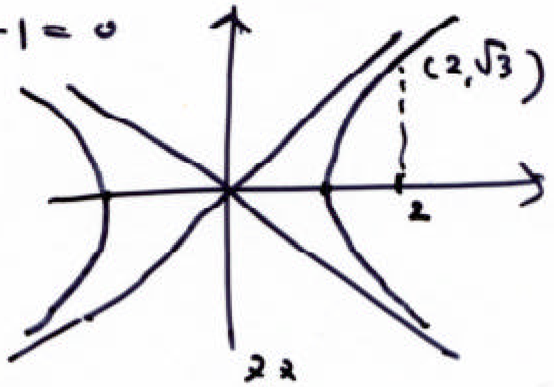
at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$g_x = 2x, \quad g_y = 2y$$

$$2 \times \frac{1}{2}\left(x - \frac{1}{2}\right) + 2 \times \frac{\sqrt{3}}{2}\left(y - \frac{\sqrt{3}}{2}\right) = 0$$

$$\left(x - \frac{1}{2}\right) + \sqrt{3}\left(y - \frac{\sqrt{3}}{2}\right) = 0$$

$$g(x,y) = x^2 - y^2 - 1 = 0$$



2次曲線

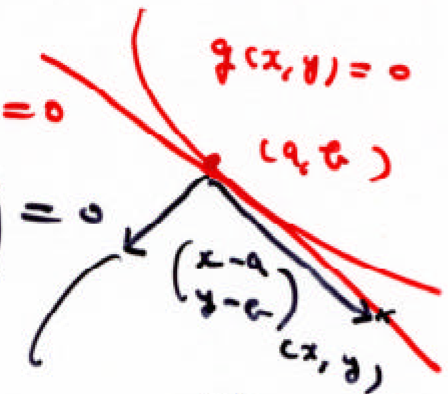
$$g_x = 2x$$

$$g_y = -2y$$

$$2 \times 2(x-2) - 2 \times \sqrt{3}(y - \sqrt{3}) = 0$$

$$\sim 2(x-2) - \sqrt{3}(y - \sqrt{3}) = 0$$

$$g_x(a,b)(x-a) + g_y(a,b)(y-b) = 0$$



$$\begin{pmatrix} g_x(a,b) \\ g_y(a,b) \end{pmatrix} \cdot \begin{pmatrix} x-a \\ y-b \end{pmatrix} = 0$$

内積

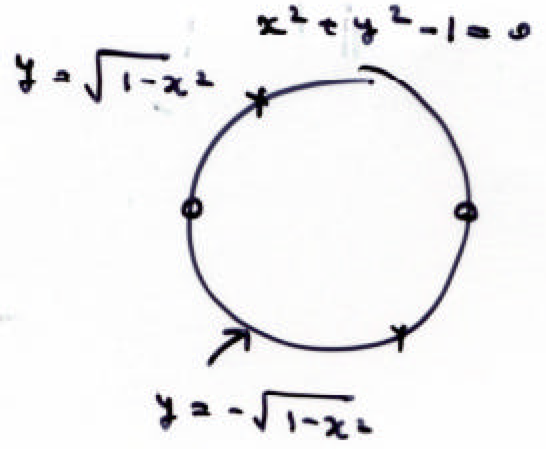
法線方向 $\longleftarrow$ $\nabla(g)(a,b) = \begin{pmatrix} g_x(a,b) \\ g_y(a,b) \end{pmatrix}$

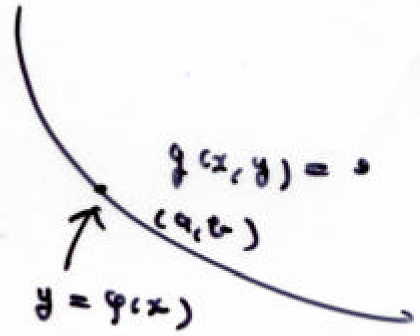
勾配ベクトル

$g_y(a,b) \neq 0$ とする.

$$y = \boxed{-\frac{g_x(a,b)}{g_y(a,b)}}(x-a) + b$$

傾き





公式 $\quad \varphi'(a) = -\dfrac{g_x(a,b)}{g_y(a,b)}$

---

g

$$\boxed{2x^2-3xy+2y^2-2x-2y}=0$$

$$2y^2-(3x+2)y+2x^2-2x=0$$

$$y=\frac{(3x+2)\pm\sqrt{(3x+2)^2-4(2x^2-2x)}}{4}$$

$y=\varphi(x)$ と書けていたとする.

$$2x^2-3x\varphi(x)+2\varphi(x)^2-2x-2\varphi(x)=0$$

$$4x-3\varphi(x)-3x\varphi'(x)+4\varphi(x)\varphi'(x)$$
$$-2-2\varphi'(x)=0$$

$$\varphi'(x)(4\varphi(x)-3x-2)$$
$$+(4x-3\varphi-2)=0$$

$$\rightsquigarrow \quad \varphi'(x)=-\frac{4x-3y-2}{4y-3x-2}$$

$g_y=-3x+4y-2$

$g_x=4x-3y-2$

$\cancel{z = e^{2x}}$

I $z = \sqrt{1-x^2-y^2}$ に対して

$(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2})$ に

おける 接平面

$$(\sqrt{t})' = \frac{1}{2}\frac{1}{\sqrt{t}}$$

II $x^2 + xy - y^2 - 1 = 0$

が… あるところで $y = \varphi(x)$ と書けたとする.

$\varphi'(x) = ?$

x と y で書く.
