$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad a, b, c, d \in \mathbb{R}$$

$$\Phi_A(\lambda) = \lambda^2 - (a+d)\lambda + \det(A)$$

の判別式 $D < 0$ の場合.

固有値 α, β $\begin{cases} \alpha = p + iq \\ \beta = p - iq \end{cases}$ $(p, q \in \mathbb{R})$

固有ベクトル

$$\alpha \longleftrightarrow \vec{v}_1 = \vec{w}_1 + i\vec{w}_2$$

$$\vec{w}_1, \vec{w}_2 \in \mathbb{R}^2$$

$$\beta \longleftrightarrow \vec{v}_2 = \vec{w}_1 - i\vec{w}_2$$

$P = (\vec{w}_1 \ \vec{w}_2)$ ← 正則となる.

$$AP = (A\vec{w}_1 \ \ A\vec{w}_2)$$

$$= \underbrace{\boxed{(\vec{w}_1 \ \vec{w}_2)}}_{P} \begin{pmatrix} p & q \\ -q & p \end{pmatrix}$$

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) \rightsquigarrow P^{-1}AP = \begin{pmatrix} p & q \\ -q & p \end{pmatrix}$$

$$\vec{z}(t) = P^{-1}\vec{x}(t)$$

$$\frac{d}{dt}\vec{z}(t) = P^{-1}\frac{d}{dt}\vec{x}(t) = P^{-1}A\vec{x}(t)$$

$$= P^{-1}AP \cdot P^{-1}\vec{x}(t)$$

$$= \begin{pmatrix} p & q \\ -q & p \end{pmatrix}\vec{z}(t)$$

$$\boxed{\frac{d}{dt}\vec{z}(t) = \begin{pmatrix} p & q \\ -q & p \end{pmatrix}\vec{z}(t)}$$ を解く.

$$X(t) = e^{pt}\begin{pmatrix} \cos qt & \sin qt \\ -\sin qt & \cos qt \end{pmatrix}$$

$$X'(t) = p\, e^{pt}\begin{pmatrix} \cos qt & \sin qt \\ -\sin qt & \cos qt \end{pmatrix}$$

$$+\ e^{pt}\begin{pmatrix} -q\sin qt & \boxed{q\cos qt} \\ -q\cos qt & \boxed{-q\sin qt} \end{pmatrix}$$

$$= e^{pt}\begin{pmatrix} \cos qt & \sin q(t) \\ -\sin qt & \cos qt \end{pmatrix}\begin{pmatrix} p & 0 \\ 0 & p \end{pmatrix}$$

$$+\ e^{pt}\begin{pmatrix} \cos qt & \sin qt \\ -\sin qt & \cos qt \end{pmatrix}\begin{pmatrix} \boxed{\begin{matrix} 0 \\ * \end{matrix}} & \boxed{\begin{matrix} q \\ 0 \end{matrix}} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -q \end{pmatrix}$$

$$= X(t)\begin{pmatrix} p & q \\ -q & p \end{pmatrix}$$

まとめ $\quad X'(t) = X(t)\begin{pmatrix} p & q \\ -q & p \end{pmatrix}$

$$= \begin{pmatrix} p & q \\ -q & p \end{pmatrix} X(t)$$

可換

$$\left(X(-t)\, \vec{z}(t) \right)'$$

$$= X(-t)\begin{pmatrix} p & q \\ -q & p \end{pmatrix} \times (-1)\, \vec{z}(t)$$

$$+\ X(-t)\cdot\begin{pmatrix} p & q \\ -q & p \end{pmatrix}\vec{z}(t)$$

$$= \vec{0}. \quad \rightsquigarrow \quad X(-t)\,\vec{z}(t) = X(0)\,\vec{z}(0)$$

$$= I_2\,\vec{z}(0) = \vec{z}(0).$$

$$X(-t)\,\vec{z}(t) = \vec{z}(0)$$

$$X(s)\,X(t)$$

$$= e^{ps}\begin{pmatrix} \cos\gamma s & \sin\gamma s \\ -\sin\gamma s & \cos\gamma s \end{pmatrix} \times e^{pt}\begin{pmatrix} \cos\gamma t & \sin\gamma t \\ -\sin\gamma t & \cos\gamma t \end{pmatrix}$$

$$= e^{p(s+t)}\begin{pmatrix} \cos\gamma(s+t) & \sin\gamma(s+t) \\ -\sin\gamma(s+t) & \cos\gamma(s+t) \end{pmatrix}$$

$$= X(s+t)$$

$$\cos\gamma s\cdot\cos\gamma t - \sin\gamma s\cdot\sin\gamma t$$

$$= \cos\gamma(s+t)$$

$$\cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \cos(\alpha+\beta)$$

$$\boxed{X(s)\,X(t) = X(s+t)}$$

$$X(-t)\,X(t) = X(0) = I_2$$

$$X(-t) = \left(X(t)\right)^{-1}.$$

$$X(-t)\,\vec{z}(t) = \vec{z}(0)$$

$$\rightsquigarrow \quad \vec{z}(t) = X(t)\,\vec{z}(0)$$

$$= e^{pt}\begin{pmatrix} \cos qt & \sin qt \\ -\sin qt & \cos qt \end{pmatrix}\vec{z}(0)$$

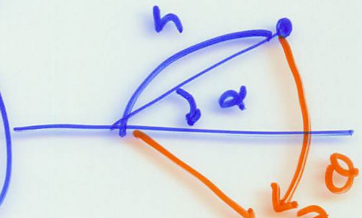
$$P^{-1}\vec{x}(t) = e^{pt}\begin{pmatrix} & \\ & \end{pmatrix}P^{-1}\vec{x}(0)$$

$$\vec{x}(t) = P\cdot e^{pt}\begin{pmatrix} & \\ & \end{pmatrix}P^{-1}\vec{x}(0).$$

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} r\cos\alpha \\ r\sin\alpha \end{pmatrix}$$

$$= r\begin{pmatrix} \cos\theta\cos\alpha + \sin\theta\sin\beta \\ \cos\theta\sin\alpha - \sin\underset{\theta}{\cancel{\alpha}}\cos\alpha \end{pmatrix}$$

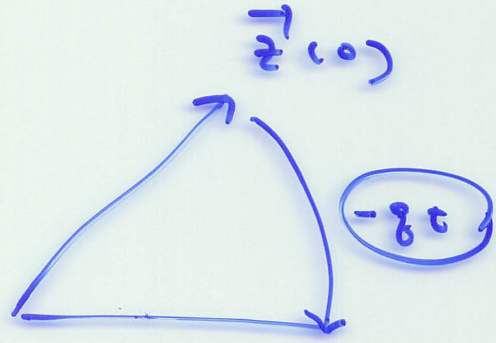
$$= r\begin{pmatrix} \cos(\alpha-\theta) \\ \sin(\underset{\alpha-\theta}{\cancel{\theta-\alpha}}) \end{pmatrix}$$



$(-\theta)$의 회전

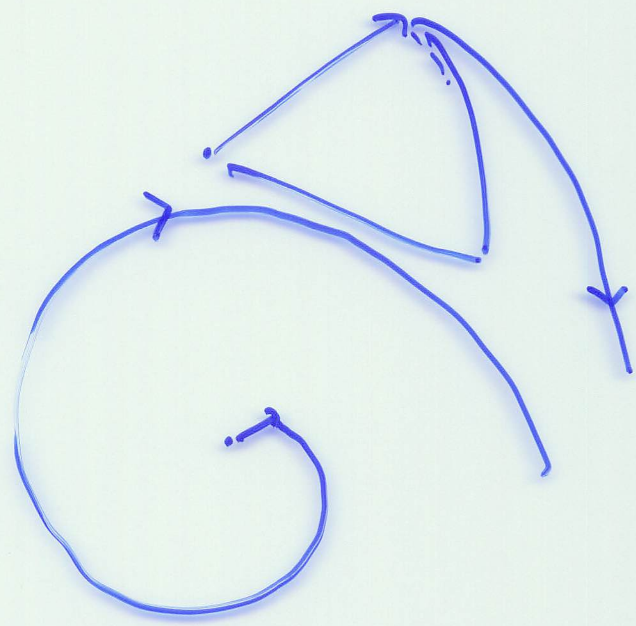
$$X_0(t) = \begin{pmatrix} \cos qt & \sin qt \\ -\sin qt & \cos qt \end{pmatrix}$$

$$X_0(t)\,\vec{z}(0)$$



$$e^{pt}\,X_0(t)\,\vec{z}(0)$$

$p > 0$



$p < 0$